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Walter K. Jackson

AN

ELEMENTARY TREATISE

ON

ASTRONOMY,

ADAPTED TO THE

PRESENT IMPROVED STATE OF THE SCIENCE,

BEING

THE FOURTH PART

OF

A COURSE OF NATURAL PHILOSOPHY,

COMPILED

FOR THE USE OF THE STUDENTS OF THE UNIVERSITY

AT

CAMBRIDGE, NEW ENGLAND.

BY JOHN FARRAR,

PROFESSOR OF MATHEMATICS AND NATURAL PHILOSOPHY.

◆

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The following treatise was selected from Biot's *Traité Elémentaire d' Astronomie Physique*, 2d edition, printed at Paris in 1811. Although the original is in three volumes octavo, all the body of the work is retained in this translation, except what relates to a description of astronomical instruments, the construction of tables, the temperature of the earth, the determination of certain astronomical periods, used in chronology, and two or three other topics anticipated in the parts of this course already published. A few only of the notes are given, and these are subjoined at the end in the form of an Appendix. The centesimal notation of the original has been changed to the sexagesimal; and alterations and additions have, in a few instances, been made for the purpose of adapting the work to the latest discoveries and improvements.

Cambridge, March, 1827.

105148

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ERRATA.

Page 34, line 18, *for* ‘368,’ *read* 360.
— 37 — 21 — ‘formed,’ *read* found.
— 38 — 12 insert in the margin Fig. 13.
— 39 — 32 *for* ‘the sides,’ *read* two sides.
— 52 — 37 — ‘$\frac{1}{30}$,’ *read* $\frac{1}{130}$.
— 64 — 2 — ‘This,’ *read* Its.
— 110 — 31 — ‘*PCZ*,’ *read* *P′CZ*.
— 130 — 23 — ‘100° 01′ 33″,6,’ *read* 36″,01.
— 138 — 18 — ‘40,’ *read* 222.
— 140 — 2 — ‘275°,’ *read* 279°.
— 141 — 26 — ‘from it half,’ *read* from half.
— 142 — 3 — ‘275°,’ *read* 279°.
— “ — 24 — ‘0^h 25′ 27″,3,’ *read* 0^h 25′ 07″,3.
— “ — 35 — ‘365^d 6^h 13′ 56″,9,’ *read* 365^d 6^h 13′ 58″,9.
— 145 — 6 — ‘sa,’ *read* as.
— 149 — 22 — ‘32° 17′ 37″,56,’ *read* 32° 17′ 39″,67.
— “ — 33 — ‘9° 43′ 52″,’ *read* 9° 29′ 3″.
— 152 — 25 — ‘0° 51′ 23″,2974,’ *read* 59′ 08″,3297.
— 166 — 2 — ‘takes,’ *read* take.
— 171 — 19 — ‘east to west,’ *read* west to east.
— 175 — 11 insert in the margin Fig. 67.
— “ — 12 *for* ‘Z,’ *read* ♎.
— 196 — 33 — ‘364,’ *read* 346.
— 221 — 2 — ‘apeears under us,’ *read* appears under.
— 232 — 26 — ‘touching to,’ *read* touching.
— 233 — 16 — ‘his,’ *read* this.
— 247 — 24 — ‘diameter,’ *read* semidiameter.
— 249 — 11 — ‘shadows,’ *read* shadow.
— 251 — 20 — ‘of a,’ *read* from a.
— “ — 21 — ‘of,’ *read* from.
— 253 — 25 — ‘*SH*,’ *read* *H*.
— 259 — 34 — ‘edge of the disc,’ *read* disc.
— “ — 35 — ‘hollowed out,’ *read* defalcated.
— 260 — 16 — ‘when it is,’ *read* as it departs.
— 262 — 15 — ‘hallowed out,’ *read* crescent shaped.
— 276 — 31 — ‘114000000,’ *read* 144000000.
— “ — 34 — ‘190000000,’ *read* 1900000000.
— 279 — 16 — ‘Ceres and Juno,’ *read* Pallas and Juno.
— 299 — 30 *dele* (App.)
— 306 — 24 *for* ‘117°,’ *read* 11°.
— 313 — 20 — ‘other,’ *read* another.
— 316 — 19 — ‘188,’ *read* 108.
— 344 — 9 — ‘0,0000,’ *read* 1,0000.

OF THE
UNIVERSITY
OF

AN
ELEMENTARY TREATISE
ON
ASTRONOMY.

GENERAL PHENOMENA.

Appearance of the Heavens.

1. Let us imagine ourselves on an elevation, in an open country, where the view is uninterrupted on every side. Upon the setting of the sun the western sky is faintly illuminated. This light fades away by degrees, the darkness increases, night comes on, and the sky seems a vast dome studded with a multitude of brilliant points. These are the *stars* which the too strong light of the sun has prevented us from perceiving during the day. The disposition of these stars seems to be immutable. It is essentially the same now that it was in periods the most remote. The different clusters are such in their configurations as the ancients described them, when they grouped them together under the name of *constellations*, and, to assist the memory, associated them with the figures of men, animals, &c. But these stars, while they preserve the same order, are carried round the heavens as by a common motion. Those towards the west, decline more and more, and disappear where the sun sets; while others, presenting themselves in the east, seem to come from beneath the *horizon*. Rising to different heights in the heavens, they descend again and set in their turn, like those which preceded them. But if, in our latitude, we place ourselves in such

a manner as to have the east on our right and the west on our left, we shall see, in that part of the heavens which is before us and which we call the north, some groups of stars which never set; such, for example, is the remarkable collection called the Great Bear. This constellation, and the greater part of those which are found in that part of the heavens, disappear only when their light is lost in that of the sun. They may be seen during the whole of the night, and followed through the lowest part of their course, for they never descend to the horizon. If observed at different times of the night, they will be found to have their positions in the heavens reversed, the natural effect of the rotatory movement which they have in common with all the other stars; and the centre about which they move is a point of the heavens situated directly north.

The light which appears in the east at the approach of day, soon becomes strong enough to eclipse the stars which have just risen in that direction; the west is now in darkness, and the scene is the reverse of that which happens at the beginning of the night. The light continuing to increase, the stars grow fainter and fainter till they at length disappear, and day sheds its brightness upon every object. The sun now presents itself, is seen in the east like the stars; it ascends, and passing through the heavens, declines to the west and disappears in the part opposite to that in which it rose, when all the phenomena of night are repeated in the manner just described.

The *moon*, of which we have not yet spoken, and which is remarkable for the magnitude of its disc, its splendor, and its *phases*, or the changes which take place in the form of the enlightened part, presents also analogous phenomena.

This rotatory motion common to all the heavenly bodies, and which is completed in the space of one day or twenty-four hours, is called the *diurnal motion.*

2. Since the stars situated in the north, near the centre of general motion, remain always far above the horizon, while others farther from this centre descend nearer to the horizon, and others still more distant sink below it, we see that their setting results from the magnitude of the circles which they describe, and which carries them below the horizon, so that they disappear from our sight. We are led by the most obvious analogy to extend this explanation to stars situated in the opposite part

of the heavens, namely, the south. These stars, after setting, continue their motion below the horizon, and, like the sun, reappear in the east. If then we conceive a physical line, or *axis of rotation*, around which all these motions are performed, we must suppose that in our latitude this axis appears elevated towards the north, and oblique to our horizon, or the plane which passing through the eye, and touching the surface of the earth, separates the visible from the invisible part of the heavens.*

We are not to consider this axis as any thing material, actually existing in space; it is to be regarded as an imaginary line, designating a series of points in space, which appear to be stationary amidst the general motion. It is the same with the *circles* which the stars seem to describe about the axis, in their diurnal revolution; by these circles we are to understand only different series of points where the stars successively appear; and upon the supposition that the diurnal motion is truly and exactly circular, we only employ this as a hypothesis proper to fix our ideas; for although it be true in all the rigor of geometry, the complete proof can only be given by means of instruments which are very exact, and principles which we cannot now consider.

3. By examining the heavens for a number of nights, we observe certain stars change their places with regard to the rest; passing from one constellation they approach toward some and recede from others, each day by imperceptible degrees. But these small changes at length become apparent, and the stars in which they take place are transferred to different parts of the heavens; they are on this account called *planets*, or wandering stars, to distinguish them from those which preserve the same relative situation, and which are called *fixed stars*.

The number of planets known at present, is ten; they have received particular names, and have also characteristic signs which serve to designate them in an abridged manner. They are Mercury ☿, Venus ♀, Mars ♂, Jupiter ♃, Saturn ♄, Herschel or Uranus ♅, Ceres ⚳, Pallas ⚴, Juno ⚵, Vesta ⚶. The first five may be perceived by the naked eye; these have been known from the remotest times. Uranus, discovered more recently, can be perceived by the naked eye only under the most favorable circumstances; the other four cannot be seen at all except by the aid of the telescope; they are hence sometimes called *telescopic planets*. It will be readily seen that their discovery is not to be ascribed to accident, or to a simple inspection of the heavens; it is very recent, and is the result of delicate observations, made methodically, and with the most perfect instruments. We shall speak particularly of them hereafter.*

The change of place of the planets among the stars is called their *proper motions*.† The moon and the sun have also their proper motions, which are recognised in the same manner; those of the sun are particularly remarkable for the effects with which they are attended.

4. If we observe this luminary near the time of its setting, for several days in succession; and, when it is below the horizon, examine the stars which follow it and which set immediately after; they are easily recognised by the figures under which they appear in the heavens; after several days they are too faint to be seen. There are other stars which follow the sun and which set immediately after it. These same stars, on the preceding days, did not set till some time after the sun; the sun has therefore advanced towards them from west to east, in a direction contrary to the diurnal motion. If we observe the heavens in the morning some time before the rising of the sun, the appearances will be the reverse of these. The stars which rise to-day, at the same time, or nearly at the same time with the sun, after several days will rise some time before it. They will appear to have receded in the heavens from east to west, or which is the same thing, the sun will have removed from them in the direction from west to east; for it is more simple to suppose the sun to have a proper motion, than to suppose that all the stars have, with relation to the sun, a common motion. By this proper motion, the sun seems to describe the whole circle of the heavens from west to east.

5. So also by observing the heavens at night they are found, in different seasons, to be altogether changed; the arrangement and disposition of the stars are different. This is a very simple

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* The planet Uranus was discovered by Dr Herschel on the 13th of March 1781; Ceres by Piazzi, January 1, 1801; Pallas by Olbers, March 28, 1802; Juno by Harding, September 1, 1803; Vesta, also by Olbers, March 29, 1807.

† Proper motion here denotes not an absolute motion necessarily, but a motion relatively to the fixed stars.

consequence of the proper motion of the sun. In that part of the heavens where it is found, the splendor of the light prevents our perceiving the stars with the naked eye; since with a telescope they may be seen during the day; but as the sun in its apparent course recedes from these stars, they will arrive above the horizon during the night, and will thence become visible. We discover already in this circumstance, the imperfection of those rude notions which we are led to form from the first view of these phenomena, and from which we might be tempted to believe that the heavens are divided into two portions that appear successively above the horizon, one being occupied by the stars, and the other by the sun. Accurate and continued observation of celestial phenomena will make known other illusions, and teach us to correct them.

Those stars only which appear in the north, and which do not set, remain visible at all seasons; but we see them successively in different positions at the same hour, according to their situation with respect to the sun; from this circumstance the proper motion of the sun becomes very sensible.

6. The motion of the sun, is not however exactly in the direction from west to east, for it is universally known that at certain times the sun rises much farther above the horizon, than at others, which becomes particularly observable by the variations of its heat, whence results the difference of the *seasons*. Also by observing the points where the sun rises and sets, it will be seen that they do not answer always to the same terrestrial objects. The first motion of the sun, from west to east, is the more considerable, since by this motion it passes successively through the whole circle of the heavens; while the second motion above mentioned, appears to be confined within certain limits of elevation and depression which are never passed. From all which it results that this luminary describes in the heavens an oblique path which is not exactly from east to west, but varies from this direction within certain determinate limits.

7. The proper motions of the planets with respect to the stars, are also directed, like that of the sun, from west to east, at least in a great part of their course. It happens, however, to each planet, at certain different and determinate epochs, that its proper motion is slower and slower, till it becomes altogether imper-

ceptible, and the planet appears stationary among the stars. After which it begins to move from east to west, that is, in a direction opposite to its general course; this produces in the planet an apparent *retrograde* motion with respect to the stars. After some time this retrogradation is diminished, the planet becomes *stationary* again, and afterwards resumes its *direct* course from west to east. These phenomena, observed from the earliest times, are called the *stations* and *retrogradations* of the planets.

The proper motions of the planets, like that of the sun, are not exactly from west to east; they depart from this direction within certain limits which they never pass; they are all comprehended within a certain zone of the heavens called the *zodiac.*

8. We observe besides, from time to time, in the heavens, certain bodies which have not been before perceived; and these at first appear very small and very dull; they are usually accompanied by a kind of nebulous appearance or luminous *tail.* They have, in like manner, proper motions among the stars; but their course is very variable; they traverse the heavens in all directions. It often happens that their brightness increases from their first appearance for a certain time, after which it decreases by the same degrees; and at length, after an interval more or less considerable, they cease to be visible. From this nebulous appearance which accompanies them, they are called *comets*, or *hairy stars.*

9. The heavenly bodies of which we have now spoken, taken together, make the subject of the science called *astronomy.* To observe and determine exactly their positions in the heavens, to trace their paths and measure their motions with precision, to ascertain the constant laws by which they are governed, to apply these laws to the determination of their future positions, or those which they have occupied at any past time,—these are the leading problems which present themselves for our solution.

10. We are not to place among the number of real stars, those luminous meteors whose appearance continues only a few moments, which are not visible in the heavens at the place from which they depart, except at the instant of their shooting, and which leave no trace of themselves in the part where they disappear. Such are the *fire balls* which sometimes present them-

selves suddenly in the heavens, and are followed by a luminous train, sending out brilliant sparks, and which, after a rapid course of a few seconds, often burst with great noise. Such also are the transient meteoric appearances which are commonly called *shooting stars*, and which seem to be nearly related to the preceding. These phenomena have been long regarded as effects purely physical, produced by vapours which are raised into the air, and which are accidentally inflamed by causes of which we are ignorant.

Of the Roundness of the Earth.

11. We have now described the most obvious phenomena which the heavens present to an insulated observer; but among these phenomena, that of the rising and setting of the stars is one of the most remarkable, and merits our first attention. What then is that limit which hides from us one half the heavens, and which we call the horizon? Is it the same for all countries, or is it different? Is it possible to arrive at it? And what shall we find beyond it?

These questions and many others, are easily solved by travellers. When navigators depart from a coast they observe that edifices and mountains sink by degrees and at length disappear as if immersed in the ocean. This effect is not to be ascribed to distance, which causes objects to appear smaller; for, when we lose sight of land from a ship's deck, we perceive it again by ascending the masts. The same takes place with respect to the ship also as seen from the shore. It declines by little and little, and finally disappears, descending below the horizon like the sun at its setting. These phenomena, which are observed continually and in all directions, prove that the surface of the sea is convex, and that it is by its interposition that distant objects are concealed. If it were a plane surface, a single mountain or a tower elevated above it, would be perceived from every quarter, at least if the spectators were not so far off as to render its dimensions insensible on account of the distance; and this would not be the case except at very great distances. The bases of elevated objects would not disappear before their summits. They would not seem to sink by degrees; and when they disappeared from the deck of a vessel they would not be visible from the top of the mast.

The horizon of the sea, which seems to terminate its surface, is only an apparent limit with respect to the situation of the observer, and is produced by the convexity of the surface of the water. Navigators whom we see depart from the shore, seem to us to go beyond that limit, but their horizon moves on with them. When they have disappeared from our view, we may ascend a mountain near the shore and see again for some time the same vessel which had before appeared to sink into the waters.

It would be a bold and important undertaking to ascertain what becomes of this apparent barrier as we advance towards it continually in the same direction. Ferdinand Magellan, a Portuguese, was the first who executed this enterprise. He embarked from a port in Portugal, and directed his course towards the west. After a long voyage he descried the continent of America, which had been previously discovered by other navigators, pursuing the same route. Not finding an opening to enable him to continue his course in a westerly direction, he sailed along the coast toward the south, till coming to its southern extremity he sailed round it and found himself in the great Southern Ocean. Then he pursued his course toward the west; after some time he arrived at the Molucca Islands; and sailing continually towards the west, he made Europe from the east, and thus arrived at the place from which he set out.

12. This great achievement, since repeated by several navigators, proves that the whole surface of the water and land is convex, returning into itself, that the heavens do not touch any part of it, and that into whatever country we travel, the general system of stars is seen to revolve round the earth in consequence of the diurnal motion.

13. From this we conclude that the heavens do not rest upon the horizon of the sea, as one might be led to believe from a hasty observation. This illusion arises from our always supposing objects in the direction of the visual rays which render them sensible to the eye. When the rays which come from a star just graze the surface of the sea, it seems to us that the star is at the extremity of that surface. If we conceive a plane

passing through the eye of the spectator, and touching the horizon of the sea, all the points of the heavens situated in this plane, must appear contiguous to the surface of the water, as if the heavens rested upon it.

14. These results show us that the earth is convex not only from east to west, but also from north to south, as is shown by voyages undertaken in this direction. On the land it is difficult to make the same experiment, because the horizon being almost always terminated by mountains more or less elevated, we might suppose that they alone hide from us all that is beyond them. But we may supply the place of such proofs by a consideration more general; and one which applies as well to the land as to the sea. It is this; the same stars attain different heights above the horizon, according to our change of place. When, for example, we depart from any place upon the land, and travel towards the south, we observe the stars situated in that part of the heavens to become more and more elevated above the horizon. The arcs which they describe in consequence of the diurnal motion are more extended. Some also which are not seen in the place from which we depart, become visible. On the contrary, the stars situated towards the north are depressed; those which described very low arcs withdraw from the view and are concealed below the horizon, precisely as in commencing a voyage edifices and mountains sink lower and lower, and finally, when we have departed a certain distance, disappear. The reverse of this takes place when we travel from south to north. By changing, therefore, our position upon the earth, and passing from north to south, or from south to north, we can always change the aspect of the heavens. These phenomena give us the clearest indications of the convexity of the terrestrial surface. The stars are here with respect to us, what edifices and mountains are with respect to navigators who depart from the coast. The only difference is that the view of the navigator is free in every direction, while ours upon the land is limited. This obliges us to have recourse to the heavenly bodies in order to avoid the objects which obstruct our view on the surface of the earth and prevent our observing its convexity. It is from the same cause, that the most elevated objects on the earth's surface, as the summits of mountains and the tops of towers, are the first to receive the sun's light in the morning, and the last to part with

it in the evening. Accordingly, while the sun is setting with respect to some countries, it is at the highest point of its course with respect to certain regions farther to the west, and at the same time just rising to others still more westerly.

15. The convexity of the earth is proved in a very striking manner, by certain phenomena which the moon presents; but the evidence derived from this source requires certain preliminary considerations. There are remarkable variations in the moon's disc which we call her *phases*. She appears successively in the form of a *crescent*, of a *semicircle*, and of a complete *circle*, after which her disc is diminished again by degrees, in the same manner as it increased. These *periodical* variations, that is, variations which take place always in the same order, are so obviously connected with the position of the sun, as to leave no doubt that the moon is a round opake body, receiving its light from the sun, of which the side turned towards us is sometimes illuminated, sometimes dark, or a part of it is illuminated and the rest dark, according to the situation of the sun with respect to us and the moon. This supposition so fully explains these appearances, as we shall see hereafter, that it becomes impossible to doubt its truth.

The moon then appearing bright only in consequence of the light received from the sun, if in its revolution round the earth it pass between us and the sun, it is evident that the sun must be concealed either wholly or in part. This is precisely what takes place. The moon interposed between us and the sun, appears like a black spot, and prevents our seeing the sun, or at least deprives us of a part of its light. This phenomenon is called an *eclipse of the sun.*

Sometimes also the moon is suddenly obscured in the heavens, and in the course of a few hours successively loses and regains its light. That side of the disc which first disappears, is the first to reappear, precisely like what happens to a body illuminated by a candle, if it pass through the shadow projected by another body. This phenomenon, which is called an *eclipse of the moon*, takes place at the time when the moon appears as an entire circle, and opposite to the sun. It is natural to conclude that the earth enlightened on one side by the sun, projects into space on the opposite side, a shadow into which the moon enters when it is eclipsed.

The form of this shadow projected upon the disc of the moon, shows us that the earth is round. When the moon is just entering this shadow, the greatest part of the disc is still illuminated by the sun. This illuminated part does not appear to be terminated by a straight line, as would happen if the contour of the earth's shadow were rectilinear. It has the form of a crescent whose convexity is turned towards the enlightened part. This convexity is an evident indication of the roundness of the shadow, and consequently of the roundness of the earth which projects it. The same appearance is also produced while the moon is emerging from the earth's shadow.

16. Comparing the results of these observations with the phenomena presented to navigators, we conclude with certainty that *the land and water together form a mass rounded on all sides and insulated in space.*

17. This conclusion is very certain, since it rests upon well established facts; still it is difficult to conceive that the earth is insulated and self-sustained in the midst of space. We are apt to generalize improperly, in this case, the idea of gravity which we have remarked in bodies situated on the surface of the earth. But it does not hence follow, that the earth itself must tend towards either this or that point in space. And when we learn from observation that the earth is self-sustained, free and insulated, there is nothing in the phenomena of gravity that ought to surprise us.

18. Moreover, since the earth is round, the different people who inhabit it have their heads directed towards different points of the heavens. There are people, therefore, who are directly opposite to us, and who have their feet opposite to ours. Places thus situated opposite to each other, are called *antipodes.* This seems a very singular arrangement, but it is nevertheless real. I have mentioned these considerations to show that we ought not to be surprised at new truths which observation and experience make known. Our surprise usually arises from this, that we are apt to regard as general what we are accustomed to. This is a prejudice which we ought to dismiss, and which vanishes in proportion as we acquire the habit of observation.

When we shall have advanced farther in astronomy, the roundness of the earth will appear by no means extraordinary; for in examining the heavenly bodies with telescopes which magnify very much, we observe in many of them phenomena which prove that they also are round. Of this number are the sun, the moon, and the planets. The roundness of the earth, therefore, which at first view, appears so singular, is only a property which it has in common with many other bodies, insulated like itself in the infinite expanse of the heavens.

19. The earth being convex, perpendiculars drawn to different points of its surface, are far from being parallel to each other; they converge towards the interior. If they all crossed each other at the same point, the earth would be spherical. In general, the manner in which they incline the one to the other, indicates the form of the curvature; for if we conceive a straight, inflexible line *AB*, to which are drawn in the same plane several perpendiculars Pp, $P'p'$, $P''p''$, &c., at the points M, M', M'', &c.; as long as this line remains straight, these perpendiculars will be parallel to each other. But if it becomes curved, as in figure 2, the perpendiculars will approach each other on the concave side, and they will diverge from each other on the opposite side; and the degree of convergence or divergence, will depend upon the degree of curvature. The direction of these perpendiculars with respect to the terrestrial surface, is therefore an important circumstance to be determined. This is indicated in each place by the direction which heavy bodies take when freely exposed to the action of gravity. For it is observed that bodies always fall perpendicularly to the surface of tranquil waters, which every where indicate the form of the earth's surface, its inequalities being left out of consideration. Fig. 1.

20. In the mean time, as this circumstance is the basis of all our knowledge respecting the figure of the earth and the celestial motions, it is important to know that its truth can be rigorously demonstrated. Experience proves that the particles of water are heavy like those of all other bodies. If we suppose the direction of gravity unknown, it is not the less certain that this force urges each of the particles in a certain direction. And when we see that they are in equilibrium, it is a proof that they are so disposed as not to be able to yield to the action of gravity; so that the particles at the surface, for example, are sustained by the incompressibility of the particles in the interior. To this end it is absolutely necessary that the direction of gravity should be perpendicular to the superior surface of the water;

for if it were oblique to it, the particles would move upon this surface, those in the interior having no power to prevent it, and the equilibrium would not be maintained. *The direction of gravity, therefore, is every where perpendicular to the surface of tranquil waters;* and this result is altogether independent of every hypothesis respecting the figure of the earth. Only, if it is spherical, all these perpendiculars tend to its centre and indicate the direction of its radii.

21. To ascertain this direction by experiment, we suspend a heavy body at the lower extremity of a line, the other end being fixed. This line takes a direction perpendicular to the terrestrial surface. The instrument is called a *plumb line.* The perpendicular thus determined is called a *vertical.* The point of the heavens directly above it is called the *zenith.* The opposite point, situated on the other side of the earth, is called the *nadir.* These too words are Arabic, and designate the two opposite points of the heavens, where the vertical would pass if produced.

22. Since all the verticals converge towards the interior of the earth, their directions are different, but this difference is by no means appreciable in places near each other, and the directions of the plumb line are sensibly parallel. They appear so because the angle which they make with each other is very small, the earth's convexity not being perceived for so small a distance.

As a consequence of this convexity, in proportion as we are elevated above the terrestrial surface upon high mountains, for example, or in a balloon, we discover a larger part of this surface; and the visual rays drawn to the extremities of the horizon make a greater angle with the vertical. This is confirmed by experience; and every one has frequent opportunity to make the experiment.

23. The depression of distant objects being different according to the heights from which they are viewed, the extent of the horizon will vary with these heights. It would be reduced to a mathematical point if the eye of the observer were placed at the very surface of the sea. For the sake of exactness amid these irregularities, it is agreed in astronomy, to consider the horizon as a plane passing through the eye of the observer perpendicularly to the vertical. This plane is supposed to be indefinitely extended in every direction. In figure 3, *HO h* represents

the horizon, and the angle *HOH'*, the inclination of the visual ray below this plane, called the *apparent depression* or *dip*. This angle is always much smaller than it appears in the figure, where, in order to render it sensible, it is necessary to enlarge the dimensions of the mountain compared with those of the earth.

24. As we change our place upon the earth, the summit of the vertical answers successively to different points in the heavens, so that different stars are seen in the zenith. The plane of the horizon which is always perpendicular to the vertical, takes all possible directions in space. This is a necessary consequence of the roundness of the earth.

25. If we conceive a visual ray extending from the point *O*, the eye of the spectator, to the star *S*, the angle *SOZ*, which the ray makes with the vertical *OZ*, is called *the zenith distance* of the star, and the complement of this angle or *HOS* is called the *altitude* of the star *above the horizon*, or simply the *altitude*.

26. By accurate observations repeated in different directions, the surface of the earth has been proved to be nearly spherical. We are enabled also to measure very exactly the variation of the earth's curvature from that of a sphere, but these results suppose operations which we cannot now explain. Our object has been to prove by phenomena that the earth is *round and insulated in space;* for the knowledge of these important truths must precede the very idea of a geometrical determination of its curvature.

Of the Transit Instrument or Meridian Telescope and the Manner of using it.

27. The first instrument which we have occasion for in astronomical observations, is called a *transit instrument;* it is used to note the passage of the heavenly bodies over the meridian at the highest or the lowest point of the circles which they describe. It consists of an astronomical telescope furnished with a fixed Fig. 4. micrometer, and mounted upon a nicely formed axis at right angles to it. This axis, remaining always horizontal, rests at its extremities in two sockets perfectly even, and set in two blocks of stone of a size and weight sufficient to prevent all agitation.

From this description it will be perceived that the telescope of the transit instrument when turned about on its horizontal axis, describes a *vertical plane*, that is, a plane perpendicular to the plane of the horizon; only the direction of this vertical plane will be different according to the points of the horizon towards which the axis of the telescope is directed. And as all the planes which pass through the vertical are necessarily vertical, there is for each point of the terrestrial surface an infinite number of planes which possess this property.

Now in which of these directions must the telescope be placed, so that the heavenly bodies, carried round by the diurnal motion, may be seen with it at the instant of passing their highest or lowest point? As all these bodies seem to have one common motion, it will be sufficient to find this position for one of them only. The sun, by the unequal length of the shadows which he casts at different times of the day, affords us a means, if not very exact, at least very simple, of approximating this direction.

26. To make these observations, we place upon the ground a table of polished stone or marble which we render perfectly horizontal by means of a spirit level placed upon it in different directions. In this we fix a straight stile, making it exactly perpendicular by means of a plumb line. This stile represents the vertical of the place. About the foot of it, as a centre, we describe upon this horizontal plane, several concentric circumferences, whose radii are taken at pleasure. The shadow projected by the stile, in the morning very long, becomes continually shorter and shorter as the sun rises; its extremity reaches successively the different circumferences which have been described, and we mark exactly the points where it crosses them. When the sun has passed its greatest height, it begins to descend towards the horizon, and the shadow increases again by the same degrees, and its extremity crosses successively the different circumferences, in a contrary order and at different points. But these points evidently correspond to heights of the sun respectively equal to those belonging to the points on the other side. We note them with care, and bisect the arc of each circumference comprehended between the two observations. A straight line drawn through the middle points of these arcs, and the foot of the stile, determines the vertical plane in which the sun is at its greatest height. This plane is called the

meridian, and the straight line in question the *meridian line*, a name derived from the word which signifies the *middle of the day*, because when the sun reaches this plane it has performed half the arc which it describes on that day above the horizon.

The extremity of the shadow is so indistinct, that it is always difficult to determine it. To avoid this inconvenience, we can substitute in the place of the stile, an inclined metallic plate with a round hole in the upper end, to allow the sun's image to pass. The centre of this image can be more exactly determined than the extremity of the stile's shadow. But then the concentric circumferences would have for their centre the foot of the perpendicular let fall from the centre of this aperture. The direction of the lower part of this plate is then altogether immaterial. Such an instrument is called a *gnomon*.

There is an inaccuracy in these methods, arising from that proper motion of the sun by which it daily advances from north to south, or from south to north, a motion whose existence has always been recognised. For, in consequence of this phenomenon, the arcs which the sun daily describes above the horizon are not strictly parallel to those which it would describe by the diurnal motion simply. But as this change takes place very gradually and slowly, its effects are hardly sensible in the interval of two observations made on the same day, and may be neglected in the preceding process, which is only an approximation, especially near the time of the sun's reaching the point of greatest departure north and south.

29. When the meridian line is thus drawn it is easy to place the transit instrument by means of two plumb lines let fall upon this line from the two extremities of the telescope, one passing through the centre of the object-glass, the other through the centre of the eye-glass. When this position is found the sockets which support the axis, are firmly fixed in the stone pillars. The axis is rendered horizontal by means of a spirit level which is attached to it; and the telescope in turning describes the plane of the meridian, so far as we are able to approximate it by these first trials, which, as will be seen, are only a necessary preparation for the use of methods more precise.

30. The micrometer of the transit instrument, or *meridian telescope*, (for it is called by both names) should be placed in Fig. 5. such a manner that the visual ray *OCS*, which passes through

the centre of the wires, may be exactly perpendicular to the axis of rotation. To effect this, the telescope is directed to some distant object; we observe the point of this object that corresponds to the centre of the wires which determines the direction of the visual ray OC; the instrument is then taken out of the supports and inverted, so that that end of the axis which was toward the west, will be east, and that which was east will be west. The telescope is then directed to the same object; if the centre of the wires answer to the same point S, it is a proof that the instrument is properly placed, and that the visual ray passing through it is perpendicular to the axis of rotation; but if this centre correspond to any other point S', the visual ray is oblique to the axis, and its deviation, doubled by reversing the instrument, will have for its measure half the angle SOS', formed by the two visual rays. The micrometer is to be moved by means of the adjust. screw so far that the centre of the wires shall correspond to the middle of the apparent interval of the two objects, and if this be exactly bisected the instrument will be rightly placed. But in case of a small error, which is almost unavoidable, we go over the process again with this new position of the micrometer, choosing another object, or if we please, directing the telescope again to the first by means of the adjusting screws of the supports. If this second trial indicate any deviations, it will be very much less than before, and may be corrected in the same manner; in this way the centre of the wires may be placed upon the visual ray perpendicular to the axis of rotation.

31. We have not yet determined the direction of the wires; it is most convenient for the observation of transits, to have a transverse wire perpendicular to the plane which the telescope describes; and the others, generally five in number, should be all parallel to this same plane, cutting the first wire at right angles. The place of the micrometer in the telescope is regulated by the artist in such a manner that this disposition of the wires takes place of itself, in regulating the optic axis, but it may nevertheless be useful to verify it, and nothing is more easy. It will be sufficient to render horizontal the axis of rotation by means of its spirit level, after which, directing the telescope to some fixed object, and moving it, we observe whether those

points of the object which are behind the wires, depart from them in this motion, or whether they follow them without sensible deviation. In the first case the wires have not the required parallelism; in the second case, they have the direction sought as perfectly as is necessary for the nicest observations.

32. The meridian telescope, being thus regulated, describes a vertical plane, which, if it is not exactly the plane of the meridian, differs very little from it. Let us direct it to the heavens at night, and begin to observe the stars situated to the south. A star enters the field of the telescope from the right. We bring it to the transverse wire of the micrometer; it follows the wire and, without quitting it, traverses successively the several intervals of the different parallel wires, and after a few seconds leaves the telescope on the left.

We here see immediately the effect of the diurnal motion which we have before noticed in a general way. The apparent course of the star from right to left in the telescope, indicates a real motion from left to right, or from east to west; for astronomical telescopes reverse the objects. Opt. 203.

Moreover, the continuance of the star upon the transverse wire of the micrometer shows that its motion is sensibly perpendicular to the vertical plane which the telescope describes. This direction is therefore horizontal. And the star is at the highest point of its revolution.

Nevertheless we are not to give to this conclusion a rigor too absolute; it will be sufficient, if the motion of the star is nearly horizontal to make it appear to us quite so, in the small interval which is occupied by the horizontal wire of the micrometer.

The different stars situated to the south, and whose transits we wish to observe, present themselves in the same manner. The meridian determined by means of the sun answers in like manner for the stars also.

We now turn the telescope to the north, towards the stars which do not set. We see some of them move in the telescope from left to right; these are the highest; those which are lowest, pass from right to left. The first in reality move from right to left, or from east to west, and the last from west to east. These are the effects of their circular revolution; the superior stars are at the highest point of their course, and the inferior at the lowest.

Moreover we shall observe a very great difference in the rapidity of the transits. The stars situated towards the south pass much more rapidly through the field of the telescope than those which are found to the north. One of the latter especially, has so slow a motion that during the time it remains in the field of the telescope, the transit of a large number of the southern stars might be observed.

These differences evidently indicate that the general motion of the stars is performed about an axis one of whose poles is situated in the north part of the heavens; indeed the general aspect of the heavens would lead to such a conclusion.

33. But the admirable invention of pendulum clocks gives us the precise measure of these different motions. At the time we observe the transit of a star we note exactly the hour, minute, second and fraction of a second, by the clock, when the star passes each wire.

If these wires are at equal distances (and they are always intended to be so placed), an arithmetical mean between the observed epochs, gives, *in time by the clock*, the precise instant at which the star passes the wire at the middle of the micrometer.* The passage is determined more exactly by the five observations than by one, because it is probable, in all cases, that the same mistake will not be made in each, but that the mistakes would be on different sides; so that, by uniting the five results, the errors would very probably serve to correct each other.

If the intervals of the wires are not exactly equal, and it is very difficult to avoid making some mistake in this particular, the arithmetical mean of the five observations will not correspond to the wire at the middle of the micrometer, but to a line which will be extremely near the truth. This will constitute a kind of ideal wire which will be the same for all the stars, and no very considerable inconvenience will result from the use of it, so long as we do not derange the micrometer; an indispensable condition, the neglect of which would lead to the greatest errors.

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* Let T be the time the star passes the wire at the middle of the micrometer, t the time employed in going from one wire to another; the epochs of transit for the five successive wires will be $T - 2t$; $T - t$; T; $T + t$; $T + 2t$; the mean of which is T.

Having thus learned to combine the indications of the meridian telescope with the measure of time by the clock, we form by means of the two instruments, a system of observations involving principles which we now proceed to make known.

Of the Uniform Motion of the Heavens and its Use in Measuring Time.

34. The transit instrument being thus regulated, if we direct it for several successive nights towards the same star, and note each time, with a good clock, the hour, minute, second and fraction of a second, at which the transit takes place, we shall find the intervals of these returns to be perfectly equal. This result, continually verified by every observer, is the foundation of astronomy.

For the sake of an example, and also to show that the uniformity which is here supposed, is not imaginary, I will state the following transits of the star, called α arietis, observed by Arago and myself, in the small island near the desert Formentera.

Dates of Observations.	Times of the transit of α Arietis by clock.	Intervals of two successive transits.
1807. Dec. 24	9^h 42′ 32,36″	
25	9 41 29,70	23^h 58′ 57,34″
26	9 40 26,72	23 58 57,02
27	9 39 23,90	23 58 57,18
28	9 38 21,38	23 58 57,48
30	9 36 15,74	23 58 57,18
1808. Jan. 2	9 33 7,68	23 58 57,31*
5	9 29 59,06	23 58 57,13
6	9 28 55,88	23 58 56,82
9	9 25 47,66	23 58 57,26

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* When several days pass without any observation, we divide the difference of time by the number of diurnal revolutions which have taken place in the interval. For example, from the 30th of December to the 2d of January, the difference of time is 71^h 56′ 51,94″, which divided by 3, gives 23^h 58′ 57,31,″ for the mean time of one diurnal revolution.

The intervals of these transits agree so nearly, that it is very natural to suppose them exactly equal, and to attribute the small variations here noted to slight irregularities in the clock, or to errors in the observations.

35. As a confirmation of the above position it may be observed, that the interval between two successive transits is constant, not for one star only but for all. In the following table are the transits of several other stars, observed in like manner at Formentera, and with the same instruments.

Dates of Observations.	Times of observed transit by clock.		Intervals of transit.
1808. Jan. 10	α Arietis	9^{h} 24′ 43,02″	
	Aldebaran	11 53 11,10	
	Rigel	12 33 37,92	
	β Tauri	12 43 6,20	
	σ Orionis	12 59 21,30	
Jan. 11	α Arietis	9 23 40,53	23^{h} 58′ 57,51″
	Aldebaran	11 52 8,34	23 58 57,24
	Rigel	12 32 35,26	23 58 57,34
	β Tauri	12 42 3,52	23 58 57,32
	σ Orionis	12 58 18,60	23 58 57,30

36. The equality of these intervals takes place not only in the plane of the meridian, but also in all other vertical planes. This truth is confirmed by the general experience of all astronomers. A proof of it may also be seen in these last observations which we have given; for, when they were made, the instrument was purposely placed in a vertical plane a little different from that in which it was before situated.

37. Whatever be the vertical in which we wish to observe these phenomena, it is necessary that the wires of the micrometer to which the transits are referred, should be exactly parallel to the vertical plane which the telescope describes. This condition is necessary in order to avoid the effect of refraction, which, as we shall soon see, increases the apparent altitudes of the heavenly bodies above the horizon, and not always to the same degree; so that they might, on this account, be brought earlier or later to the wires of the micrometer, if they were inclined to the horizon. This inconvenience will not exist, if the wires are parallel to the vertical plane of the star; for then the instant when

it reaches any one of them, will be the same as if there were no refraction, the only effect of which is to cause it to pass at a higher or lower point.

38. We ought, in the mean time, to state that, if we thus observe a star situated very near the axis of diurnal rotation, the one, for example, which we call the *pole star*, and if these observations are made with great accuracy, we shall find between the consecutive transits some appreciable differences, which may sometimes amount to half a second of time; that is, to about $\frac{1}{172800}$ of the duration of an entire revolution. These differences are produced by very slight changes which take place in the apparent position of all the heavenly bodies, even the fixed stars, according to laws of which we shall speak hereafter; at present it is sufficient to say that the cause is well known, and the amount exactly determined.

39. This constant equality does not hold with respect to those heavenly bodies which have a proper motion, such as the planets and comets; the duration of their diurnal revolution is longer or shorter than that of the fixed stars; longer, if their proper motion is from west to east, in a direction contrary to the diurnal motion; shorter, if their motion is from east to west. It will be perceived, moreover, that these differences ought to be unequal for the different planets, according as their proper motions are more or less rapid; but however small these differences may be, if the observations are continued for several days, they will so accumulate as to become perceptible; and it is thus that we distinguish those heavenly bodies which have a proper motion from those which have not, or which have only a very slight one. For there is not a star in the heavens in which a proper motion, nearly imperceptible, may not be discovered by a long series of observations. But, although these motions are continual, their extreme slowness is a reason for distinguishing them from other proper motions, as those of the planets and comets, which are incomparably more rapid. The latter, after two days' observations, become sensible; the others are hardly recognised after intervals of a month, and the changes which they produce *upon two successive returns* of the same star, or of any two stars compared together, cannot be directly perceived by any means whatever.

40. This constant equality in the revolutions of the fixed stars

is observed in all countries and at all times. In supposing it perfectly exact, and it is natural to adopt this supposition, allowance being made for slight inaccuracies of observation, it affords us a *unit of time* as perfect as can be desired; for the representative of this unit is common to all parts of the earth, and is always present to observers, and, what is of the greatest importance, it is absolutely unchangeable. This point is established by theoretical considerations, by which it appears that there has been no appreciable variation from the time of the ancient astronomers to the present.

The unit of time then, exactly defined, will be the interval between two successive returns of a star to the same vertical plane, corrected for *precession*, *aberration*, and *nutation*, of which we shall speak hereafter. This interval is called a *sidereal day*, and is divided into twenty-four hours.

41. The mathematical connexion which we have been able to establish between all the results of the celestial motions, shows that the sidereal day, is by its nature, one of the most invariable elements of the solar system. In order to conceive how astronomers have been able thus to reduce all these phenomena to a calculable *theory*, it is necessery to know that the motions of the heavenly bodies are subject to several great general laws, which were made evident by a comparison of observations, and which are susceptible of mathematical calculation. They are called *Kepler's laws*, because they were discovered by that great astronomer. Guided by these laws, Newton was able to determine the expression for the mechanical forces, which are necessary to give rise to them. He discovered that, in order to produce the results which Kepler had observed, it is necessary that the heavenly bodies should mutually attract each other, in the direct ratio of their masses, and in the inverse ratio of the squares of their distances. This discovery, the most important that has ever been made in the sciences, enables us to calculate *a priori* all the motions of the heavenly bodies, as being the consequences of a general law; and the results of calculation are always found to correspond so exactly with observation, that a more wonderful agreement can hardly be conceived. *Universal attraction* then is to be regarded as a primary fact, more exact than the observations themselves, and we are induced to follow out its consequences as affording a clew to all the phenomena of the

solar system. This is the leading object to which the ablest geometers, Euler, Lagrange, and Laplacé, have directed the labors of their lives. Laplace has combined the fruits of these researches in his great work entitled *Mécanique Céleste,* all the celestial phenomena being there deduced from the primary fact of attraction, according to the laws of mechanics. The same author, in another work entitled *Exposition du Système du Monde,* has brought together all the results demonstrated in the *Mécanique Céleste,* and has undertaken to develope their relations without the aid of calculation, yet always by mathematical reasoning.

42. It is to the revolutions of the fixed stars, as a unit of time, that we are to refer all astronomical clocks. If we thus consider the observations of article 34, we shall see at once that the clock gives less than twenty-four hours for the interval between two successive returns of the same star, or a sidereal day. Thus the clock *loses with respect to sidereal time.* To obtain the amount of this *loss,* it will be sufficient to take the difference of the times of the successive transits; for, if the clock had given exactly twenty-four hours for a sidereal day, the successive transits would have returned always at the same hour, minute, &c. We shall thus find for the daily loss the following values;

Dates.	Times of the transits of α Arietis.	Differences of the times or daily loss.	
1807. Dec. 24	9^{h} 42′ 32,36″		
25	9 41 29,70	1′ 2,66″	
26	9 40 26,72	1 2,98	
27	9 39 23,90	1 2,82	
28	9 38 21,38	1 2,52	
30	9 36 15,74	1 2,82	mean of 2 days.
1808. Jan. 2	9 33 7,68	1 2,87	mean of 3 days.
5	9 29 59,06	1 2,87	do.
6	9 28 55,88	1 3,18	
9	9 25 47,66	1 2,74	mean of 3 days.
	Mean loss	1′ 2,72″	

Here the times of the transits are earlier each day than the preceding, since there is less than twenty-four hours between two transits. It would be the reverse if there were more than twenty-four hours; and the time of each successive transit would be later. The sign of these differences, therefore, shows whether the clock loses or gains with respect to sidereal time, the differences themselves being the amount of loss or gain.

By taking the mean of all the preceding observations, the daily loss of the clock in a sidereal day will be 62,27″. The variation of the particular results from this mean, are so very inconsiderable that we ought to attribute them less to the clock than to errors of observation.

43. The consecutive returns of a star to the same vertical, will thus show us the daily average rate of going of the clock. The successive transits of different stars, show whether this rate is uniform; for, in this case, the interval of these transits, whatever stars be taken, are equal among themselves.

If by observations of this kind, we note carefully the general rate of gain of the clock, and its going for particular intervals, it is evident that we shall be able to judge of its performance with exactness, and infallibly detect the smallest irregularities. We shall see hereafter that the same purpose may be effected without the aid of the transit instrument.

44. From what has been said, it will be seen that the constant revolution of the stars gives us a unit of time, and that their succession which is also constant, gives us the parts of this unit. All clocks regulated by this common measure, may be compared with each other, whatever be their rate of going; and we can always substitute for them an ideal clock that shall exactly divide the sidereal day into 24 hours.

Thus far we have the transits of the stars observed by means of the meridian telescope. Indeed no other element is necessary in determining the duration of the sidereal day, and the measure of time. The meridian telescope serves also for observing the transits of all the heavenly bodies; but as many of them exhibit a disc of a sensible extent, the transits of their centres over the several wires of the micrometer, are not so easily determined as that of the stars which appear only as points. To avoid this inconvenience, we observe the transit of the first limb of the disc over each wire, and the transit of the second limb. The arithmetical mean between these two observations, is evidently the time of the transit of the centre. We make these two observations for each of the wires of the micrometer, and the arithmetical mean of all the results gives the transit of the centre of the heavenly body over the middle wire. In the case of the sun, we must place before the eye a coloured glass to diminish the intensity of the light which, concentrated by the telescope, would be liable to injure the sight. Ordinarily the coloured glass is fitted to the tube of the telescope containing the eye-glass, to avoid the inconvenience of holding it in the hand.

In night observations it is necessary to illuminate the wires of the micrometer. For the feeble light of the stars whose transits are to be observed is not sufficient to enable us readily to distinguish the wires. Sometimes we place before the object-glass of the telescope, an inclined mirror, perforated in the centre to allow the rays of the stars to pass; and which by its obliquity, reflects into the telescope the light of a lamp properly situated. But it is much more convenient to have an opening in the horizontal axis of the instrument, and to place before this opening a lamp, the light of which may be reflected upon the wires of the micrometer, by means of a small plane mirror, placed obliquely in the tube of the telescope. By this means the
Fig. 12. light is always in a right direction, whatever be the position of the telescope. We place also before the lamp a prismatic glass whose position, variable at the will of the observer, enables him to regulate the intensity of the light by the variation of its thickness.

Exact Determination of the Meridian by the Measure of Time. Axis of Rotation of the Heavens.

28. 45. The transit instrument being placed in a vertical plane as above explained, and the clock being well regulated to sidereal time, if a superior and inferior transit of a star which does not set, be observed, there will almost infallibly be found a difference of time in the intervals of these transits; that is, one will be longer and the other shorter than a semi-revolution of the heavens, by the same quantity. Yet if the process which we have described be exactly followed, in placing the instrument, the

difference of the intervals of the transits cannot be very considerable; it would amount at the most to only a few minutes of time. But it is easy to render it less by altering a very little the supports of the axis, so as to turn the telescope a little towards the east or west, in the direction necessary to establish the equality. This alteration of the supports is not made with the hand, but with a screw furnished with a graduated dial plate, and an index which points out upon this dial the smallest motion which can be given to it. As by means of this apparatus the supports may at any time be moved in any direction and to any amount, a few trials will be sufficient to place the telescope in the position required. Care should be taken to keep the axis always horizontal; this is done by means of a regulating screw by which the supports of the axis are elevated or depressed till the spirit level shows the axis to be perfectly horizontal. All these conditions being fulfilled, the telescope of the instrument, or to speak more exactly, the centre of the wires of the micrometer, will be found in a vertical plane which will divide the revolution of the observed star into two parts corresponding to intervals of time perfectly equal.

But it is also found that this bisection, thus determined by a single star, is the same for all those stars of which the two opposite transits can be observed; so that the intervals of the transits over the centre of the wires is twelve hours for these, as well as for the first. This result is the same in whatever part of the earth we may be placed. Consequently, if it be supposed that the rotatory motion of the heavens is uniform and circular, a supposition which, as we shall see hereafter, is fully confirmed by the phenomena, the vertical plane, determined in this manner, will contain the axis of rotation of the heavens; for it is only with respect to a plane drawn through this axis that the equality of the intervals of the transits can take place.

This plane is the meridian. We at first determined it by approximation; but it is now fixed with all the precision that can be desired for the measure of time.

46. This property of dividing the revolution of the stars into two parts of equal duration can be directly verified with respect to the stars which do not set. The others, if we suppose the plane of the meridian indefinitely produced, make their inferior transits below the earth whose interposition prevents our observ-

ing them. But in following their course above the horizon, through the whole extent of the arcs which they describe, we discover on each side of the meridian, a symmetry, a correspondence, which evidently shows that the second transit, though it cannot be seen, actually takes place, like that for stars which are still visible, after a semi-revolution.

One very simple and very exact proof of this symmetry, is, that the time of the transit of any one star over the superior meridian, as observed by the clock, is exactly intermediate between the times when the star is at the same height above the horizon on each side.

47. The meridian, the perpendicular to the meridian, and the vertical, form at each place on the surface of the earth, three rectangular axes to which all points in space may be referred. But as observations in astronomy are always made by means of angles, we make use of angular co-ordinates in preference to rectilinear co-ordinates.

The direction of a visual ray, produced to any object, is determined by its distance from the zenith, and by the angle which the projection of this ray upon the horizon, forms with the meridian line. This latter angle is called the *azimuth* of the object. It is reckoned either from the north or from the south point, and each way from the meridian. So also the angle formed by a vertical plane with the plane of the meridian, is called the azimuth of the vertical plane. It is readily seen that this angle is equal to that which is formed by the horizontal projection of the meridian and that of the vertical plane, since these projections are always perpendicular to the common intersection of the two planes.

48. When the laws of the diurnal motion are known, the azimuth of a star may be found by a very simple calculation from the observation of its altitude and the time which elapses between its meridian transit and its arrival at the vertical plane in which it is observed. The azimuths may thus be directly measured by means of a graduated horizontal circle upon which the direction of the meridian is determined.

49. If there were a star precisely in the axis of rotation it would appear constantly stationary in the plane of the meridian. There is none seen exactly in this situation; but we discover a conspicuous one in the northern part of the heavens, whose motion is so very slow and the changes of position so inconsider-

able, that they can hardly be perceived without the aid of instruments. This star is therefore not far from the axis of rotation. It is called the *pole star.* It is readily recognised by its vicinity to the Great Bear, which contains seven very bright stars, four being disposed nearly in a square, while the other three form a kind of *tail.* If through the two stars of the square the farthest from the tail, we conceive a straight line to be drawn, or rather a visual plane, this line or plane will pass very near the pole star, which is designated by *P* in the figure. Fig. 7.

The pole star terminates another group composed of seven stars similar to the Great Bear, except that the two have opposite positions in the heavens. This is called the *Little Bear;* and the pole star is the brightest of the collection. Among the different groups of stars which are observed in the heavens, there are none which present figures like these.

We propose now to inquire into the laws of the diurnal motion; and shall first endeavour to ascertain the axis of revolution of the heavens.

50. If, by the diurnal motion, all the stars describe circles about the axis of rotation of the heavens, as appearances seem to indicate, an observer placed precisely upon this axis, would see all the stars in their meridian transits, pass equally distant above and below it; so that the direction of the axis would be exactly in the middle between the two zenith distances. Let HZH' be the plane of the meridian containing the axis of rotation; HH' its projection upon the plane of the horizon, or the meridian line; OZ the vertical; OS, OS', the visual rays drawn from the observer at O to the star in its two transits; and let OP be the direction of the axis of rotation which is also supposed to pass through the eye of the observer. This being supposed, if the star describe a circle about the axis OP, the diameter SS' of this circle will be divided by the axis at the point P, into two equal parts, and the angles POS, POS', the one above and the other below this axis, will be equal. Whence it follows that the angle POZ, or the distance of the axis of rotation from the zenith, will be equal to the half sum of the zenith distances of the star at its two meridian transits, or what amounts to the same thing, will be an arithmetical mean between these two distances. Fig. 8.

51. This reasoning will apply to all the stars whose entire revolution can be observed. Thus in comparing their superior

and inferior transits, they ought all to give the same mean result, and consequently the same zenith distance to the axis of rotation. It will be readily seen that this circumstance is peculiar to the position of the observer which we have supposed. If the axis of rotation did not pass through the eye of the observer, the result would be different, and the half sum of the zenith distances would not be constant. These distances, observed at different places on the earth, will show whether any one point of its surface is situated in the axis of rotation of the heavens.

52. It is very remarkable that these results are not peculiar to any one place. In whatever part of the earth we make the observation, and many places have been used for this purpose, the same agreement is constantly found, allowance being made for small errors in the observations; *the axis of rotation of the heavens seems always to pass through the eye of the observer.*

53. This singular appearance can be explained only in one way; and that is, by supposing that the axis of rotation of the heavens passes through the interior of the earth; it being admitted at the same time that the dimensions of the earth, compared with the distance of the stars, are almost infinitely small. Then indeed, the visual rays drawn from the different points of its surface to the same star, ought to appear parallel; or, what amounts to the same thing, the earth, as seen at the distance of the stars, would seem but a point.

54. This result is confirmed by all astronomical phenomena. The stars, when seen through powerful telescopes, those, for instance, which magnify 200 times, do not present a disc of any appreciable extent. They appear as brilliant points, exhibiting no sensible difference in their apparent magnitude, when seen as if they were 200 times nearer, which proves that their distance is immense. The disc of the sun and that of the moon, being seen in the same manner, appear very much larger than they do to the naked eye, especially the moon, upon whose surface are perceived mountains and cavities. The discs of the planets likewise, when seen through a telescope, are considerably enlarged, and some of them, as Mercury and Venus, present phases like those of the moon. These bodies are, therefore, very much nearer to us than the stars. Also, by observing them in different parts of the earth, we discover in their respective positions, sensible differences of aspect. Thus, when the sun is eclipsed by

the moon, the eclipse is never general for the whole earth, as it ought to be if the sun and moon were at an infinite distance. There are always certain places where the entire disc of the sun is seen; in others it is seen partially eclipsed, as by the interposition of a round body upon one portion of its surface; in others, it is seen more eclipsed, according as we approach nearer to the straight line which joins the centres of the sun and moon.

55. Observations of the meridian altitudes of circumpolar stars, will determine the distance of the zenith of the observer, from the axis of rotation of the heavens, the complement of which distance will be the inclination of this axis to the horizon of the place, or the *altitude of the pole.* And as, on account of the roundness of the earth, the plane of the horizon of each place takes different directions, the axis of rotation appears differently inclined to the horizon in different places.

If the diurnal motion of the stars be really circular about this axis, there ought to be between the successive positions of the same star above the horizon, certain geometrical relations depending on this law. And this is what actually takes place with the greatest exactness, as we shall see hereafter.

Exact Determination of the Laws of Diurnal Motion. Proof that this Motion is circular, and that it is uniform. Measure of Time by Celestial Arcs.

56. At any place on the earth, let NEM be the plane of the horizon, NOM a meridian line, EO a perpendicular to the meridian line, OZ the vertical; and let OP be the axis of rotation of the heavens passing through the eye of the observer at O. Produce the visual ray OS to any star S; the angle POS, formed by this ray with the axis of rotation, is called the *polar distance* of the star. If the star move in a circle about the pole, this distance will be invariable, and the angle POS will remain constant in every situation of the star above the horizon. Fig. 9.

57. We propose to find this angle for any situation. This is easily done when we know the azimuth of the star, its distance from the zenith, and the distance of the zenith from the pole, that is, the angles NOA, ZOS, and ZOP. It consists simply in

the resolution of a spherical triangle. For if, from the point *O* as a centre, we suppose a sphere to be formed whose surface cuts the axis of rotation, the visual ray and the vertical, in the points *P*, *S*, and *Z*; the arcs *PZ*, *ZS*, *PS*, which join these points upon the sphere, will constitute a spherical triangle in which we know the sides *ZP* and *ZS*, or the distances of the pole and the star from the zenith, and the angle *PZS*, formed by the meridian *PZ* with *ZS*, the arc of a vertical circle passing through the star, for this angle is equal to the azimuth *NOA* of the star reckoned from the north. With these data, we find by calculation the third side *PS*, or the polar distance of the star.

58. In performing this calculation for any star in any point whatever of its course, the same value is always found for the polar distance *PS*; *the diurnal motion of the stars about the pole is therefore exactly circular.*

We have indeed a very simple verification of this conclusion, in the fact that the constant value in question is precisely the polar distance of the star when observed directly in the plane of the meridian; for then the vertical plane of the star is the meridian itself. If the star pass the meridian to the south of the zenith, the angle *PZS* or *NOA* is equal to two right angles, and the polar distance *PS* is the *sum* of the zenith distances of the star and of the pole, as in figure 10; if, on the contrary, the star pass the meridian to the north of the zenith, the angle *NOA* is nothing, and the polar distance is the *difference* of the zenith distances of the star and the pole, as in figure 8.

59. Having thus proved that the diurnal motion is exactly circular, we can make use of this result as a principle or a geometrical condition in calculating the situation of a star at any instant, its altitude simply being observed at that instant; provided its polar distance, as observed in the plane of the meridian, is known. For then in the spherical triangle *PZS*, we know the three sides *PZ*, *PS*, and *ZS* the zenith distance of the star at the moment of observation. We can, therefore, calculate the three angles of this triangle, namely, *PZS* or *NOA*, the azimuth of the star, and the angles *ZSP* and *ZPS*, namely, those at the star and at the pole.

60. The polar distance *PS* of each star being constant during the whole of its diurnal revolution, as we have just shown, it follows *that the visual ray* OS *extended to any star, describes in its*

diurnal motion, the convex surface of a right cone whose base is a circle, formed by the revolution of a perpendicular let fall from the star to the axis of rotation of the heavens. The apex of this cone seems to be situated in the eye of the observer, but is readily referred to the centre of the earth, since we know that when compared with the distance of the stars, the dimensions of the earth are insensible.

61. The result to which we have now arrived, indicates simply a geometrical relation among the visual rays, drawn to the same star, in the different parts of its diurnal course, but it does not determine any thing with respect to the distance of the stars in the direction of these rays. We are left then in complete uncertainty as to this point, and ought consequently to form no opinion on the subject. But as the mind inclines to dwell on some fixed idea, we involuntarily suppose all the stars placed at a great distance in the direction of the rays by which they are seen; as our senses afford no means of comparison by which we are able to infer that some of them are farther and others nearer, we unconsciously suppose them at the same distance. Hence the spectacle of the spherical starry vault, so beautiful in a calm clear night. If, in conformity to this illusion, we conceive a plane passing through the apparent place of each star, perpendicular to the axis of rotation of the heavens, the star in its diurnal motion will not depart from this plane; and its course thus projected upon the heavens, will be the circumference of a circle. In this sense it may be said, that *the circles which the stars appear to describe in consequence of the diurnal motion, are situated in planes perpendicular to the axis of rotation of the heavens.*

62. It only remains to ascertain whether the circular motion in question be uniform or variable. To discover this, let S, S', S'', be the successive positions of a star upon its circle, after it passes the meridian at the point M. The arcs MS, MS', MS'', will be perpendicular to the plane of the meridian and to the visual planes passing at each instant through the star and the axis of rotation. The angles ZPS, ZPS', ZPS'', formed by these planes with the plane of the meridian, will therefore have for their measure the arcs MS, MS', MS''. Fig. 11.

The values of these angles are easily found by means of the spherical triangle ZPS, the three sides of which are known. By performing the calculation, the angle ZPS, or the angle at the pole, is found to be proportional to the time elapsed after the meridian transit of the star; so that the entire circumference is to the number of degrees contained in this angle, or in the arc *MS'* which measures it, as the time of an entire revolution, or a sidereal day, to the time elapsed after the meridian transit. Each of the arcs *MS*, *MS'*, *MS''*, is therefore described by the star, like the entire circumference; that is, *the diurnal motion of the stars is uniform throughout the whole extent of their revolution.*

63. This truth once ascertained, it may be used as a principle or a geometrical condition in finding the angle *ZPS* at the pole, having the time elapsed from the star's passing the meridian, without recurring to an observation of its altitude. For, let D be a sidereal day, and t the interval of time between the star's passing the meridian and its arriving at the place proposed, we shall have the value of P by the formula $P = 360° \frac{t}{D}$. Reciprocally, if the angle at the pole be obtained from an observation of the star's altitude, we can find the time; thus,

$$t = \frac{P.D}{360°}.$$

64. To the visual planes *POS*, *POS'*, *POS''*, we give the name of *horary planes*, because the arcs *MS*, *SS'*, *S'S''*, which they intercept upon the diurnal circle of the star, answer to hours and the fractions of an hour into which the time of the entire revolution, or the sidereal day, is divided. For example, if the angles of the horary planes be 15°, they will intercept upon the diurnal circle of the star, arcs which are equal to 15°. There will therefore be 24 of these arcs in the entire circumference, since 24 × 15 is equal to 360; consequently, each of these will be described by the star in the twenty-fourth part of a sidereal day. This is readily seen from the formula

$$P = 360 \frac{t}{D};$$

for, making $P = 15°$, it gives $t = \frac{D}{24}$.

65. Generally, if we wish to represent the divisions of the day by the angles of the horary planes, or by the arcs which these planes intercept upon the diurnal circle described by the star, it will be found that 15° answer to one hour, 15' of a degree to

one minute of time, and 15″ of a degree to one second of time. Thus to convert any number of degrees, minutes, and seconds of an arc, into hours, minutes, and seconds of time, we divide them by 15. On the contrary, to convert the time into an arc, we must multiply by 15.

The divisions of the circle and of the day having this connexion, they may be represented the one by the other, so that we have the measure of a celestial arc if we know the time in which it is described.

Thus, the diurnal arc intercepted between the meridian and the horary plane of a star, or which amounts to the same thing, the angle formed by the plane of the meridian with the horary plane of a star, *being converted into time*, will express the hours and parts of an hour which have elapsed from the meridian transit of the star, or which must elapse before the transit. It is for this reason that these angles are called *horary angles*.

66. There is nothing in these relations which determines the time at which the horary reckoning is commenced. By general consent, we begin in civil reckoning at the time the sun passes the inferior meridian, and count twenty-four hours to the next inferior transit. But the motion of the sun being subject to variation, as we shall see hereafter, this system is likewise variable, and requires some modification to render it applicable to astronomy. Since we are obliged to confine ourselves to sidereal time till we have explained these inequalities, we may always take for the commencement of sidereal time the meridian transit of a known star, or of a determinate point in the heavens, and continue to count the hours from the instant of this transit.

Of the Celestial Sphere and its principal Circles.

67. The different questions which we have discussed in the last section will be found to depend upon the solution of spherical triangles. It is the same with all which have for their object the angles formed by the visual rays extending from the eye to the different points of the heavens. If, about the observer, considered as a centre of celestial motion, we conceive a sphere of any radius whatever, whose surface cuts perpendicularly all the

visual rays drawn to the different stars or other heavenly bodies, the points of intersection of these rays being united by arcs of great circles, spherical triangles will be formed upon the sphere. The different parts of these triangles being calculated and connected together by the known ratios which constitute spherical trigonometry, will give the relative situations of the stars thus observed.

68. It is of no consequence at what distance the visual rays are cut by this sphere; the same relations of position and magnitude will subsist between them. We may, therefore, easily conceive it to be described with an immense radius extending beyond all the stars, and we may in imagination attach them to the concave surface of this same sphere, the centre of which is occupied by the observer; this construction forms what is called by astronomers the *celestial sphere.*

Each observer being placed in the centre of his sphere, there may be as many centres as there are points upon the terrestrial surface; and the apparent situations of the stars, that is, the points of the celestial sphere to which they are referred ought to be different for each observer. This is true in fact with respect to most of those heavenly bodies which have a proper motion. But as to the fixed stars of which we ought first particularly to treat, since all the visual rays leading to them at the same instant from all points of the earth are considered as parallel, it follows that the different observers would project them upon these spheres in points exactly corresponding. Or, which amounts to the same thing, if we give to the sphere in question a radius so large that the radius of the earth may be considered as nothing in comparison with it, and the earth itself be considered a point, there will be but one celestial sphere for all these observers, and the visual rays drawn from their eyes to one and the same star, infinitely distant, must be considered as meeting at the same point in its surface. But if these visual rays, being directed to a heavenly body which is nearer, make among themselves sensible angles, they will tend to different points upon the surface of the celestial sphere.

69. We must not forget that the spherical form which we here suppose does not actually exist with respect to the absolute distances of the different heavenly bodies. They may be, and are, placed at very unequal distances. The celestial sphere is a

purely geometrical conception intended to fix our ideas and facilitate our reasoning, whenever we wish to compare the angles formed at the eye of the observer by the different visual rays.

70. We shall now present, under this point of view, the geometrical constructions which we make use of in fixing the positions of the heavenly bodies.

All planes passing through the eye of the observer, pass through the centre of the celestial sphere; and the sections of the sphere thus formed are *great circles.* Such, for example, is the plane of the horizon, that of the meridian, and those of the horary and vertical circles. The visual angles observed in these planes, are measured upon the great circles which result from their intersection with the celestial sphere; and hence the arcs may be substituted for the angles in the course of reasoning which we have occasion to employ.

Moreover, all the circles of the celestial sphere are conceived to be divided into degrees, minutes, and seconds, and the measure of the visual angles is indicated by means of these divisions. If, for example, the visual rays extending to two stars situated in the meridian, make an angle of 10 degrees, we say that they are formed in the meridian at 10 degrees distance from each other. It is the same with respect to all visual angles. Each has its vertex in the eye of the observer, that is, in the centre of the celestial sphere; they will, therefore, always intercept, upon the surface of the sphere, arcs of great circles which serve to measure them, and by means of which they may always be compared.

71. This is not the case with the circles which the stars describe in their diurnal motion. These have all their centres situated in the axis of rotation of the heavens, but not in the centre of the celestial sphere; they are therefore not great circles Only one of them answers to this character; it is that whose plane, always perpendicular to the axis of rotation of the heavens, passes at the same time through the centre of the celestial sphere. This is called *the celestial equator* and its plane *the plane of the equator.* It is so called because it divides the celestial sphere into two symmetrical portions, relative to the extent of the circles which the stars describe. These circles have a less radius than that of the celestial sphere, and are called *small circles.* They are called *celestial parallels,* because their planes being

perpendicular to the axis of rotation of the heavens, are parallel to the plane of the equator.

These celestial parallels may indeed be considered as resulting from the intersection of the celestial sphere, and the conical surface described by the visual ray, directed to a star during the whole of its revolution.

72. We can now fix, by geometrical data, the position of the plane of the equator and that of the parallels which we have just described.

The position of the equator is determined by its intersection with the plane of the horizon, and its *altitude*, that is, the angle which it makes with the horizon. *Its intersection* OO′ *with the plane of the horizon, is a straight line perpendicular to the meridian.*

It is scarcely necessary to remark, that a degree of a parallel is not equal in length to a degree of the equator, or of a great Top. 35. circle; still the arc of a parallel is readily compared with the arc of a great circle or with the arc of another parallel.

73. *The altitude of the equator above the horizon*, or the angle *EOH*, *is the complement of the altitude of the pole.* For, the angle *EOP* being a right angle, the angles *HOE*, *POH′*, are together equal to a right angle.

74. The zenith distances and azimuths afford the means of determining the positions of the stars at any place upon the earth; but these are inconvenient on account of being different in different countries. For, in consequence of the roundness of the earth, the planes of the horizon and meridian to which altitudes and azimuths are referred, take all possible directions in space, and accordingly the positions of the heavenly bodies, thus expressed, do not admit of being compared. The equator and horary planes present us a far preferable system of co-ordinates, since, occupying a determinate situation in the heavens, they offer to astronomers, situated upon every part of the earth's surface, a uniform means of expressing the results of their observations.

75. To determine then, the situation of any star upon the celestial sphere, it is sufficient to know the horary plane or horary circle in which it is found, and its position upon that circle or in that plane. The whole is, therefore, reduced to determining these two elements.

The position of the star upon its horary circle, is determined when its polar distance is known, or its complement, that is, its *distance from the equator*, which is called the *declination*; for this reason the horary circles are called *circles of declination.*

The position of the horary plane is determined by the angle which it makes with a known horary plane, assumed at pleasure and which is supposed to be fixed. This, for example, may pass through any star that shall be designated. If we suppose several horary planes to be drawn through different stars, they will make greater or less angles with the first. Each of these will, therefore, be distinguished by the angle which is peculiar to it, and which has for its measure the arc of the equator comprehended between it and the first horary plane. This arc is called the *right ascension.* It is determined by observing the time which elapses between the passage of the star in question over the meridian, and that of the horary plane from which right ascension is reckoned. This time, converted into degrees, is the right ascension of the star; it is counted always from west to east, and from zero to an entire circumference. The declination is reckoned from zero to 90°, and is called *northern* or *southern*, according as the star is north or south of the equator.

76. When these two co-ordinates, the right ascension and declination, are known, we can find by trigonometry all the relations of position and distance which exist among the points to which they are referred.

For example, if we would know the arc which measures the distance between two stars, we find, by means of their declinations, their distances from the same pole. We then take the difference of their right ascension; this will be the angle comprehended between the horary planes in which they are found. Thus, the two polar distances and the arc which joins the stars, will form a triangle of which we know the sides and the included angle. The third side is easily calculated. The distance of two stars may also be calculated from their polar distances and their azimuths.

77. The point of the equator from which right ascension is reckoned, depends upon the motion of the sun, or rather, upon its situation at some determinate epoch; for it is to the sun, as the natural regulator of days, years, and ages, that astronomers refer all their observations. But this choice, purely arbitrary,

does not in the least affect the relative situations of the heavenly bodies; it has no influence on their declination, or the difference of their right ascension. It determines only the absolute right ascension. It ought not, therefore, to occasion any change in the laws of their motion as deduced from their relative situations at different times. Accordingly until we have the data by which this origin is fixed, we may suppose the right ascensions of all the heavenly bodies to be referred to one, as to a fixed and determinate origin.

78. By observing the meridian transits of the stars, and measuring their meridian zenith distances, their right ascensions and declinations are determined. Tables may therefore be constructed, showing the situation of all the stars thus observed, and the order in which they succeed each other in their transits. These are called *catalogues of the stars.*

79. The right ascension of all the stars being determined relatively to one of them, in the above manner, if we wish to refer them to that point of the equator upon which astronomers have agreed as the origin, it will be sufficient to know the difference of right ascension between this point and any star; this difference added to the *relative right ascension* of the several stars, will give their absolute right ascension. For example, it is sufficient to know that, on the first of January 1810, this point of the equator passed the meridian $1^h\ 56'\ 29'',53$ before the star α arietis, or in other words, that the *true* right ascension of this star, at this instant, expressed in sidereal time, was $1^h\ 56'\ 29'',53$, or $29^\circ\ 7'\ 22'',95$; we say at this instant, because the precession, nutation, and aberration, render the right ascension variable.

80. That point which is the origin of right ascension, is also the origin of sidereal time; that is, the time is reckoned zero when this point is on the meridian. It is generally designated by the sign ♈, the characteristic of the constellation aries. This will be explained hereafter.

81. From these definitions, nothing is easier than to find for any instant the sidereal time in any place where the altitude of the pole is known. It will be sufficient for this purpose, to observe the zenith distance of any known star, and to calculate its horary angle, which we suppose to be reckoned from the superior meridian in the direction of the diurnal motion from 0 to 360°. By adding this angle to the right ascension of the star,

and subtracting an entire circumference if there be one, the remainder, converted into time, will express the distance from the meridian to the point in the heavens which is assumed as the origin, that is, the *sidereal time.**

Terrestrial Equator.—Meridians and Parallels.—Different Aspects of the Celestial Sphere above the Horizon of different Countries.

82. The principal circles of the celestial sphere being defined, and their respective positions determined, the aspect under which they are presented to observers, depends upon the inclination of the horizon of different countries, and consequently upon its direction in space; moreover this last particular depends upon the curvature of the terrestrial surface. On this account it is important to consider the variable situation of the celestial sphere.

This application of our results is general, whatever be the figure of the earth; but it is so much simplified by considering the earth as spherical, that we shall begin our explanation upon this supposition.

As we have perceived that the form of the earth is convex in all directions, we may always regard it as nearly spherical. We are confirmed in this conclusion by observing the horizon of the sea from a mountain or any other elevated situation; its depression is always found to be the same on all sides, in whatever country it is observed. This is peculiar to a spherical curvature, since it is only a sphere that can thus be touched on all sides by the surface of a right cone having a circular base; which surface would be formed by the visual rays extending to the different points of the horizon from any elevated station. The horizon of the sea is particularly favourable, on account of the uniformity of its surface, being interrupted by no inequalities, an advantage which the land does not afford. But even in the latter case also, there seems to be an equal depression in every direction, especially when one is elevated to a great height; for then the distant mountains which border the horizon, cause only small undulations, that do not prevent our recognising the sphericity of the curvature. And though this observation cannot be considered as strictly rigorous, on account of the small extent which the visual rays embrace, yet it is sufficient for a first approximation, or rather for a first enunciation; since, after having established what would take place, if the earth were spherical, we shall proceed to determine, in general, the results which must take place, whatever be its curvature.

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* Let $M♈M'$ be the equator; MM' the projection of the plane of Fig. 14. the meridian; OS the projection of the meridian of the star; $♈S$ will be the right ascension which we call A, and MS will be the arc of the equator corresponding to the observed horary angle, which we call P; adding these arcs we have $P + A = M♈$. This sum, converted into sidereal time, will give therefore the distance from ♈ to the meridian. We have no entire circumference to reject, as we should have had, if the star had been on the other side of the meridian, at S' for example; in this case we should have $MSM'S' = P'$, $♈MS' = A'$; consequently, $P' + A' = 360 - MS' + ♈MS'$; and by subtracting 360°, the remainder will be $♈MS' - MS'$, or $♈M$, as before. The formula will apply in either quadrant, as the arcs are always counted in the same direction from 0 to 360°.

83. If the earth be spherical, its centre may be regarded as the centre of the celestial sphere; for, from what we have already observed with respect to the immense distance of the stars, the centre of that sphere may be supposed to be placed at any point whatever of the earth. Then, an observer at the very centre of the earth would see all its points projected upon the surface of the celestial sphere, each following its vertical; and the geometric divisions which we have just traced upon this sphere, would apply also to the terrestrial surface. We shall begin, therefore, by developing these relations upon the hypothesis that the earth is a sphere.

In this case, the plane of the equator and that of the meridian being drawn through the centre of the celestial sphere, which is here that of the earth, will cut the terrestrial surface, forming great circles to which we have given names analogous to those which correspond to them in the heavens. The intersection of the surface of the earth by the plane of the equator, is called the *terrestrial equator*. The axis of rotation of the heavens, considered in this relation, is called *the axis of the earth*; it passes through the earth's centre, and the two opposite points where it meets the surface, are called *the poles of the earth*. Finally, the intersection of this same surface by the horary planes, gives the *terrestrial meridians*, or simply the *meridians;* which cut each other at the poles, since they all pass through the axis of rotation.

84. The earth is so small that two planes which touch it at its two poles, and which are parallel to the equator, would pass through the same stars, and meet the celestial sphere in the equator itself; or, at least the arc intercepted by them upon this sphere, as seen from the earth, would be too small to be perceived. The planes of the celestial parallels, which intercept considerable arcs upon the celestial sphere, on either side of the equator, will not, therefore, cut the surface of the earth; they will pass infinitely without it. But if we imagine cones which have their common vertex at the centre of the earth, and for their bases the different celestial parallels, these cones will cut the surface of the earth, considered as spherical, in circles whose planes will be parallel to that of the equator, and for this reason they are called *terrestrial parallels.*

85. These different parallels may therefore be classed and distinguished from each other, according to the celestial parallels to which they correspond; or, what amounts to the same thing, according to the angle which their vertical makes with the plane of the celestial equator. This angle is called the *latitude of the parallel.*

In figure 15, *ZO* is the vertical at the point *O* upon the parallel *OO'*. The line *OE* being supposed to be parallel to the equator, the angle *ZOE*, or its equal *ZNE*, is the latitude of the point *O*. If the earth be spherical, the vertical *ZO* passes through the centre, for all the verticals will meet in this point. But this circumstance is peculiar to a spherical figure. Whatever be the form of the earth, the latitude is nothing at the equator, since the vertical will be found in the plane of the celestial equator, if the earth be spherical, or parallel to this plane, let the figure be what it may. At the pole, the latitude is equal to 90°, because there the vertical *CP* is parallel to the axis of rotation. The latitude varies between these limits from 0° to 90°.

86. To find the latitude of a place, it is sufficient to measure the altitude of the pole above the horizon. Let *P* be the pole, *OH* the horizon, perpendicular to the vertical, and *OE* a straight line parallel to the equator, and drawn in the plane of the meridian *POZ*. The angles *EOP*, *ZOH*, are equal, each being a

right angle. Taking away the common part *ZOP*, there remains *EOZ* equal *POH*; that is, generally, *the latitude of a place is equal to the altitude of the pole above the horizon.*

87. The north pole of the earth is situated in the icy sea between northern Russia and Greenland. The south pole is opposite to it beyond New Holland. They are both surrounded with ice which has hitherto baffled all attempts to reach them.

The places which lie in the equator are better known This circle cuts the island of St Thomas in the Gulf of Guinea, crosses Africa, passes through Sumatra, Borneo, and the north part of New Guinea; thence it traverses the Pacific Ocean to Peru, over South America through the mouth of the river Amazons, and crosses the Atlantic Ocean to the place first mentioned.

88. The axis of the earth, which is that of the diurnal motion, being differently inclined to the horizon in different countries, there hence results a remarkable difference of appearance in the general motion of the stars, which, from what has been said, may be easily foreseen.

At the equator, for example, those stars which are in front of an observer facing the east, will rise vertically in the heavens, pass his zenith, and set directly behind him. The arcs which they describe will all be in the same plane perpendicular to the horizon; this plane is that of the equator.

The stars situated on the right and left will move in circles parallel to the equator. They will therefore describe circles of the celestial sphere, but small circles, since their planes do not pass through the centre of the sphere. The magnitude of these circles is less as they are farther from the equator. The stars, therefore, which describe these small circles do not rise so high above the horizon. Finally, towards the south and towards the north, are stars discovered which describe arcs so small that their motion is hardly sensible; and the points of the heavens where they are seen, appear to remain at rest amid the general motion. These are the celestial poles.

89. Such are the phenomena which the motion of the heavens presents to an observer at the equator. But in most parts of the civilized world the appearances are different, as will be readily seen; for, by changing our place on the earth, the direction of our horizon is changed, and we have our zenith successively at different points in the heavens. Under the equator the observer

sees the two poles. He ceases to see both upon advancing towards the north or south. If he travel towards the north, for example, his zenith approaches the north pole, and recedes from the south pole, which sinks beneath the horizon and is hidden by the convexity of the earth. By continuing his progress towards the north, the stars which surround the south pole, decline by degrees, and become invisible. The other pole, on the contrary, rises by the same degrees above the horizon, and the stars which surround it are more and more elevated, till the entire circles which they describe appear above the horizon. Thus these stars *will not* set; and were it not for the two great light of the sun, they would be constantly visible. The phenomena are reversed if we travel towards the south.

90. *These* different appearances have suggested to astronomers different denominations for designating the different countries, from the position of the celestial sphere in relation to the horizon. A country is said to be in a *right*, an *oblique*, or a *parallel sphere*, according as the celestial equator is perpendicular, oblique, or parallel to the plane of the horizon. The first takes place at the terrestrial equator, the last at the poles, and the other in every other part of the earth.

91. The altitude of the pole above the horizon is always the same at any given place. The earth's axis and equator, therefore, answer always to the same points of its surface. But they answer successively to different points of the sphere of the fixed stars. The star which is situated at the extremity of the Little Bear, and which is called the pole star, because it is in the vicinity of the pole, was very far from it in the time of Hipparchus, about 2000 years ago. It gradually approaches during a long period, and will at length recede in the same manner. In consequence of this change, the stars situated in the celestial equator, recede from it, and others are brought into their place. In the mean time the altitude of the pole above the horizon remains the same. This progressive motion is one of those which we have already indicated as altering the positions of all the stars. It is called the *precession*; we shall determine the laws of this motion hereafter.

Determination of the Figure of the Earth and its Magnitude.

92. In the preceding sections, we have recognised and established the convexity of the earth; we are now to inquire how the form of its curvature can be exactly determined.

This problem would seem at first view to be very difficult, and, in some respects, incapable of solution when we consider the irregularities of the terrestrial surface, intersected as it is by so many rivers, mountains, and vallies. But these variations which to us appear so great, are in fact extremely small and as nothing in comparison with the entire dimensions of the earth. We shall by and by see a mathematical proof of this truth; we shall here confine ourselves to the physical considerations which render it sensible.

We remark, in the first place, that the figure of the habitable earth differs little from that of the waters of the ocean, and is only a kind of continuation of the same surface. We have in fact already shown that the surface of continents is convex. This is evident from the progressive manner in which daylight advances. It is also perceived by the apparent elevation and depression of the stars, as we approach the north or south. But continents are surrounded on all sides by the sea, and indented by innumerable creeks and bays. Thus, for example, the continent of America is separated into two parts which are joined by only a narrow neck of land. So also the eastern hemisphere is separated into a great number of parts by seas, as the Mediterranean, the Red Sea, the Euxine, the Baltic, &c., which are only ramifications of the oceans with which they communicate. The interior of a continent, therefore, is nearly on a level with the waters which surround them. It hence follows that the general surface conforms very nearly to the convexity of the ocean.

This becomes more evident, if we consider the course of the rivers by which continents are intersected, many of which, as the Amazons, the Mississippi, the Rhine, the Danube, the Wolga, and the Nile, flow through a very great extent of country. These show the small degree of inclination of the surface of the countries through which they flow, that is, the inconsiderable difference of curvature of these countries from that of the sea.

It will now be seen that an inquiry into the form of the earth is by no means so difficult as might at first appear; the small inequalities of the terrestrial surface have comparatively no influence on its general figure.

93. To obtain an exact idea of this figure, it is necessary to measure it successively in different directions. We begin, therefore, by examining it in the direction of the meridian, since this direction is *the first* with which observations make us acquainted.

The process would be easy if the terrestrial meridians were plane curves. For the very simple considerations presented in the second section, teach us that the curvature of a plane 19. curve, is always indicated by the angles, more or less acute, which are formed by perpendiculars drawn to its different points. To apply this principle to the earth, it will be sufficient to take upon the same meridian several points at any known distances from each other, for instance, at equal distances, and determine the angles which their verticals form with each other; for these verticals are perpendicular to the curve of the meridian.

If the earth be exactly spherical, the curvature of its meridians will be every where the same. All the verticals will meet at the centre; and when the angles formed by these verticals are equal, the arcs which they intercept upon the terrestrial surface, must also be equal in whatever part of the meridian they are observed. In figure 16, let C be the centre of the earth, AA', $A'A''$, &c., equal arcs measured upon its surface in the direction of the meridian. In general, if the meridians are circular, the angles made by the verticals with each other are proportional to the arcs comprehended between them.

On the contrary, if the earth is not spherical, it will be made manifest; for where the convexity is greatest the verticals will meet at a less distance from the surface, and where it is least convex, they will meet at the greatest distance from this surface. So that, to measure the same angle between these verticals, it will be necessary to take a greater arc in the second case than in the first. In the curve $AA'BB'$, the angle C formed by the Fig. 17. perpendiculars AC, $A'C$, is equal to the angle C' formed by the perpendiculars BC', $B'C'$; but the curve being more convex in E than in P, the arc BB is greater than AA'.

94. Although we do not know *a priori*, whether the terrestrial meridians are plane curves; yet, as this is the most simple hypothesis, it is natural to begin with it, and then see if it agrees with observation. But we can at the outset foresee that it approaches so near to exactness, that we have never been able to measure the quantity by which it deviates, or even to be very certain that it does really deviate. We ought, moreover, to mention that the form of the earth, deduced from observation, deviates so little from that of a sphere, that the difference is by no means sensible for any small extent, even from the most accurate observations; so that to perceive this difference, it is necessary to compare the results of observations made in very distant parts of the same meridian, or in places whose latitudes are very different. To find, therefore, in each place, the value of a degree of the meridian, that is, the length of the terrestrial arc corresponding to an angle of 1° between the verticals, it is not necessary to take two points whose verticals make just an angle of one degree; this would be very difficult and almost impossible. But we shall proceed upon the supposition that the earth is spherical for a small extent about the points of observation, which will allow us to consider she arcs of the meridian proportional to the angles formed by the verticals. If, therefore, the angle observed between the verticals is a, and if M is the length of the terrestrial arc comprehended between them, we shall have the proportion $a : M :: 1^\circ : \frac{1^\circ . M}{a}$, and the quantity $\frac{1^\circ . M}{a}$ will express the length of a terrestrial degree, in that latitude, such as it would be were it directly measured.

95. The measurement of a degree of the meridian requires therefore, two distinct operations. The first consists in determining the direction and length of an arc on the terrestrial meridian. The second has for its object the measure of the angle contained between the verticals at the extremities of this arc.

We shall commence with this last, because the process which it requires is easily described. We have only to observe the latitude of each of the extreme points of the arc to be measured. The difference of these latitudes is the angle which the two verticals make with each other.

Fig. 18. For, let ZO, ZO', be two verticals at the points O, O', of the meridian OO', and let us suppose that the verticals produced

towards the interior of the earth, meet at the point *C*. Draw the horizons *OH*, *O′H′*, perpendicular to these verticals, and the visual rays *OP*, *O′P′*, which, being directed towards the same celestial pole, will be parallel. The angles *POH*, *P′O′H′*, will represent the altitude of the pole above the horizon of each place; and, if *OH″* be drawn parallel to *O′H′*, the angle *HOH″* will be the difference of these two altitudes. But the angle *HOH″* is equal to the angle *HCO* or *Z′CZ*. Consequently, *if at two points situated on the same terrestrial meridian, the altitude of the pole, or latitude of the place be measured, the difference of these altitudes or latitudes, will give the angle formed by the two verticals at these points.* Fig. 18.

96. We proceed now to trace the meridian line. It is proposed to continue it for a considerable extent over the earth's surface. The transit instrument placed exactly in the meridian, affords a very simple and exact means of effecting this object.

The instrument having been regulated by observing circumpolar stars, or by any other method, direct it towards some distant terrestrial object. Upon this object note the point which is on the vertical wire in the middle of the telescope, and this point will be in the meridian. A line drawn from the observer, or from the centre of the telescope, to this point, will therefore mark the direction of the meridian. We may thus note two points *N* and *S*, in a horizontal direction, the one north and the other south of the instrument.

Remove the instrument to the vertical of the point *N*, and fix it at the foot of this vertical, that is, at the point where it meets the terrestrial surface, and direct the telescope to the signal *S*, or to the point of the first station. When the point falls on the wire of the telescope direct it towards the north, and place in the direction of the wire another signal *N′*; this will be in the first meridian produced. By removing the instrument to the foot of the vertical *N′*, and regulating it as before, we can determine towards the north a third signal *N″*, and so on to an indefinite extent. If we designate by *A* the first station, and by *F*, *F′*, *F″*, the feet of the several verticals, the series of points *A*, *F*, *F′*, *F″*, will indicate upon the terrestrial surface the meridian of the first station produced, which may be continued from one side of a continent to the other.

97. The line drawn in this manner will be a terrestrial meridian only when these meridians are plane curves; in any other case the celestial meridians of the points A, F, F', F'', will be different. But, we shall soon see by the results of the observations, the difference is so inconsiderable that it may be neglected in the small extent ordinarily embraced by observations made in the same country.

It is moreover necessary to suppose that the surface of the country is throughout horizontal, that is, that its convexity is the same with that of the surface of the sea, and without interruption from elevations or depressions. This circumstance may take place, though rarely, upon the borders of the sea. It is thus, that the direction of the meridian has been traced in Pennsylvania; and the interval of the two stations being measured by a chain was found to be 538077,94 feet, the temperature being 61°,25 by Fahrenheit's scale.

The latitude of the northern extremity of the arc was found by accurate observations to be	39° 56′ 19″
The latitude of the southern extremity	38 27 34
Difference, or the arc of the meridian comprehended between the two stations	1° 28′ 45″

98. Now, if we suppose the earth sensibly spherical for a small extent about the observed points, the angles may be supposed proportional to the arcs, and the length of a degree will be $\frac{1^\circ \times 538077,94}{1^\circ\, 28'\, 45''}$, or 363771 feet, or 68,896 miles.

If the earth were exactly spherical, we might hence calculate its dimensions; for the value of a degree being 68,896 miles, the entire circumference would be 360 × 68,896 = 24803 miles. This, divided by 3,1416, gives for the earth's diameter 7895 miles. But these results must be considered as conditional till we have compared the values of degrees in different latitudes.

99. The above process is undoubtedly the most simple that can be imagined; but the circumstances under which it is to be employed very rarely occur. It requires a level open country of considerable extent. And when this cannot be met with, astronomers are obliged to resort to a different method to attain the same end. For this purpose, a series of spherical triangles

is constructed extending nearly in the direction of the meridian The first triangle *ABC* has for its vertex the point of departure Fig. 12. *A*. The two other angles *B* and *C* have for their vertices any two visible objects. This first triangle is horizontal. We observe its angles, measure one of the sides *AB*, and determine the angle which it makes with the meridian. From these data we readily calculate the two other sides *AC*, *BC*, the point *M*, where *BC* meets the meridian, and the distance *AM* comprehended between this point of meeting and the point of departure *A*. We then form a second horizontal triangle *CBD*, with the two objects *C*, *B*, and a third *D*, which can be seen at these two stations. On account of the roundness of the earth, the second triangle is not in the same plane with the first, and has nothing in common with it except the side *BC*. The meridian line *AM*, in the first triangle, if produced, will be elevated above the plane of the second. But it can be reduced to this plane by making it turn about *BC* as an axis. In this motion the angle *M'MC* is not changed, because *MM'* describes a conical surface, and as the two triangles *ABC*, *BCD*, are very nearly in the same plane each point of *MM'* describes only a very small arc of a circle, which may be considered as a straight line perpendicular to the plane of the triangle *BCD*. So that the whole process is reduced to bringing down the line *MM'* according to a vertical plane passing through the point *M'*. The meridian thus produced, meets the side *BD* of the second triangle, in a point *M'*, whose position and distance are determined as in the preceding case. This series of triangles may be continued in this way through any extent.

Such is the nature of the curve traced by operations which are called *geodesic*. Its first side is a tangent to the surface of earth; its second side is the first produced, and reduced to the horizon, and so of the rest. It is evident that this construction gives the same result as the direct measurement of an arc of the meridian in a level country.

100. To execute literally such an operation, it would be necessary to find a series of points situated at precisely the same distance above the terrestrial surface, and disposed in such a manner as to serve for the vertices of successive triangles. Then, in reality, the sides of the triangles would be the chords of the same spherical surface, the form of the earth being considered as spherical in the small extent usually embraced by operations of this kind. The measured angles will be those formed by the chords, and the planes of the different triangles will follow the convexity of this sphere, so as to be always horizontal. It is not possible always to obtain signals of the same elevation; while some are in the horizon, others will be elevated above it. But this difficulty is remedied by *reducing* all these angles *to the horizon*, and considering the several stations as projected upon the same spherical surface, and as forming upon this surface a series of spherical triangles the angles of which are known. With these data, and the length of one of the sides directly measured, we can easily calculate all the other sides, all the chords, all the angles comprehended by them, and finally the length of the meridian itself. Top. 174.

101. This method has been employed by many astronomers, and the length of a degree of the meridian has been determined in different countries and different latitudes.

Places of Observation.	Latitude.	Length of a degree in English miles.	Observers.
Peru	0°00′ 00″ N.	68,732	Bouguer
Pennsylvania	39 12	68,896	Mason and Dixon
Italy	43 01	68,998	Boscovich and Lemaire
France	46 12	69,054	Delambre and Méchain
England	51 29 54½	69,146	Mudge
Sweden	66 20 10	69,292	Swamberg.

102. Thus it appears that the degrees of the terrestrial meridian are unequal; therefore the earth is not spherical. The length of a degree increases as we approach the poles. The convexity of the meridian is therefore diminished towards the same parts. Consequently the earth is an oblate spheroid whose greatest diameter is that of the equator. By comparing the degrees in different latitudes, their increments from the equator to the pole are found to be nearly in the ratio of the squares of the sines of the latitude.

But it will be seen at the same time that this spheroid deviates but little from a sphere; for the maximum difference between two degrees, does not amount to the $\frac{1}{80}$ part of the length of one of them.

103. The measurements that have been made in the northern hemisphere, prove, as we have seen, that the meridians are not exactly circles. There is even reason to believe that the portion of the earth on the south side of the equator is not symmetrical with that on the north. A degree measured at the Cape of Good Hope, in latitude 33° 18′ 30″ south, is found to be 69,076 miles, and consequently greater than a degree measured in France in latitude 46° 12′ north.

It would appear, therefore, that strictly considered, the figure of the earth is actually irregular, and very complicated.

104. *Beset* with these irregularities, all we can do is to consider what curve will most nearly correspond with actual observation. This curve will differ from a circle. It must be *elongated at the* equator where the degrees are less than in any *other part*, and compressed at the poles where the degrees are *greatest*. The ellipse is the most simple curve which answers to these conditions, the transverse axis being in the plane of the equator, the extremities of the conjugate at the poles, and the centre at the centre of the earth. But this is not all; there is an infinite number of ellipses which may be so placed, among which we must choose that which best answers to the lengths of the degrees.

To have a complete idea of the operation in question, it is necessary to know that through each point of an ellipse or of any other curve, a circle may be drawn touching it, which shall approach to a coincidence with it more nearly than any other circle. This is called the *osculating circle*, and is confounded with the ellipse for a small extent about the point of contact, so that for this space we may suppose the degrees measured on the circumference of the circle instead of being referred to the ellipse. The radius of the osculating circle varies with the curvature of the ellipse, but its value may be calculated for each latitude in a function of the two axes of the ellipse. This value being multiplied by 2 × 3,1416, the ratio of the circumference of a circle to radius, and divided by 360, will give the length of a degree. Accordingly, by putting this expression equal to the observed lengths of two degrees, measured in two different latitudes, we shall have two equations by which we can determine the two axes of the ellipse; these are the two unknown quantities of the question to be solved; then, every thing being known in the expression for any degree, we calculate this expression for each latitude. The result of the calculation is found to approach so near to observation, that the difference may be safely neglected, even in the most delicate inquiries.

105. To arrive at the result in question it is necessary to combine together degrees taken in very different latitudes, and measured with great care. On both these accounts the arc measured at the equator by Bouguer and La Condamine, and that measured in France by Delambre and Méchain, are to be preferred. By combining them together in the manner above indicated, Delambre found for the ellipse the following results.

Semitranverse axis or radius of the equator	3963,2 miles.
Semiconjugate axis or radius at the pole	3950,4
Difference of the two axes . . .	12,8

This difference, divided by the semitransverse axis, is called the *oblateness* of the earth, since, according as it is greater or less, the terrestrial spheroid is more or less oblate, or compressed at the poles. The oblateness, as deduced from the above values, is $\frac{1}{309}$ nearly.* This result is proved to be very near the truth in several different ways. The lunar motions, depending upon the particular figure of the earth, give for the oblateness $\frac{1}{305}$; and, from observations on the pendulum in different latitudes, it is computed to be $\frac{1}{315,7}$†. It will be perceived that the terrestrial ellipse differs but very little from a circle, as might be foreseen from the small difference in the lengths of degrees.

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* Or more accurately 308,65, the original measures in toises being retained.

† The oblateness deduced from observations on the pendulum, is often stated to be $\frac{1}{335,78}$ from Laplace's *Mécanique Céleste.* But a mistake has been detected in Laplace's calculations by Dr Bowditch, which reduces it to $\frac{1}{315,7}$. See *Mem. Am. Acad., &c.*, vol. iv. p. 30. Later and more exact observations on the pendulum, as reported by Biot and Arago, make the oblatenes $\frac{1}{304}$. Laplace, after an elaborate discussion of the measurements and observations belonging to the different methods above referred to, for obtaining this important element, concludes that all the phenomena appertaining to the point in

According to the preceding values the fourth part of the terrestrial meridian, considered as elliptical, has for its length 6214 miles. From the equation of the ellipse, we can, in like manner, calculate the length of the terrestrial radius for any given latitude. One of these is particularly worthy of attention, namely, that which answers to the latitude of 45°; since, being near the middle between the equator and the pole, it may be considered as the mean radius of the earth. It is found to be 3956 miles. Moreover, it appears that the length of a degree in this latitude is 69,05 miles.

106. Hitherto we have studied the figure of the earth only in the direction of the meridians. To understand it fully, it is necessary to examine it in the direction of the parallels of latitude. This object is effected by tracing in the direction of the parallels *geodesic lines*, by the process above employed in the case of the meridian.

It is important to remark, that these geodesic lines are not terrestrial parallels. They have indeed a property in common with the parallels, which is that of cutting the meridian at right angles at the point of departure. But the parallels preserve this property with respect to all the other meridians which they meet; whereas the perpendiculars under consideration, possess it only in the case of the first. Hence, according as they depart from this plane, they deviate more and more from the parallel with which they coincided at the commencement. For example, the earth being supposed to be spherical, the perpendiculars to the meridian are great circles, the planes of which pass through the centre of this sphere, and all the perpendiculars to the same meridian intercept each other at the poles of this meridian 90° from the point of departure.

107. If the supposition that the earth is a spheroid of revolution is not altogether exact, it does not deviate much from the

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question, taken together, indicate an oblateness equal to $\frac{1}{306,75}$. See *Recueil d'Observations Géodésiques*, &c., *par MM. Biot et Arago*, p. 574, &c.

truth; and the error thus occasioned is altogether insensible in nearly all the applications which we have occasion to make.

In order to attain to precise notions as to the consequences to Fig. 20. be derived from such a supposition, let *MEOP* be an ellipse representing the terrestrial meridian. The semitransverse axis *CE* will be the radius of the equator; and the semiconjugate axis *CP* will be the radius at the pole. For any point *O*, situated on the meridian, *CO* will be the terrestrial radius, and, drawing the normal *NOZ*, this will be the vertical of the place, and *Z* the zenith, as given by the plumb line. It is accordingly about this vertical that the zenith distances of the heavenly bodies are observed, and not about a terrestrial radius, as in the case of the sphere. For this reason, and in conformity to the general definition of article 85, the latitude on a spheroid will be equal to the angle *ENO*, comprehended between the normal and the equator; whereas on a sphere it is the angle *ECZ'* formed by this same axis and the terrestrial radius. All these differences proceed from this, that the terrestrial radius and the vertical are two distinct and separate lines, except at the pole and equator where they are confounded. But between these points the angle *ZOZ'* varies according to laws easily calculated, and its absolute values depend on the oblateness of the earth. If the oblateness is very small, the greatest value is found very nearly in the latitude of 45°; and is equal to $11' \, 9'',39$, the oblateness being supposed $\frac{1}{308,65}$, as given by a comparison of the degrees measured in France and at the equator.

108. The effect of this oblateness may be wholly neglected in navigation, and all degrees of latitude be considered as equal. For the difference arising from this cause, even in a long run, can never occasion a considerable error, or expose one to danger. This conclusion is of great importance, for by observing each day the meridian altitude of any star, seamen find by the increase or diminution of these altitudes, how much they have advanced to the south or to the north, and what course they are pursuing.

Considerations on the Apparent Diurnal Motion of the Heavens.

109. We have now discovered in a manner the most satisfactory, that this earth which seems to us so vast, is, at it were, a globule in space, and nearly of a spherical form. We have discovered, moreover, by combined observations of altitudes and transits, that the whole system of heavenly bodies seem to turn circularly and uniformly about an axis passing through the interior of the earth, according to a direction that we have determined. This motion takes place without disturbance. The angular distances of the stars remain invariably or almost invariably the same; some of the heavenly bodies however, as the sun, moon, planets, and comets, are exceptions to this law. Their relative distances vary, and their apparent diameters undergo considerable changes, which, magnified by the telescope and measured by the micrometer, become sensible, even after short intervals of time. With this exception the bodies in question, like the rest, are subjected to the general laws of diurnal motion. The enlargement of the discs, by the telescope, indicates that the moon, sun, planets, and comets, are incomparably nearer to us than the stars. Moreover, the variation in their apparent diameters, proves that these bodies are not always at the same distance from the earth. But it does not teach us whether the sun, planets, &c., are really in motion, the earth being at rest, or whether the earth approaches to and recedes from these bodies. When the sun seems to us to come nearer the earth, a spectator in the sun would, in like manner, suppose himself at rest, and that the earth was in motion towards him. The same may be said of the diurnal motion of the heavenly bodies. We cannot infer that they actually revolve; for the appearances would be precisely the same on the supposition that the earth turns on its axis. Carried along with the bodies which surround us, the sea and the atmosphere, we seem to be at rest. And when the earth by its rotation presents us successively to different points of the heavens, we imagine that it is really the heavens which revolve about us.

110. We have frequent examples of this sort of deception. We often attribute our own motion to external objects. A tra-

veller, seated at the bottom of a carriage in rapid motion, sees the trees on each side approach swiftly towards him. Another person, situated in the same carriage, and looking out behind, will see the same trees flying from him.

In the evening when the sky is partly covered with clouds, driven along by the wind, if we look at the moon or a star in their neighbourhood, it will seem to move in a contrary direction, and it is very difficult, if not impossible, to resist the illusion.

111. Several causes conspire to make us judge erroneously of motions which take place without us. 1. We suppose our eyes at rest when the motion by which we are carried, is not the actual effect of our own volition, or an immediate result of the action of our organs. The illusion is so much the stronger as the motion is more rapid, especially if we are not apprized of it by any sudden jar. 2. We are led, by habit, to leave out of consideration the motions of which we ourselves partake, since they do not prevent our taking hold of objects which are displaced at the same time with us. 3. Among distant objects it is always the small ones which seem to be in motion. Those which appear larger and more difficult to be moved, we are apt to think immoveable. Thus the moon and stars, appearing very small compared with the masses of cloud which often surround them, the motion is referred to them and not to the cloud. We are, moreover, deceived in this particular by habit, small objects being for the most part in motion among large ones, as birds, &c., among trees and mountains. But these false judgments are easily corrected when we have acquired a just idea of motion; and, if it is not always possible to guard against illusions thus produced, we can at least discover the error and avoid its consequences.

112. We hence conclude that whatever motion the earth may actually have, if all the objects situated on its surface partake of this motion, we cannot perceive it; and to us it is as though it did not exist. But, on the other hand, it is equally necessary that this motion should appear general and common with respect to all visible objects, removed from the earth, that are foreign to its motion. Thus the true method by which we are to learn whether a motion is to be attributed to it, is to consider and observe whether among the bodies separated from us, like the planets, we discover a common motion which belongs to them all. For, a motion which is observed, for example, only

in the moon, and which does not take place with respect to the planets or the stars, can only be attributed to the moon. There is nothing more general in this respect, than the diurnal motion which comprehends all the heavenly bodies, and is from east to west. Thus, from appearances merely, this motion may with as much reason be attributed to the earth simply, in a contrary direction, as to the rest of the system, the earth being excepted.

The same may be said with respect to other motions, which, though small, are general, and whose existence we have recognised. These motions cannot well take place among the stars, for the appearances would be precisely the same if the equator and the axis which is perpendicular to it, remaining fixed with respect to its surface, were gradually displaced in the heavens. The height of the pole above the horizon of each place would be constantly the same. All terrestrial objects would preserve their respective positions, and we should imagine ourselves still at rest in the midst of them. The stars and other bodies would appear to us, therefore, to perform in the heavens, all the motions which the earth is supposed to have in the opposite direction. Appearances themselves present absolutely nothing by which it can be decided which of the two hypotheses is true. In this uncertainty the only proper course to be pursued is to study carefully the phenomena of the celestial motions, and to see which of the hypotheses explains them in the simplest manner.

Physical Consequences of the Earth's Oblateness. Length of the Pendulum vibrating Seconds in different Latitudes.

113. Although the oblateness of the earth is very small, it is extremely important on account of the consequences which it involves.

We see, in the first place, that the action of gravity, always perpendicular to the surface of the earth, or in the direction of the vertical, does not tend to the centre of the earth. These directions deviate from the centre, by a very small quantity, according to the degree of oblateness. There is, therefore, a connexion, a necessary connexion, between the figure of the earth and gravity. By pursuing this connexion we may perhaps discover whence arises this oblateness, and to what probable cause it is to be attributed.

114. We observe generally, that gravity acts upon all terrestrial bodies by a kind of force which draws them towards the earth. This force exists upon the tops of mountains and at the bottom of the deepest excavations. We may, therefore, consider the entire earth as composed of an infinite number of material particles united and concentrated by gravity.

If there was a time when these particles were not in a solid mass, if they were ever in a yielding state which would allow greater freedom to their motions, they must have arranged themselves in the manner required by the forces which are exerted upon them; and if we suppose them drawn together by gravity alone, they would unite in a spherical mass, like drops of water or mercury. Now it appears from a great number of facts in natural history, that this state has really existed, and that the earth was once a fluid. But since it has not taken the spherical form, we must conclude that some other cause has been exerted upon its particles and contributed to their actual arrangement. This may be easily explained if we suppose the earth to revolve upon its own axis once in twenty-four hours, for then its oblateness might be ascribed to the *centrifugal force* derived from its rotatory motion.

It will be seen that if the earth revolves, its different parts would have a tendency to recede from the axis of rotation. This centrifugal force, increasing with the velocity, would be greatest in the equatorial parts which describe the largest circle. It would be nothing at the poles which are fixed, and it would decrease by insensible degrees from one limit to the other. The earth, by the action of this force, ought therefore to become flattened at the poles and protuberant at the equator. Thus oblateness would be a necessary consequence of such a rotation, and therefore, as it is found actually to exist, it is a strong proof of the rotation in question.

115. There are striking analogies which confirm us in this hypothesis. Among the planets there are several, as Jupiter, Saturn, and Mars, whose oval discs afford unequivocal evidence of their spheroidal figure. Observing them with care, we discover upon the surface of their discs, spots whose regular change of place and periodical return, leave no doubt of the

rotatory motion of the planet about its shorter diameter. Why then should not the earth, which resembles the planets in its spheroidal figure, have like them a rotatory motion about its axis? So far from this conclusion appearing extraordinary, we should, on the other hand, be surprised if the fact were otherwise.

116. We are led by this induction to another consequence not less important. If the earth were spherical, the attraction exerted by the mass upon the different parts of its surface, or what is the same thing, the weight which draws bodies towards its centre, would be every where the same. But its spheroidal form destroys this equality, and the attraction ought to increase as we proceed from the equator to the pole, according to the square of the sine of the latitude; this is proved by calculations which we cannot here make known. Now, if the earth revolves about its shorter axis, another cause must combine with the preceding. The action of the centrifugal force will diminish the effect of attraction. But this diminution varies also from the equator to the pole, since, by approaching the pole, the centrifugal force, always perpendicular to the axis of rotation, becomes more and more oblique to the earth's radius, and consequently to the direction of gravity. It is also easy to prove by mechanical principles, that this diminution is proportional to the square of the sine of the latitude like that of attraction. Thus, from the effect of these causes, the absolute weight which we observe, that is, the excess of attraction over that part of the centrifugal force which is in the direction of the radius, ought to vary from the equator to the pole according to the same law. Consequently, in passing from the equator to the poles the fall of bodies ought be accelerated proportionally to the square of the sine of the latitude, and the same bodies ought to become heavier, according to this ratio.

The vibrations of the pendulum afford a simple means of verifying this fact. If the fall of bodies is accelerated, the vibrations must be more rapid, and the increase of weight may be calculated from their rapidity. For, it has been shown in mechanics that the lengths of pendulums vibrating in the same time, are proportional to the intensity of this force. Now, by carrying the same pendulum to different parts of the earth, it has been found to move faster as it departs from the equator; and this accelera- Mech. 346, 347, compared.

tion, the laws of which have been determined with great exactness, is in fact proportional to the square of the sine of the latitude as required by the rotation of the globe.

117. To place this fact in a clear light, it will be sufficient to state the lengths of the pendulum vibrating seconds, as observed in different latitudes.

Stations.	Latitudes.	Length of the seconds Pendulum †.
St Thomas	0°24′ 41″ N.	39,02074 E. inches.
Trinidad	10 38 56	39,01884
New York	40 42 48	39,10168
Hammerfest	70 40 05	39,19519
Spitsbergen	79 49 58	39,21469

It will be seen from these results, that the seconds pendulum increases in length as we depart from the equator; whence it necessarily follows that gravity increases as we proceed in the same direction; for, since the length of the pendulum is greater, if gravity remained the same, the vibrations would be slower. Moreover, the variations of their lengths are very well represented by increments proportional to the square of the sine of the latitude, as may be readily seen by the numbers given in the preceding table. The increase of gravity from the equator to the pole, is therefore a new proof of the rotatory motion of the earth. Besides, it is shown by calculation, that the length of the seconds pendulum, in different places, is proportional to the force of gravity by which it is moved. Whence, the ratios expressed in the above table, ought to represent the successive weights of the same mass transferred successively to different latitudes.

118. The rotation of the earth becomes further evident from another remarkable phenomenon, namely, the deviation of bodies which fall from a great height. To understand this, let us imagine a heavy body placed at a great distance from the earth's surface, at the top a high tower, for example. If the earth is at rest the body will fall at the foot of the tower in the direction of the vertical. But if the earth turns upon itself, bodies which partake of this motion will have at the commencement of their

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† The solar day of 86400″, and not the sidereal day, is here used.

fall, a velocity of rotation exceeding that of the base of the tower, on account of its greater distance from the axis of motion. Thus, when a body falls by the combined effect of this horizontal motion and that of gravity, it ought to meet the earth a little in advance of the vertical in the direction of the earth's motion; and consequently, after its fall, it will be at a little distance to the east of the tower; this is confirmed by experiment.

The extent of this variation has been calculated for different heights upon mechanical principles, and theoretical results are found perfectly to agree with those of observation.†

Of the Atmosphere and its Effects.

119. The air, notwithstanding its transparency, intercepts a sensible portion of light, and reflects it like all other bodies. But the particles which compose the atmosphere being extremely small and far removed from each other, we cannot perceive them, except when they are united in a great mass. In this case, the many luminous rays which they send back, produce on our organs a sensible impression, and we see that the colour is blue. Indeed the air gives a bluish tint to objects before which it is placed. This tint is very sensible in the case of distant mountains, and it is so much the more so as they are more distant. In order to represent distant objects it is necessary to diminish their brightness, and to reduce the proper colour by a mixture of blue, more or less deep. It is moreover the proper colour of the air which constitutes the blue colour of the sky, that azure vault which appears to surround us on all sides, and to which all the stars appear to be attached. According as we rise into the atmosphere this colour becomes less striking. The brightness, which it diffuses all around us, diminishes with the density of the air which reflects it, and upon a high mountain or in a very elevated balloon the heavens appear almost black.

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† This is not the place to bring forward all the arguments which we are able to produce on this question. As astronomical phenomena multiply, they present new proofs, which at length leave no doubt of the truth of the doctrine here advanced.

The air is not self-luminous. The light which it sends to us, comes from the sun and other heavenly bodies. This colour proves that it reflects the blue rays in a greater quantity than the others. The air surrounds the earth like a bright veil, which multiplies and propagates the light of the sun by an infinite number of reflections. It is by means of it that we enjoy the light when the sun is no longer above the horizon. After the rising of this luminary there is no place so secluded, provided the air can gain admission, which does not receive more or less light, although the rays of the sun do not reach it directly. If there were no atmosphere, each point of the terrestrial surface would receive only the light which comes to it directly. On ceasing to look at this luminary and the objects illuminated by its rays, we should be immediately enveloped in darkness. The solar rays reflected by the earth, would be lost in space, and we should be continually exposed to severe cold. The sun, even when near the horizon, would shine with the whole of its light, and instantly upon its setting we should find ourselves in profound darkness. In the morning, when this luminary appears, day would succeed to night in the same sudden manner.

We are able to judge of these effects by what is already known to take place on the tops of high mountains, where the air is very rare. It is notorious that extreme cold prevails in these regions. Very little light is received, except what comes directly from the sun and the other heavenly bodies. The air reflects so small a quantity at these heights, that by standing in the shade one may see the stars at midday.

120. On the general level of the earth's surface the influence of the atmosphere is such that the rays of the sun, light up the whole heavens and, by innumerable reflections, are diffused in all directions. In the evening, when the sun sinks below the horizon, its light is still showered upon us from the higher regions of the atmosphere under the name of *twilight*, and we pass by slow and imperceptible degrees from the full splendor of day to the obscurity of night. A similar phenomenon takes place in the morning, while the sun is yet below the horizon, and the gradual approach of day is governed by the same beneficent influence which tempers its evening decline.

121. The duration of morning and evening twilight depends therefore, other things being the same, on the height of the

atmosphere, or more correctly, on the height of those parts of the atmosphere whose density is sufficient to reflect a sensible portion of light. It accordingly varies with the state of the air. Its duration is increased on this account in summer, when the air is more dilated by heat. It is also for the same reason, longer in the evening than in the morning.†

122. To complete the results made known in the preceding sections, and to give to them all the evidence and rigor which belong to established truths, we must determine the influence of the atmosphere on the phenomena of the heavens; and if, in ascertaining the true effect of the diurnal motion of the celestial sphere, we have already supposed the observations freed from all optical illusions, that we may avoid a *petitio principii*, we must demonstrate that the corrections thus required, can be deduced from data purely physical, founded on the constitution of the atmosphere, independently of any knowledge of the motions of the heavenly bodies.

123. The atmosphere being composed of an infinite number of strata whose density increases as they approach the earth, the rays of light in traversing the atmosphere are affected in the same manner as if they passed successively through different media; and in passing obliquely they ought to be inflected towards the earth, according as the density of the atmosphere is increased. This is exhibited in figure 21, where the broken line $RR'R''R'''$, represents the successive directions which a ray of light would take in traversing the successive strata of the atmosphere A, A', A'', A'''.

But as the density of the atmosphere continually increases towards the earth, the broken line above supposed to be described by a ray of light, becomes a curve concave towards the terrestrial surface.

This ray, therefore, comes to the eye of the observer at O, in the direction $S'O$; and as we always see an object in the direction in which the ray enters the eye, the star S will appear to be at S'. While the apparent zenith distance is ZOS', the real zenith distance is *ZOS*. The difference *SOS′* of these two angles is called *astronomical refraction*.

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† These differences are not very considerable. The great length of the summer twilight over that of winter, is mainly owing to another cause. See *Top.* art. 98.

The effect of this refraction is to give to the heavenly bodies an apparent elevation greater than their real elevation.

124. It will be readily seen that the curve described by a ray of light, coming from a star, will be in a vertical plane passing through the star. For, as the atmosphere does not extend far above the terrestrial surface, and as refraction is, in this case, towards a perpendicular to the surface of the refracting medium, Opt. 32. the ray, once in this plane, cannot depart from it. Whence we conclude, that *the effect of refraction is always in a vertical direction, that it augments the apparent altitudes of the heavenly bodies, and diminishes their zenith distances.*

125. These effects diminish continually as the incident ray approaches to a perpendicular. Thus, refraction becomes *nothing at the zenith*; for the ray in this case passes the several media in a direction perpendicular to their surfaces, and consequently will not be turned out of its course.

On the other hand, the refraction of rays incident upon the same surface, increases with their obliquity. *Refraction will therefore be the greatest possible in the horizon;* and from thence to the zenith it will go on diminishing according to a determinate law.

126. It is found that the quantity of this deviation from the entrance of the ray into the atmosphere to its incidence upon the eye of the observer, does not depend upon the degrees by which the density of the atmosphere increases toward the earth, but upon the total augmentation of this density; that is, upon the pressure of the atmosphere, and the temperature at the place of observation; circumstances which are indicated by the barometer and thermometer.

127. The formula which expresses the law of refraction under these circumstances is extremely simple, and may be enunciated in the following manner. *Under the same pressure and at the same temperature, the refraction is proportional to the tangent of the apparent zenith distance of the heavenly body diminished by three times the refraction.*[*]

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[*] Or more exactly, three times and a quarter. That is, Z being the zenith distance, and r the corresponding refraction, we have

$$r = A \operatorname{tang} (Z - 3\tfrac{1}{4} r).$$

128. The coefficient of this proportion depends upon the refracting power of the air under a given pressure and at a given temperature. But it has been found by accurate experiments, that, when a ray of light passes out of a vacuum into air, the temperature being that of melting ice, and the pressure of the atmosphere being 29,92 inches, the sine of refraction is to the sine of incidence as 1 : 1,0002943321. With this datum we obtain the coefficient of the refraction under these circumstances; it is found to be equal to 60″,666.

This value varies with the refracting power of the air and consequently with its density; for it has been shown that the refracting power of the air and its density are constantly proportional to each other. The density of the air is proportional to the pressure which it sustains, and inversely as the bulk which it occupies. This bulk being taken for unity, or represented by 1, for the temperature of 32° of Fahrenheit's scale, increases 0,00208 for each degree of the same scale, the pressure remaining the same. Consequently, *in order to reduce the coefficient 60″,666, to a given temperature and pressure, it is necessary to divide it by unity increased or diminished by so many times* 0,00208 *as there are degrees in the temperature above or below* 32°, *and to multiply the result by the direct ratio of the pressures.* Mech. 470.

The coefficient, determined by our experiments, answers to the latitude of 45°. If we would find it for any other latitude, always considering the pressure as 29,92, it will be necessary to make a small reduction. For, gravity varying with the latitude, the weight of a column of mercury equal to 29,92 inches, varies in the same ratio, and consequently the coefficient should follow this proportion. But, g representing the force of gravity at the latitude of 45°, and g' that of any other latitude L, we have $g' = g(1 - 0{,}002837 \cos 2L)$. The coefficient 60″666 must, therefore, be increased or diminished according as the latitude is greater or less than 45°, the correction being in all cases $-60'',666 \times 0{,}002837 \cos 2L$, or $-0''17 \cos 2L$. Mech. 473.

A being a constant quantity for the same pressure and the same temperature. Under a pressure of 29,92 inches, and at the temperature of melting ice, the value of A has been found by exact experiments to be equal to 60″,666; we have, therefore,

$$r = 60'',666 \,.\, \text{tang}\,(Z - 3{,}25\, r).$$

The above corrections being made, the coefficient $60'',666$ will serve for all parts of the earth, even for mountains as well as planes. It is also adapted to all states of the air. Opt. 82.

129. The formula for refractions being thus established, and the corrections which it requires being made known, the refraction may be calculated beforehand for every degree and minute of latitude, and for all altitudes from 11° to the zenith, with the corrections required for the ordinary variations of atmospheric pressure and temperature which belong to any climate. *A table of refractions* being thus formed, it may be directly consulted, without having recourse, in each case, to calculations by the formula. For, if the observed zenith distance does not answer exactly to any one of the numbers in the table, it will fall between some two of them, and the correction required may be found by a simple proportion. The same method may also be used with respect to variations in the height of the barometer and thermometer. By means of such corrections, depending upon these two instruments, the same table of refractions is applicable to elevated situations, as well as to the general level of the earth's surface.

It will be perceived from what has been said, that the indications of the hygrometer may, in these experiments, be entirely disregarded. Opt. 82.

130. Such are the laws of atmospheric refraction for heights greater than 11°. When the rays of light make a less angle with the horizon; it becomes necessary to introduce into the calculation the law according to which the density varies at different heights. Top. 161. It will be perceived that this law must be modified continually by winds and other causes, even by those which the observer cannot foresee or appreciate. On this account all astronomical observations which have not refraction directly for their object, and which are not of a nature to be independent of it, are made above 11° of altitude. If it sometimes becomes necessary to observe below this limit, we correct the observations by the tables which are extended for the purpose even to the horizon. But in this case much less reliance is to be placed on the results.

Of Parallax.

131. The methods of observation which we have now established, relate particularly to the fixed stars, and are founded upon the supposition that the stars are at a distance nearly infinite, so that the visual rays coming from the same star to different parts of the earth may be considered as parallel. This supposition cannot be made with respect to those bodies which compose the solar system, since they are very much nearer to us, and are therefore seen in different directions in different parts of the earth. It is accordingly necessary to consider more particularly the effect of this difference of situation, and to find the means of applying the proper correction.

132. When different observers in different parts of the earth direct their attention to the same planet, they by no means refer it to the same point of the heavens. Let *L*, for example, be this body, *C* the centre of the earth, *O*, *O′*, the positions of two observers; *LO*, *LO′*, the direction of the visual rays. As the heavenly bodies are referred to the extremities of the lines in which they are seen, the first observer will see the object in *l*, upon the celestial sphere, and the second in *l′*. The difference in these two results will depend upon the angle *OLO′*, subtended at the body by the chord of the terrestrial arc which joins the two places of observation. This angle is called *the parallax*. In the case of the stars it is absolutely insensible, as is evident from our observations; since we have found that the visual rays *OL*, *O′L*, may be considered as parallel without any appreciable error. Fig. 22.

133. There are certain phenomena which show the effect of parallax independently of any astronomical calculations. For example, in the preceding figure, if the moon is in *L*, and the sun in *S*, much farther from the centre of the earth, the two observers will at the same time see the two heavenly bodies, and perceive the interval which separates them; but at the point *E*, situated upon the line *SL*, which joins their centres, this distance will seem to vanish, and the sun will be eclipsed. So also when a cloud passes between us and the sun, we are enveloped in its

shadow, while places at a small distance suffer no obscuration; the difference of these effects depends upon a real parallax.

134. To avoid the irregularities resulting from this cause, and render all observations comparable with each other, astronomers have agreed to refer them to the centre of the earth, considered as spherical or spheroidal; and they regard as the *true place* of the heavenly body in the celestial sphere, that in which it would appear if observed from this point. The point in the celestial sphere to which a body is referred when seen from the surface of the earth, is called its *apparent place.*

135. For the sake of greater simplicity we shall begin by considering the earth as spherical, and endeavour to find the true place of a heavenly body when the apparent place is known. Fig. 23. Upon this supposition let C be the centre of the earth, and O the position of the observer on its surface; the radius CO produced will be the vertical. Now, if the observer measure the zenith distance of the heavenly body, he will find it equal to the angle LOZ; if he had observed it from the centre, he would have found it equal to LCZ or to $LOZ - CLO$, because the angle LOZ is the exterior angle of the triangle LOC. The angle CLO, formed by the two visual rays drawn to the body, is called the *parallax in altitude; by subtracting it from the apparent zenith distance, we have the true zenith distance.* We suppose however, that the observed distance is previously corrected for refraction.

136. The two visual rays OL, CL, being in the same vertical plane, it will be readily seen that the effect of parallax is entirely in a vertical direction, like that of refraction; parallax, however, is subtractive, considered with reference to the zenith distance, while refraction is additive. The effect of the parallax in altitude, is therefore, to depress a heavenly body in the vertical circle in which it is found; this effect is contrary to that of refraction, which makes it to appear more elevated.

137. This depression depending upon the angle OLC, is by no means the same for all altitudes. The distance of the body from the earth being supposed constant, it is the greatest possible Fig. 24. at the horizon, as $OL'C$. The angle $OL'C$ is called the *horizontal parallax.* If from the point L' there be drawn two tangents to the circle OO', which represents the contour of the earth, the horizontal parallax will be the half of the visual angle

OL′O′, under which the earth will be perceived by an observer situated in the heavenly body; or, what amounts to the same thing, it will be with respect to this observer, the apparent semidiameter of the earth.

The parallax in altitude, or the angle *OLC*, diminishes as the star is more elevated above the horizon, and becomes nothing when the body is in the zenith, since the visual ray directed to the centre of the earth, is the same as the visual ray of the observer. The law of this diminution is easily calculated by the rules of trigonometry, on the supposition that the distance of the body from the earth's centre, does not sensibly vary in the interval of a diurnal revolution; it is found that *the parallax corresponding to any apparent altitude, is equal to the product of the horizontal parallax by the sine of the apparent zenith distance.*†

138. The whole process, therefore, reduces itself to determining the horizontal parallax. But the mere inspection of the triangle *OL′C*, shows that the sine of this parallax is equal to $\frac{CO}{CL'}$ or to $\frac{R}{D}$, calling *R* the radius of the earth drawn to the observer, and *D* the distance of the body from the centre of the earth. If this distance can be determined, as we have the radius of the earth, the parallax will be known.

In order to this, the most simple and natural process is that which is employed in trigonometry to find the distance of an inaccessible object, by observing it from the two extremities of a known base. Two observers at *O*, *O′*, situated under the same celestial meridian at a known distance asunder, observe, at the same time, the meridian altitude of the body at *L*, or its zenith distance. Then, in the quadrilateral *LOCO′*, the three angles *O*, *O′*, and *C*, are known; the last being formed by the two verticals at the places of observation. We have, moreover, the lengths of the sides *OC*, *O′C*, which are radii of the earth. We can, therefore, construct the quadrilateral, and calculate the diagonal *CL*, which is the distance in question of the heavenly body from the centre of the earth. The radius of the earth, divided by this distance, will be the sine of the horizontal parallax. Fig. 25.

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† The process by which this and other results are obtained will be given in an Appendix to this volume.

This method, reduced to calculation, leads to the following very simple result. *The horizontal parallax is equal to the angle at the body in question divided by the sum of the sines of the two zenith distances, if the body is between the zeniths of the two observers, or by the difference of these sines, if the body is found on the same side of the zenith with respect to them both.*

139. It is proper to remark that the angle *OLO′*, or the angle at the planet or other heavenly body, may be determined immediately from the differences of declination observed between a given star and the planet, and this is the method used by astronomers, since these differences can be easily observed with very great exactness. To exhibit this procees let λ be a star which passes the meridian at the same time with the planet, and Fig. 27. through the two places of observation, draw to this star two visual rays which may be considered as parallel, since the parallax of the stars is insensible. Then, the angles $LO\lambda$, $LO'\lambda$, will evidently be the differences of the observed declination, and *OLO′*, being the exterior angle of the triangle *LOS*, will be equal to the *sum* of these differences. If the planet be on the same side of the zenith with respect to the two observers, the angle *OLO′* will be equal to the difference of the angles at *O* and *O′*.

We have supposed that the star passes the meridian at the same time with the planet or other heavenly body. This is convenient in explaining the method, but it is not necessary in practice; for, if the star passes before or after the planet, their declinations may be separately observed and the difference taken. In order to this, we should choose stars very near the parallel of the planet, so that without deranging the instrument, the transit of each may be observed, and their difference of declination taken with a simple micrometer. This operation may indeed be performed with very great exactness, since it is independent of all those causes of error which affect the absolute heights of bodies above the horizon, and only supposes that the intervals of division of the instrument are exact in the small extent embraced by the observation; a condition which it is now very common to see realized in instruments constructed by good artists.

Let us take, for example, the observations of Wargentin and Lacaille for the parallax of Mars. They compared this planet with the star λ in the constellation Aquarius. Mars being in the meridian of the Cape of Good Hope, Lacaille found that the northern limb of its disc was 26″,7 north of the star; its zenith distance was 25° 2′, confining ourselves to minutes. At the same time and under the same meridian, Wargentin, at Stockholm, observed Mars, while on the meridian, to be 68° 14′ distant from the zenith; the northern limb of its disc being 6″,6 south of the star; the angle at the star was, therefore, 26″,7 + 6″,6, or 33″,3. The sine of 68° 14′ is 0,9287, that of 25° 2′ is 0,4231, and their sum is 1,3518. This is the denominator of our formula which gives $P = \frac{33'',3}{1,3518} = 24'',64$, for the value of the horizontal parallax of Mars at the instant of observation. It will be seen that the numerator of this expression, or the angle OLO', being very small, it is not necessary to know with the greatest exactness the sines of the zenith distances, nor indeed the zenith distances themselves; in this consists the great advantage of the method.

140. If unity be divided by the sine of the horizontal parallax, the quotient will be the distance of the heavenly body from the earth, expressed in a multiple of the radius of the earth; for $D = \frac{R}{\sin \Pi}$. From the results which have just been given, the distance of Mars from the earth at the instant of these observations, was equal to 8373 times the radius of the earth.

141. The condition that the two observers are exactly under the same meridian, seems to limit very much the application of the above method. But this is not indispensable. Indeed, it will not be at all necessary if the heavenly body which is observed, has no motion in declination, since then its meridian altitude will be the same for all places upon the same parallel. One of the observers, however, will see the transit after the other; but the zenith distance, measured by the second for instance, may always be considered as having been observed in the same latitude under the meridian of the first. The case is not quite so simple if the declination is variable, because there is then necessarily some small change in the time which it occupies in passing from one meridian to the other. But it is easy to obviate this inconvenience by observing for several days in succession the meridian altitude of the body. For the successive differences of these altitudes, showing the diurnal change of declination, we can ascertain with very great exactness the small variation which it undergoes during the time of its passage from one meridian to the other. By means of this correction the two observations can be reduced to the same meridian, and the parallax determined as we have stated above. To render the application of this method more exact, we endeavour to diminish these corrections as much as possible. To this end we select meridians which are near each other, and observe the body when in its greatest declinations, because then the motion in declination is less than under any other circumstances. We shall by and by see the reason of this; at present, however, we adopt the fact as a result of observation.

142. By a method entirely similar the parallax of the moon is determined. The moon's distance, as computed by astronomers, is about 60 times the radius of the earth; whence it will be seen that it is much nearer to the earth than Mars. The moon's distance from the earth being different at different times, its parallax is subject to considerable change, the determination of which has very much exercised astronomers; and they have employed different processes in order to determine it with the greatest precision. But, as most of these processes suppose a thorough knowledge of the motions of the moon, we shall, at present confine ourselves to the above method, which may be considered as the basis of all the others.

143. The parallax of the heavenly bodies makes known not merely their distances from the earth; but, taken in connexion with their apparent diameters, it gives their magnitudes also. *If the apparent diameter of a heavenly body be divided by double its horizontal parallax, we shall have, supposing it spherical, the ratio of its radius to the radius of the earth.* For, double the parallax is nothing else than the apparent diameter of the earth, considered as spherical, to an observer situated in the centre of the body; and, at equal distances, the apparent dimensions of bodies are proportional to their real dimensions. This theorem will be of use in comparing the dimensions of the planetary bodies with those of our globe. Fig. 28.

144. The effect of parallax being to depress the heavenly bodies, it does not affect their altitudes merely, it alters also their horary angles and their polar distances. The changes which it produces in these two particulars, are called the *parallax in right ascension*, and the *parallax in declination*. But these are easily deduced from the parallax in altitude.

Fig. 29.

Indeed, if through the eye of the observer, the visual rays OZ, OL, OP, be drawn to the zenith, to the body, and to the pole, these three lines produced will form upon the celestial sphere, the vertices of a spherical triangle PZS having reference to the apparent place of the body. Now, if, from the centre of the earth, three other visual rays CZ, CL, CP, be drawn to the same points, the two first being parallel to OP and OZ, on account of the infinite distance of the stars, may be considered as meeting the celestial sphere in the same points P and Z, that is, the angle OPC may be considered as nothing. But the third ray CL, differing sensibly from OL, on account of the parallax of the body L, will meet the celestial sphere in a point S' different from S. From this there results a new spherical triangle PZS', reference being had in this case to the true places; these two triangles will have in common the arc PZ, the distance from the zenith to the pole, or the complement of the latitude, and the angle PZS, equal to the azimuth of the body reckoned from the pole; for the arcs ZS', ZS, the true and apparent distance being comprised in the same vertical plane, the azimuth PZS, or the angle which the meridian makes with this vertical, is the same for the apparent places as for the true. But since the parallax in altitude SS', is determined by what precedes in a function of the horizontal parallax and the zenith distance, it follows that one of the two triangles being given, the other is completely determined. Indeed, they may be deduced the one from the other by the trigonometrical formulas, and we thus obtain the parallax in right ascension and declination. (*App.*)

145. The values of these reductions are necessary in a great variety of cases; they are indispensable in the calculation of eclipses, and occultations of the stars. They serve also to find the true distance of the moon from a star, in calculating the longitude. It is evident, indeed, that the parallax changes all these elements; but it is always easy to find by spherical trigonometry, the variations which they undergo, after knowing the parallax in altitude, since this fixes and determines the true place of the body in the celestial sphere; we shall accordingly defer these different calculations till they shall become necessary; it is sufficient for our present purpose to have stated the principle.

146. Thus far we have supposed the earth spherical. In this case the horizontal parallax of a heavenly body, placed at an

invariable distance, is the same to all observers, whatever be their situation upon the earth's surface. But this result does not take place, if the terrestrial radii are unequal, for the horizontal parallax is the angle under which the terrestrial radius, corresponding to the place of the observer, is seen from the centre of the body in question. So that two important questions present themselves. Has the oblateness of the earth any sensible influence upon the parallax? In what way are we to allow for it?

These questions are easily answered. We have already stated that the sine of the horizontal parallax is equal to the radius of the earth divided by the distance of the heavenly body. If this distance is constant, the terrestrial radii being unequal, the sines of the horizontal parallaxes will be to each other as the corresponding radii, and the parallaxes themselves being very small angles, will be very nearly in the same ratio; the equatorial parallax is therefore greater than the polar parallax, by a quantity equal to $\frac{1}{309}$ of its total value.

The moon's parallax is greater than that of any other of the heavenly bodies; its equatorial parallax elevates it 1° or 3600″. By taking the 309th part of this quantity, we have 11″,6 for the excess of the equatorial above the polar parallax; the difference is less for the intermediate radii, and varies in the ratio of the square of the sine of the latitude. But this difference, though very small has a very considerable influence upon several astronomical phenomena; for example, upon the time of an occultation of a star by the moon, and even upon the occurrence of the phenomenon. It is necessary, therefore, in many cases to have regard to it.

147. To this end we must adopt some hypothesis respecting the figure of the earth. We shall suppose it a regular spheroid, since we have found that this figure represents so nearly the variations in the lengths of degrees; and, were it merely an approximation, it would be sufficient for the object which we have in view. The first thing then to be remarked, is, that the terrestrial radius does not coincide with the vertical, and the *apparent zenith* Z, situated in this vertical produced, differs from the *true zenith* Z′, Fig. 30. situated in the radius produced. But, it is from the vertical that the zenith distances are observed, since this is the line indicated by the direction of gravity. Thus, to bring observations to what they would be if they were made from the centre of the

earth, two corrections are necessary, one to reduce them to the *true zenith*, the other to reduce them to the centre of the earth.

148. These corrections are easily made when the heavenly body is observed in the plane of the meridian, for example, in L or L'. In this case, the apparent zenith distance is $L'OZ$, if the observed body is on the side towards the pole, or LOZ, if it is on the side towards the equator. This is the *apparent* distance given by the observation. The *true* distance sought is $L'OZ'$ or LOZ'; to obtain which it is necessary to add to the apparent distance, the angle which the radius makes with the vertical, when the body is observed on the side towards the pole, or to subtract the same angle, when the body is observed on the side towards the equator.

149. Knowing the zenith distance LOZ', referred to the true zenith, the parallax in altitude answering to this distance is found in the same manner as if the earth were spherical, except that we employ here the particular value of the terrestrial radius OC, which answers to the place of observation. This parallax subtracted from the zenith distance LOZ', referred to the terrestrial radius, will give the *true zenith distance*, such as it would be if it had been observed from the centre of the earth. (*App.*)

150. If the heavenly body is observed out of the meridian, the lines OZ, OZ', and OL, indefinitely produced, will form upon the celestial sphere the vertices of a triangle in which we know the side ZS, which is the observed apparent zenith distance, the side ZZ', which measures the angle made by the radius with the vertical, and the contained angle $Z'ZS$, which is the supplement of the apparent azimuth of the body, reckoned from the pole, since it is formed by the vertical plane SOZ with the plane of the meridian; we can therefore calculate the third side $Z'S$, or the apparent distance of the heavenly body from the true zenith. Fig. 33.

We can also eliminate the azimuth by means of its value in a function of the apparent polar distance and the apparent horary angle; and, by this means, $Z'S$ is found to be expressed entirely in quantities easily deduced from observation.

Knowing the apparent distance $Z'S$, we find the parallax in altitude for this distance, and for the terrestrial radius OC; this parallax, represented by the small arc SS', will have for its expression, $\Pi \,.\, \sin Z'S$, Π being the horizontal parallax for the radius OC. And thus is known the true distance $Z'S' = Z'S - SS'$, such as it would be if it had been observed from the centre of the earth. (*App.*)

151. Now if we would have the parallax in declination and right-ascension, nothing is more easy; for, by referring all to the true zenith Z', the circumstances become absolutely the same as in the case of a sphere represented in figure 29. The expressions thus found for the parallax will apply to the cases under consideration; only, as the zenith distance of the pole enters into these formulas, it will be necessary to use the true zenith distance Z', that is, to augment the apparent zenith distance of the pole, by a quantity equal to the angle $Z'OZ$, formed by the vertical with the radius of the earth. By means of these results, nothing is easier than to obtain the elements of the apparent place of the heavenly body, its true place, referred to the centre of the terrestrial spheroid, being given. (*App.*)

152. The apparent diameter of the moon in the horizon is found to be a little less than at the zenith, if accurately measured with a micrometer; this is the effect of parallax, and is easily demonstrated. To this end, let us consider the distance of the moon from the centre of the earth as constant; it will then describe its daily circle around this centre; and the moon being Fig. 32. in the zenith at Z, will be nearer to the observer at O, than when it is in the horizon at L; for in the circle LZl, the perpendicular OZ is less than the half-chord OL. The difference of these two distances produces a sensible effect upon the apparent diameter, which is augmented as the moon rises. (*App.*) The law of this increase is readily calculated, and its total value from the horizon to the zenith is about $\frac{1}{60}$, because in the passage from one of these positions to the other, the distance of the observer from the moon is diminished by a quantity evidently equal to the radius of the earth, which is about its sixtieth part. The exact agreement of this law with the phenomena, in the rising and declining of the moon, leaves no doubt that they are to be ascribed to the cause which we have just assigned; and it is readily seen that, if the distance of the moon from the earth's centre is not strictly constant, as we have supposed, the variations which this distance undergoes while the body is passing from the horizon to the zenith, are too small to produce, in this interval, any considerable change in its apparent diameter.

153. In what precedes we have not mentioned the parallax of the sun. This is in fact so small, that it cannot be ascertained precisely by any of the above methods; and astronomers were for a long time ignorant of its value. It has, however, been determined, not by direct observation, which is too liable to inaccuracy, but from certain ratios, which exist between the distances of certain planets and the earth itself from the centre of the sun. It will be seen that this indirect method could not have been discovered till the planetary motions were known with great exactness. We shall, on this account, defer, for the present, the explanation of the process. The sun's parallax, as thus deduced, is found to be 8″,8, which gives the mean distance of this luminary from the earth equal to 23578 times the radius of the earth, or 93275068 miles. Nothing can be determined with precision respecting the dimensions of the solar system, so long as any uncertainty remains respecting this element. Consequently, if we would follow the order of discovery, we must refer this value to its proper place in the inquiry we are pursuing, and return afterwards to the several results to be deduced from it. Such indeed has been the mode of proceeding by which astronomy has advanced. We have been able to make progress only by a series of successive approximations. But this course alternately direct and retrograde, would be embarrassing and immethodical if too closely pursued. We will merely observe, that the value of the sun's parallax being very small, the approximate results, obtained without considering it, will differ very little from the definitive results at which we arrive by employing it in a second approximation. But, this truth being well understood, it is a much shorter course to admit provisionally the use of so small a correction, whose necessity is recognised, and whose value will hereafter be very exactly determined, to the end that we may arrive at once at the definitive results. The same may be said with respect to the small corrections relating to the precession, aberration, and nutation, the application of which, for the sake of greater brevity and method, is taken for granted before the subject is explained.

THEORY OF THE SUN.

Of the proper Motions of the Heavenly Bodies and the Means of determining them.

154. In the foregoing part we have determined, from certain facts, the form of the earth, its dimensions and the place which it occupies in space. We have found in its configuration fixed data for ascertaining exactly the position of different points in the heavens. Furnished with these results, we can follow all the motions of the heavenly bodies, and determine very exactly the laws by which they are governed. But there is only one sure method of attaining this end, and as it is important that this should be well understood in the outset, we shall here explain it.

155. When a body moves rapidly near the earth, we can, if we observe it, know nearly its direction and the course it pursues. But the motions of the heavenly bodies, at the distance at which they are placed, are too slow to be perceived by the eye. They can be ascertained only by comparing their positions at different times; in this way we find that they have changed their places in the heavens.

In like manner, when we wish to trace a curve, the form and law of which are unknown, a number of points are determined by observation; these are then united by a continued line. So also in determining the motions of the heavenly bodies, we observe each day the point in which they are found on the celestial sphere, and afterwards ascertain the form of the trajectory, from the condition that the body has passed successively through all these positions in the order in which they were observed.

The place of the heavenly body is found for each day, by observing its meridian altitude, and the instant of its meridian transit. The altitude makes known its declination; the time of the transit will give the right ascension with respect to the point of the equator which is chosen for its origin. These data will determine the point where the body is found upon the celestial sphere. The place of a moving body being found at successive periods, its path in the heavens is thus determined. This may be effected with very great accuracy by calculation; we are thus furnished with the means of tracing the true curve which the body describes; and are enabled to fix in imagination its path on the celestial sphere.

In order to a complete knowledge of the motions of the heavenly bodies it only remains to determine the variations in their distances. For this purpose we make use of their apparent diameters, which increase as they approach, and diminish as they recede. Or, if their diameters are very small, as is the case with the comets and most of the planets, they are compared with other celestial bodies, such as the sun whose course is previously determined.

These observations are multiplied and compared with each other. They are corrected by taking the most favorable time for each in which the quantities sought, show themselves in an insulated state and in their greatest increase. We at length arrive at a very exact knowledge of *the state of the heavens;* we learn what is constant, and what is changing every day, or every year, or in periods still more considerable.

Then the business of practical astronomy is finished, and that of theoretical astronomy begins; we compare similar phenomena for the purpose of discovering their relations, that is, the great laws to which they are subjected, and which are, as it were, the common source from which they are derived.

We next seek, by the principles of mechanics, what force is required to be exerted upon the heavenly bodies, in order that they may observe these laws, and that their motions may agree with observation. This force is determined, and it is found to be the same for all, namely, one which draws them towards each other in the inverse ratio of the squares of the distances, and which is consequently called *attraction*; not that by this we would express the nature of the power; we would only indicate the manner in which it acts. The effects of this power, modified by the different distances of the heavenly bodies, accounts for all astronomical phenomena, which are found to be explained in their minutest details; and astronomy thus becomes a great mechanical problem, the elements of which are given by observation.

We may then retrace our steps, reduce to numbers the formulas of the celestial motions, deduced from a knowledge of their causes, and form what are called *astronomical tables.* By these tables we know precisely what will be the state of the heavenly bodies at any future time, and what it has been at any time past. These furnish to navigators the means of knowing their course, to geographers, the means of determining the position of places, to husbandmen the rules for regulating their labors, to nations the epochs for fixing their history. Astronomy is thus made to produce its proper fruits, which, like those of all the other sciences, are general utility and the improvement of society. But to attain completely this end, the greatest accuracy is required; this is the constant aim of astronomical labors. It is difficult to imagine the degree of precision to which the science is brought. We may judge of it however by this single fact; if a telescope were to day directed to a determinate point in the heavens, we are able to tell, many years hence, the hour, minute, and second, when a given star will be seen directly in the centre of the telescope, so as to cover a wire finer than a hair.

Of the Sun's Circular Motion.

156. To measure exactly the right ascension and declination of the centre of the sun, we employ the methods of observation which are explained at length in the foregoing part. The declination of the sun is obtained by observing the meridian distance of his upper limb from the zenith, and that of his lower limb. These values are corrected for refraction, and their half sum is the true zenith distance of the sun's centre. Lastly, this result is diminished for parallax, and we have the distance such as it would be, had it been observed from the earth's centre. The zenith distance of the equator is known by previous observations, that is, by the latitude of the place. The difference of these two distances is the declination of the sun.

In making these observations, we note the contact of the upper and lower limb of the sun with the horizontal wire of the telescope. In this case we have no regard to the breadth of the wire; in reality the observed zenith distance of the upper limb is too small by a quantity equal to half the diameter of the wire, and the distance of the lower too great by the same quantity. These errors destroy each other in calculating the mean distance of the sun's centre; they are added in measuring his apparent diameter. The breadth of the wire must therefore be subtracted from the result.

By thus taking from day to day the meridian altitudes of the sun, and comparing them, we know its motion in declination.

157. To obtain the motion in right ascension, we observe each day the instant of the sun's meridian transit, and compare it with that of a known star. This is done by observing the contact of both the western and eastern limb with the vertical wire of the transit instrument, and taking the mean between the two times. The error resulting from the breadth of the wire is destroyed in this case, as in that of the zenith distance. The time which intervenes between the transit of the star and that of the sun, shows the difference of their meridians. This difference, calculated daily, gives the motion of the sun in right ascension.

By repeating continually these two kinds of observations, we shall have all that is necessary for determining the law of the sun's motion and its course on the celestial sphere.

Dates of Observations.	Distance of the centre of the ⊙ from the zenith.	Apparent diameter of the ⊙.	Declination of the ⊙.	Times elapsed from trans. of Lyra to the trans of ⊙.
Dec. 1806	°	°	°	h
18	72,23898	0,54294	23,40175 S	22,79970
20	72,28744	0,54305	23,45022	22,92768
22	72,30213	0,54322	23,46491	23,49576
27	72,20199	0,54322	23,46476	23,86592
Jan. 1807				
1	71,90716	0,54322	23,06995	0,23090
2	71,82453	0,54322	22,98730	0,30874
9	71,03846	0,54311	22,20124	0,82320
10	70,89636	0,54311	22,05913	0,89388
11	70,74694	0,54306	21,90972	0,96638
25	67,95238	0,54234	19,11517	1,96150
28	66,19391	0,54217	17,35669	2,17052
February				
3	65,53727	0,54166	16,70006	2,57884
7	64,33069	0,54128	15,49344	2,84740
11	63,05777	. . . . .	14,22055	3,11240
13	62,39541	0,54067	13,55818	3,24356
15	61,71969	0,54044	12,88246	3,37414
16	61,37629	0,54033	12,53907	3,43864
26	57,78166	0,53905	8,94444	4,07628
March				
1	56,65378	0,53867	7,81657	4,26424
3	55,89325	0,53839	7,05593	4,38888
17	50,42876	0,53634	1,59154	5,24878
19	49,63906	0,53589	0,80184	5,37026
20	49,24406	0,53578	0,40684	5,43094
21	48,85123	0,53572	0,01401 S	5,49300
22	48,45584	0,53561	0,38137 N	5,52134
23	48,06092	0,53545	0,77630	5,61268
25	47,27560	0,53511	1,56342	5,73376
April				
8	41,95915	0,53300	6,87807	6,58258
9	41,49381	0,53284	7,34341	6,64346
25	35,86896	0,53050	12,97327	7,63056
May				
1	33,96214	. . . . .	14,87508	8,00868
2	33,66008	0,52956	15,17714	8,07218
16	29,72612	0,52789	18,94910	8,97824
17	29,65707	0,52778	19,18015	9,04414
25	28,00309	0,52700	20,83414	9,57662
27	27,64693	0,52678	21,19029	9,71124
June				
12	25,71262	0,52578	23,12460	10,80540
13	25,64833	0,52573	23,18889	10,87450
20	25,38420	0,52545	23,45302	11,35948
22	25,37243	0,52545	23,46479	11,49758
24	25,38740	0,52539	23,44982	11,63620

Dates of Observations.	Distance of the centre of the ⊙ from the zenith.	Apparent diameter of the ⊙.	Declination of the ⊙.	Times elapsed from trans. of Lyra to the trans. of ⊙.
July 1807	°	°	°	h
9	26,36406	0,52533	22,47313	12,67006
10	26,48368	0,52539	22,35354	12,73828
11	26,60778	. . . . .	22,22044	12,80642
22	28,38917	0,52545	20,44805	13,54484
25	28,99283	0,52594	19,84439	13,70576
28	. . . . .	. . . . .	. . . . .	13,94304
August				
10	33,03956	0,52706	15,79766	14,78140
11	33,33070	0,52717	15,59651	14,84474
12	33,62681	0,52727	16,11041	14,91788
20	36,13383	0,52811	12,70339	15,40754
23	37,13206	0,52850	11,70516	15,59260
September				
1	. . . . .	. . . . .	. . . . .	16,14216
3	41,01378	0,52983	7,82344	16,26292
4	41,38141	0,53000	7,45580	16,32334
16	45,91871	0,53211	2,91851	17,04368
17	46,30526	0,53239	2,53196	17,10350
21	47,85564	0,53261	0,98158	17,34284
22	48,24555	. . . . .	0,59167	17,40268
23	48,63556	. . . . .	0,20166 N	17,46260
October				
3	52,62849	0,53422	3,70127 S	18,06458
4	52,92581	0,53433	4,08859	18,12518
19	58,58747	0,53667	9,75025	19,04792
20	58,04978	0,53683	10,11256	19,11060
25	60,72294	0,53755	11,88571	19,42692
November				
2	63,39667	. . . . .	14,55947	19,94326
3	63,71472	. . . . .	14,87749	20,00824
December				
3	. . . . .	. . . . .	. . . .	22,08238
8	71,51362	0,54238	22,67640	22,44572
10	71,71538	0,54238	22,87816	22,59202
17	72,19362	0,54289	23,35639	23,10744
28	72,23164	0,54294	23,39422	23,18134
Jan. 1808				
4	71,84105	0,54311	22,82383	0,43802
6	74,44715	0,54311	22,60993	0,58460

Note.—For the sake of greater compactness, the angles in the above table are expressed in degrees and decimals of a degree, and the times in hours and decimals of an hour. In the subsequent application of these observations, the usual denominations are preserved.

158. For the sake of having always a fixed point of departure, astronomers have usually referred the transits of the sun to those of the star α in the constellation Lyra, which being very bright and situated at a much greater distance from the equator than the sun ever attains, may be observed at all seasons with the transit instrument, even when it passes the meridian at the same time with the sun. The intervals between the transits of the sun and star are reckoned in sidereal time, from west to east, and they express the time which elapses from the transit of Lyra to the next succeeding transit of the sun. It is here supposed that the position of the star in the heavens remains invariably the same, that is, that a correction is applied for its small variations. If we would follow rigorously the course of discovery, it would be necessary to refer, each day, the transit of the sun to the apparent transit of the star, as it is actually observed, that is, affected by precession, aberration, and nutation. But then the small changes of place which the star undergoes by virtue of these three causes, during a series of observations for one year, are referred entirely to the sun, whose motion appears to be affected be these same irregularities. This course, however, will finally lead to the same results as the other, and is in reality that which is followed by astronomers. For the effects of the irregularities in question, almost imperceptible in the interval of a year, become very sensible after a series of years; and when the errors are thus suffered to accumulate, the same application of the first results, will show the necessary corrections. But by making at first these small corrections, we have the advantage of obtaining the final results from the first approximation, instead of being obliged to come at them by successive trials, and pursuing step by step the path of discovery. This anticipation will not affect at all the accuracy of the reasoning, for it will have no influence upon the considerations by which we discover the existence and measure of the small apparent motions to which all the stars are subjected.

159. Having given this necessary explanation, we shall at first consider the declinations of the sun as furnished by a table of regular observations, and examine by what laws they vary. If we follow the order in which they were observed we shall see that the first, belonging to the month of December 1806, were south; they attained their greatest value on the 22d of this

month. Then the declination of the sun was 23° 27′ 54″. From that time they diminish, the sun approaches the equator, and the change of declination from south to north, shows that the sun passed the plane of this circle about the 21st of March. This is the time of the *equinox*, when the length of the day is equal to that of the night all over the earth. The sun continues its course in the same direction; it recedes from the equator and approaches the north pole; the northern declinations increase, and finally attain their maximum about the 22d of June, and this maximum is very nearly the same as that for the southern declinations. From this time the sun begins to return towards the equator, and the declinations diminish. It reaches the plane of the equator about the 24th of September. There is now a second equinox. The sun continues to descend, the declinations become south, and increase to their accustomed limit, which happens, as before, about the 22d of December. From this time they again diminish in the manner above described, and the sun moves towards the equator by the same degrees. These phenomena are continually repeated every year according to the same laws.

160. If we take the differences of these declinations, it will be seen that they are to a certain degree regular and symmetrical on opposite sides of the equator; but the progress of these variations is unequal. They are more rapid when the sun is near the plane of the equator, and it is there that their value is the greatest. They diminish as the sun departs from this plane, and become insensible near the greatest declinations. At this time the meridian altitude of this luminary changes very little from day to day, and appears almost stationary. Thus the parallels which the sun describes at this time, are called the *solstices*. They are also called the *tropics*, from a Greek word which signifies *to turn*, because the sun, coming to these points, seems to turn back and retrace its steps. That which is to the north is called the *tropic of cancer;* the other the *tropic of capricorn*. These denominations seem to have been given by a people situated north of the equator, who seeing the sun return toward the south, after it had passed the northern tropic, gave to this parallel the sign Cancer or the crab, an animal which frequently moves backward. On the contrary, the sun appearing to them to ascend from the southern tropic, and mount again to

the equator, they gave to this parallel the sign of Capricorn or the goat, this animal being fitted for climbing. The two parallels are situated at equal distances from the equator, and this distance in 1807 was 23° 27′ 54″, as may be seen by the observations contained in the table given above.

161. We now come to the motion in right ascension. If the sun at first pass the meridian at the same time with the star with which it is compared, it will the next day be later, and it will recede from it daily in the direction from west to east. This is proved by the intervals of the transits, which always exceed the duration of a sidereal day. But the march of the sun in this respect is not uniform. It is sometimes slower and sometimes faster, as may be readily seen by comparing the intervals of successive transits as found in the table.

This comparison, made at different times of the year, gives the following values, for the daily retardation of the sun with respect to the star, as measured by their respective transits, in sidereal time.

December	22	266″,54
February	11	236,02
March	26	218,30
May	15	237,24
June	23	240,91
July	28	237,46
September	17	215,57
November	1	236,02

The smallest of these values is 215″,57, and the greatest 266″,54. Between these two we often meet with the value 236″65 within a small fraction; and this may be considered for the present as the mean quantity about which the daily retardation varies, or the difference between a sidereal and a mean solar day, expressed in sidereal time. Reduced to mean solar time, it is equal to 236″, or 3′ 56″.

The sun, advancing every day towards the east through a space expressed by the above quantity, thus passes successively through all degrees of right ascension. After having made the entire circuit of the heavens, it again passes the star which we have supposed immoveable; and the daily differences of transit recommence in the same order. The motion of this body from west to east, considered as parallel to the equator, takes place in a regular manner, though not uniformly, and in the interval of a year.

162. From these considerations we are led to recognise two proper motions in the sun, the one parallel, the other perpendicular to the equator; or, which amounts to the same thing, we may suppose a single motion oblique to the meridians and parallels, which will at once produce the two effects.

163. If we trace each day upon a globe the right ascension and declination of this luminary, we shall find that it thus describes a great circle of the celestial sphere, as represented in figure 36, in which EAA' designates the equator, AA', AA'' . . . the right ascensions reckoned from the point A as their origin, and AS, $A'S'$, $A''S''$ . . . the declinations, observed from the point O. The series of points S, S', S'' . . . . , determine the great circle, oblique to the equator, which the sun describes upon the celestial sphere. This result is confirmed by calculation, which is incomparably more exact than any graphical construction, the successive positions of the sun being determined by spherical trigonometry.

The circle described by the sun is bounded north and south by the two tropics. It is called the *ecliptic*, because an eclipse never happens except when the moon is in or near the plane of this circle.

164. If the ecliptic is really a great circle of the celestial sphere, it must cut the equator, which is also a great circle, in two opposite points, that is, in two points whose right ascensions differ by a semicircumference. Let us examine the table of observations, to see if this condition is fulfilled. We observe that on the 21st of March at noon, the declination of the sun was $0^\circ,01401$ or $50'',45$, south, and the next day at noon it was $0^\circ,38137$ or $22'.52'',95$, north. The sun, therefore, must have passed the plane of the equator between these two observations, and at a point of time approaching much nearer to the first than to the second. The precise moment of this phenomenon is easily determined. For, by taking the interval of the two transits, we see that it was $24^h\ 3'\ 38'',07$, and during this time the declination of the sun changed $50'',45 + 22'\ 52'',95$, or $23'\ 43'',4$; by supposing also that the change was made in a uniform manner,

which, for so short a time is very near the truth, we find by a simple proportion the interval of time necessary to complete the 50″,45. This will be

$$\frac{24^{h}3'\ 38'',07 \times 50'',45}{23'\ 43'',4}, \text{ or } 51'\ 9'',88.$$

This quantity added to noon of the 21st of March, will give the time at which the sun was in the plane of the equator. Now, since in the interval of two meridian transits of the sun, at this season, its right ascension with respect to Lyra changed 3′ 38″,07, it is clear that in 0^{h} 51′ 9″,88 the change will be

$$\frac{3'\ 38'',07 \times 51'\ 9'',88}{24^{h}\ 3'\ 38'',07}, \text{ or } 7'',7.$$

This quantity being added to 5^{h},433 or 5^{h} 29′ 29″,6 will give 5^{h} 29′ 37″,3 for the difference of right ascension between the sun and Lyra at the moment of the equinox.

We may arrive at this result more directly, by establishing at once the proportionality between the corresponding changes of declination and right ascension; for then from noon on the 21st of March to the instant of the equinox, the change of right ascension will be equal to

$$\frac{3'\ 38'',07 \times 50'',45}{23'\ 43'',4}, \text{ or } 7'',7,$$

as we have just found by a process more circuitous.

If we perform a similar calculation upon the observed declinations of the 22d and 23d of September for which the difference of declination is 23′ 24″,02, and of right ascension, 3′ 35″,65, the interval of the two transits being 24^{h} 3′ 35″,65, we shall find that the sun must have entered the plane of the equator at 12^{h} 13′ 13″,49, after noon of the 23d of September, that is, on the 24th at 13′ 13″,49 in the morning; and the difference of right ascension of the sun and Lyra at that time was

$$17^{h}\ 27'\ 45'',33 + 1'\ 51'',54 = 17^{h}\ 29'\ 36'',87.$$

By comparing this result with that of the 21st of March, it will be seen that the right ascension of the sun has varied in this interval 11^{h} 59′ 59″,57, or very nearly twelve hours, that is, one half of a circumference. By neglecting the small difference 0″,43, which may be attributed to errors of observation, we see that the two points in which the sun's path meets the equator, are diametrically opposite, as they ought to be, on the supposition that the orbit is plane. We may make the same calculation respecting any two points opposite in right ascension, that is, whose right ascensions differ by twelve hours, and we shall always find that they answer to equal declinations on opposite sides of the equator. We may easily make this comparison with respect to the observations of January 9, and July 11; of February 7, and August 11; of May 1 and November 3; which are already very nearly suited to our purpose, and which may be exactly reduced by means of the diurnal motion of the sun at each of these epochs. This opposition shows the perfect symmetry of the plane of the orbit on the two sides of the equator.

165. The two intersections of the equator with the ecliptic, are called the *equinoxes* or *equinoctial* points, because when the sun passes these points, the day is equal to the night throughout the earth. This luminary is then in the plane of the equator, which divides the earth into two equal parts. Thus, if we consider the rays that come to us from the sun, as parallel among themselves, which is very near the truth, the earth will evidently be enlightened from one pole to the other, and the circle which separates, upon the surface, the light from the shade, is a meridian whose plane is perpendicular to the solar rays. This meridian turns with the sun in consequence of the diurnal motion, and each parallel is found to be enlightened during half a revolution of the heavens.

Fig. 28.

The plane of the equator and that of the ecliptic cut each other in a straight line which passes through the equinoctial points, and which is therefore called the *line of the equinoxes*. This must not be confounded with the equator as traced upon the earth, called by navigators the *equinoctial line*, or simply *the line*.

Of the equinoxes, that which the sun passes in ascending from the southern tropic towards the northern, is called the *vernal equinox*, and is ordinarily designated in astronomy by the sign ♈. The second equinox, through which the sun passes in descending from the northern tropic towards the southern, is called the *autumnal equinox*, and is designated by the character ♎. These denominations are derived from the division of the year into seasons, depending upon the equinoxes and solstices.

166. Astronomers usually take the point ♈ of the equator, or the vernal equinox, as the origin from which to reckon the right ascension of the sun and all the heavenly bodies. They here

place the first point of the astronomical sign called *aries*, designated by the character ♈. By means of the results which we have obtained, it is easy to conform to this usage. We found by the observations of 21st of March, that the vernal equinox, referred to the meridian of Lyra, had a right ascension of $5^h\ 29'\ 37'',3$. If we would make this point the origin of right ascensions, we have only to subtract $5^h\ 29'\ 37'',3$, from all the right ascensions of the sun, referred to the meridian of Lyra. Consequently the right ascension of Lyra itself, referred to this equinox, and reckoned in the same direction as the others, will be the complement of this quantity to 24^h, or $18^h\ 30'\ 22'',7$. We can find, in like manner, that of all the other stars whose difference of right ascension with respect to the sun or Lyra is known.

As astronomers have fixed upon the point ♈ as the origin of right ascensions, they would naturally select the same point of the equator as the origin of *absolute sidereal time*. This they have done; and at any place, absolute sidereal time is measured at any instant by the horary angle comprehended between the vernal equinox and the plane of the meridian. There is, however, a want of uniformity with respect to the commencement of the sidereal day. Some have placed it at the instant of the transit of the equinoctial point over the superior meridian, and they then reckon 0^h of sidereal time. This method is used in the ancient tables. Others think that the sidereal day should rather be made to commence at the instant of the transit of the equinoctial point over the inferior meridian, which would be conformable to the general practice of society, the day being considered as beginning at midnight. This change has been introduced into the new astronomical tables, published by the French board of longitude.

167. To complete the determination, in the heavens, of the position of the plane of the ecliptic, it is only necessary to know the angle which it makes with the plane of the equator; for the position of a plane is determined when we know its intersection with a fixed plane, and its inclination to it. In this inquiry we neglect the diurnal motion of the celestial sphere, which being common to the equator and ecliptic, has no influence upon their respective positions. Let C be the centre of the earth, EQQ' the equator, ESe the ecliptic, Ee the common intersection of these two planes, or the line of the equinoxes. We draw a me- Fig. 37.

ridian *PSQ*, whose plane shall be perpendicular to this common intersection. This meridian will cut the plane of the equator in the right line *CQ*, and that of the ecliptic in the line *CS*, and the angle *SCQ* will be the obliquity of the ecliptic required.

Now of all the visual rays *CS*, *CS'*, *CS''*, which can be drawn successively from the centre of the earth to the sun, at different times of the year, *CS* is that which makes with the equator the greatest angle; consequently *the obliquity of the ecliptic to the equator is equal to the greatest declination of the sun.*

168. To determine this with the greatest accuracy, it is sufficient to observe the meridian altitude of the sun on the day of the solstice, if the solstice happen at midday. As this circumstance takes place at only one terrestrial meridian, the observation may be next to impossible. But it should be remarked, that when the sun approaches the tropic, its meridian altitudes vary very little from day to day, and on the day in which it arrives at the highest parallel, it remains nearly at the same distance from the equator. Thus, in whatever place the greatest declination of the sun is observed, it may as a first approximation, be considered as equal to the obliquity of the ecliptic; we say as a first approximation, for it will be soon seen that there are methods by which we can calculate what ought to be added to the result of direct observation to make it what it would have been if the observation had been made at the precise instant of the solstice.

In our table of observations, the greatest declinations of the sun are 23°,46491 = 23° 27′ 53″,67 and 23°,46479 = 23° 27′ 53″,25. The obliquity of the ecliptic may therefore be considered in a first approximation as equal to 23° 27′ 53″,46 for the year 1807.

The obliquity of the ecliptic may also be obtained without knowing the latitude, by observing the zenith distances of the sun when on the meridian at the two solstices, and taking half the difference. For the visual rays, drawn from the centre of the earth to the two solstitial points, must make equal angles with the equator.

In our table of observations, for example, the zenith distances of the sun at the two solstices, are 72°,30213 = 72° 18′ 7″,67, December 22, 1806, and 25°37243 = 25° 22′ 20″,75 on the 22d of June, 1807. Their difference is 46° 55′ 46″,92, the half of which 23° 27′ 53″,46 is the value very nearly of the obliquity in 1807.

169. When the obliquity of the ecliptic is known, it will be sufficient to observe one declination of the sun in order to find the position of the equinoctial points. Indeed, by representing the meridional position of the sun, on a certain day, by S', the declination $S'Q'$ may be known by observation. Then in the triangle $S'Q'E$, right-angled at Q', we have the side $S'Q'$ and the opposite angle $S'EQ'$, equal to the obliquity of the ecliptic. We can, therefore, by the rules of spherical trigonometry, calculate the side EQ', which is the right ascension of the sun, referred to the equinoctial point E.

Fig. 37.

Top. 82.

If, moreover, on the same day, the difference of right ascension between the sun and a star which passes the meridian after it, be observed, and the arc EQ' be added to the result, we shall have the right ascension of the star with respect to the equinoctial point E. The position of this point upon the equator, will, therefore, be very rigorously determined, as also that of the opposite equinox e, which is 180° distant. To avoid the small errors which are likely to attend these observations they should be many times repeated for the same stars; then, by taking the mean of all the results, the distance of the equinox from each star will be known, and consequently the position of the line of the equinoxes will be determined with great precision.

170. A straight line perpendicular to the plane of the ecliptic and passing through the centre of the earth, is called *the axis of the ecliptic*, from its analogy to the axis of the equator. The two opposite points in which this straight line produced, meets the celestial sphere, are called *the poles of the ecliptic.* That which is situated on the north side of the equator is called *the north pole;* the other is called *the south pole.*

The axes of the equator and the ecliptic being both perpendicular to their respective planes, the angle which they make with each other, is equal to the inclination of these planes, or 23° 27′ 53″. Thus the angular distance of the poles of the ecliptic from the plane of the equator, is equal to $90^\circ - 23^\circ\ 27'\ 53''$ or $66^\circ\ 32'\ 7''$.

The two celestial parallels which have this declination on the two sides of the equator, are on this account called *the polar circles.*

It is easy to find in the heavens the place of the poles of the ecliptic; for we already know the parallels on which they are situated. We know, moreover, that they are in the same plane perpendicular to the equator and the ecliptic, and consequently perpendicular to the line of the equinoxes. The intersection of this plane with the equator, will therefore be 90° degrees distant from the equinoctial points; thus the right ascension of the south pole of the ecliptic will be 90°, and that of the north pole 270°; these right ascensions being reckoned from the vernal equinox, and from west to east, in the direction of the proper motion of the sun.

The north pole of the ecliptic is the only one which can be seen in Europe. It is now situated in the constellation Draco, between the two stars ζ and δ, and it is a little nearer to the latter star.

171. The position of the sun's path being thus completely determined, the sun's place in the ecliptic can be calculated by knowing simply his right ascension or declination. For, in the spherical triangle $S'EQ'$, the position of the sun is determined Fig. 37. by the angle $S'CE$, which the visual ray $S'C$ makes with line of the equinoxes. This is what is called *the longitude of the sun.* It is easy to calculate this angle or the arc ES' which measures it, when we know the obliquity of the ecliptic and the right ascension or declination of the sun.

172. By repeating from day to day the same observation and calculation, we find successively the arcs described by the sun in the ecliptic reckoned from the equinox, or the longitudes of the sun.

The longitudes, like the right ascensions, are reckoned from the vernal equinox, and from 0° to 360°, in the order of the sun's motion. By taking their difference from day to day, we know the sun's daily progress in the plane of the ecliptic.

With a sufficient number of observations of this kind, tables may be formed which shall show the sun's longitude and declination, for any future day and moment. These constitute what are called *tables of the sun.* They may be calculated, if desired, in sidereal time, reckoned from the transit of the equinoctial point over the meridian of any assumed place. This astronomers have done, as we have already stated. In these tables, the differences of longitude from day to day may be determined with great accuracy, since they are the same every year, and return in the same order, which admits of their being corrected by time. If there be any uncertainty it will take place with respect to the epoch at which the sun will have a certain longitude; for example, at the instant of the equinox. There may also be a doubt respecting the true value of the obliquity. So that, to perfect the tables, it will be necessary to apply ourselves to the rectification of these two elements.

The investigation of the sun's motions, has then, like all astronomical problems, two distinct parts; the formation of tables from the first observed results, and the correction of the tables, on the supposition that their elements are nearly known.

173. The first thing to be carefully determined is the obliquity of the ecliptic. Fifteen days before and after the solstice, the observer begins to take the meridian altitudes of the sun with the proper instrument, for the purpose of determining the declinations. That which is observed on the day of the solstice will be the greatest possible if the solstice happen at midday; but, it may happen half a day sooner or later; during this interval the longitude changes about 29′ 42″, on the supposition that the daily motion of the sun is 59′ 24″, which is very nearly the mean value. But an error of 29′ 42″ in longitude will give an error of 6″,48 only in declination at the time of the solstice, because very near the solstice, the declinations of the sun becoming perpendicular to the ecliptic, a small change in longitude produces a much slighter change in declination. Such then is the greatest error which can be committed in determining directly the solsticial declination by an observation made on the day of the solstice.

But this error may be corrected by calculating from tables already formed, the sun's distance from the solstice at the time of the observation; for from this we may infer with great exactness, how much the observed declination falls short of the solsticial declination. By adding this quantity to the declination observed on the day of the solstice, we shall have the solsticial declination, that is, the obliquity of the ecliptic, with all the accuracy which can be attained by observation; with the same precision as if the solstice had taken place at midday. The calculation of this reduction, like that of altitudes observed near the meridian, does not require very precise data; it is sufficient to know approximately the obliquity of the ecliptic and the longitude of the sun.

A similar calculation may be made with the same accuracy for the preceding and following days. We can thus combine the observations of twenty or thirty days, and *reduce them to the solstice*, by the aid of the tables; and their mean result will give the solsticial declination with the greatest precision, especially by calculating the reductions to the solstice from the modern tables which are already so perfect.

The obliquity of the ecliptic, deduced from these observations, is subject to one cause of error, which is indeed extremely small, but of which it is necessary to free ourselves in order to arrive at the greatest exactness. This error arises from the fact, that the sun's path in the heavens, or the ecliptic, is not strictly plane. Nevertheless, as the deviation is very small, astronomers preserve, for the sake of simplicity, the idea of a plane orbit, and they regard the departures of the sun from this plane on either side, as small inequalities of which they take account in such a manner as to reduce this luminary by calculation to the plane of the ecliptic. Indeed, this reduction renders all observations comparable among themselves. The theory of attraction has taught us that these small oscillations are to be ascribed to the action of the moon and planets, which draw the earth's centre from the plane of the ecliptic, and cause the small irregularities which we refer to the sun; since we consider ourselves as immoveable, and this luminary seems to us to make these departures from the plane of the ecliptic in the opposite direction. Theory gives, at the same time, the law of these changes; and they are found in the tables of the sun, under the title of *the sun's motion in latitude*. As they affect the declination of this body, their operation belongs wholly to the obliquity determined from the solsticial declinations, and it is therefore necessary first to free ourselves from these effects in order to obtain the true obliquity.

Finally, to reduce all our results to terms exactly comparable, it is farther necessary to correct them for the small periodical variations which the obliquity undergoes, and which sometimes augment and sometimes diminish it. We shall hereafter speak of these oscillations. In the mean time, it may be thought necessary to have regard to them, in order to obtain the values of the *mean obliquity* which are alone capable of being compared together.

It is by this method, thus corrected and applied with all imaginable care, that M. Delambre has found the mean obliquity of the ecliptic for 1800, to be 23° 27′ 57″.

174. We pass now to the equinoxes. During an entire month, half before and half after the equinox, we observe the altitude of the sun each day, near the meridian, and thence determine its meridian declination for each day. With this declination and the known obliquity, we calculate the longitude of the sun.

This longitude being compared with that of the tables, shows the error to which they are liable. But from the whole of the observations upon which they are constructed, especially from the regularity of the algebraic formulas by which they are calculated, it is evident that the error cannot suddenly vary from one day to another, and therefore that it ought to be nearly the same for several days in succession. If then the observation of the meridian altitude with which the tables are compared, be considered exact, the correction deduced, should be applied to all the following days; and with the tables thus corrected, the instant of the equinox may be exactly calculated. But as it is not to be expected that a single observed altitude, will give the the error of tables with all the required precision, these observations, as we have before said, should be repeated for several days before and after the equinox; and each observation, being calculated by itself, gives one value for the error of the solar tables. Having thus twenty or thirty observations of this error which very nearly agree, we take an arithmetical mean of the whole. From the tables, corrected for this mean error in longitude, we calculate the instant of the equinox, which is easily done, since at this instant the longitude ought to be zero for the vernal equinox, and 180° for the autumnal equinox. In this calculation it is to be understood that reference is had in the tables to the small perturbations by which the sun is drawn out of its uniform path, and particularly to the small inequality which causes it to oscillate from one side to the other of the plane of the ecliptic, as we have said in speaking of the obliquity.

175. These methods suppose always that the zenith distance of the equator at the place of observation, that is, the latitude, is accurately known. Still there may be an apprehension of uncertainty in this particular. To remove all doubt, an observation of the opposite equinox is also made, and a mean taken between the two errors of the tables which result from them. It will be

readily seen that the error in the latitude, being the same at the two equinoxes, would have opposite effects on the sun's longitude, according as this body is approaching to, or receding from the equator.

Of the Calendar.

176. The motion of the sun, determines the different periods employed by society for the distribution of time. The choice of these periods and the order of this distribution, constitute what is called the *calendar*.

177. The interval of time employed by the sun between two successive returns to the same equinox, or in general, to the same point of the ecliptic, is called the *tropical year*. Its duration has always been a subject of interest. It is indeed a natural measure of the labors, which require long intervals, and which depend on the change of the seasons; the knowledge of it is necessary to the husbandman, the merchant, and the traveller; and great pains have accordingly been taken to determine it.

What presents itself at first, as the simplest course, is to find how many solar days the year contains, no regard being had to their inequality. For this purpose, it would be sufficient to trace on a horizontal plane a meridian line, and having erected a vertical style of invariable height, to examine every day, at noon, the length of the shadow. It would be shortest on the day of the summer solstice, and the number of days intervening between two successive returns of the sun to the same solstice, would give the entire duration of a revolution. We should thus find the tropical year to contain about three hundred and sixty-five days; and it was in this way that the first astronomers appear to have observed it.

This result might have suffised for a time; but its want of exactness must necessarily have become apparent, by the accumulation of errors. If we observe the same solstice during several successive years, we shall find that it arrives later than it ought, if the year were exactly 365 days; the error is nearly fifteen days in sixty years. We hence perceive that the year is a quarter of a day longer than our first approximation made it, and its length was accordingly fixed at $365^d\ 6^h$.

This computation, though more correct than the preceding, is still far from being exact. Hipparchus, comparing an observation of the solstice made by himself, with one made by Aristarchus, a hundred and forty-five years before, found that the last solstice happened half a day sooner than it ought if the year had been $365^d\ 6^h$; the error then was 12^h in a hundred and forty-five years, or 4′ 58″ a year. Hence we obtain the length of the year equal to $365^d\ 5^h\ 58'\ 2''$.

178. This estimate also needs correction; and the true mean year, as given by the last tables of M. Delambre, is $365^d.\ 5^h\ 48'\ 51'',61$. The ancient estimates erred from want of exactness in observing the solstices. Indeed, as the meridian altitudes of the sun increase at this time, by insensible degrees; the shadow of the style has a corresponding change, and it is impossible to decide the precise instant when the sun arrives at the solstice. We avoid this inconvenience, if we determine by observation, two epochs, in which the sun's meridian altitude is precisely the same, and always increasing or decreasing equally. The interval of these two epochs, gives the mean length of the sun's entire revolution. Observations of this kind are always made most advantageously near the equinoxes, because then the meridian altitudes of the sun change very sensibly from day to day. Thus Hipparchus observed many equinoxes for this purpose. But the imperfection of his instruments was such as not to admit of his determining the length of the year from observations embracing so short a period. Several centuries were necessary to compensate for this uncertainty. Notwithstanding all the improvements in modern instruments, astronomers still make use of the same method in finding the true length of the tropical year.

179. If the sun's proper motion were perfectly uniform, it would be sufficient to observe two equinoxes with great accuracy, proceeding in the manner already described; and the number of solar days and fractions of a day, which intervened between these two epochs, would be the true length of the year. But the inequalities in the sun's proper motion, impair this simplicity, and render the problem much more difficult. For it hence results that this body does not always return to the same equinox, after intervals of time exactly equal; and as the causes and effects of these variations can be estimated only by the theory of universal gravity, it will be seen that we must

apply ourselves to that before we can completely free the sun's motion from these irregularities, and consequently before we can determine with the greatest accuracy the mean duration of the tropical year. Here then is a new example of the successive approximations to which we are obliged continually to have recourse.

We may nevertheless avoid a part of these difficulties and arrive immediately, without a *petitio principii*, at an estimate very near the length of the mean year. We shall find by what follows, that the inequalities of the sun's motion are of two kinds. Some, which are called *periodical*, are entirely completed in the space of a year, or a small number of years; and after this they compensate each other by returning through the same degrees; so that they make the sun oscillate continually about a mean state from which it departs but little. The other inequalities, on the contrary, extend through periods of such a length that they are termed *secular;* and they have been continually increasing or decreasing from the times of the most ancient astronomers. Their effects then must inevitably be perceived in comparing ancient observations with our own. But it is not the same with respect to periodical inequalities; these must compensate each other a greater or less number of times, according as the epochs compared are farther removed from each other. Consequently, all their intermediate effects ought to disappear, when we compare equinoxes, which are separated by a long interval; and there ought to remain of their influence only what belongs to the first and last epoch. Now this influence is much weakened in the mean result, where it is divided among the number of years comprehended; and we might imagine observations so far removed from one another, that the effect of the extreme errors would be imperceptible.

The mean year, determined in this manner, by comparing one of our equinoxes with an ancient one, observed, for example, by Hipparchus, might then be considered as independent of the periodical inequalities of the sun's motion, and affected only by the secular inequalities. But, besides that these are very small, we conceive that they could be determined and corrected only by a second approximation. Consequently, we must not expect to escape them in any other way. Having thus anticipated their existence, we will suppose, for the sake of brevity, that regard has been had to them, and shall henceforward make use of the definitive length of the mean year, namely, $365^d\ 5^h\ 48'\ 51'',61$. This kind of anticipation, of which we have already availed ourselves, detracts nothing from the rigor of the reasoning, provided it is properly understood, and regarded only as a means of avoiding the necessity of retracing our steps, to effect the successive approximations, which astronomers have employed, and of which it is sufficient to perceive well the necessity.

180. To apply the above results to civil life, and render their use general, we must present them disengaged from the fractions, which accompany them, and which would render it too difficult to remember them.

The first idea that presents itself is to neglect these fractions. We should then have years of three hundred and sixty-five days. These were formerly in use, as we have already said; but the want of exactness, being apparent after very small intervals, caused them to be given up, when the world had become better informed on the subject of astronomy. The principal inconvenience arose from carrying the beginning of the year successively into the different seasons, one day in four years, and one year of three hundred and sixty-five days and a quarter, in 1508 years, so that after this interval, we should have fallen back one year, and the seasons would return to their former places.

The Egyptians appear to have known this period; but they made it 1460 years, because they supposed the year $365^d\ 6^h$. This was called the *sothiac period.*

181. To avoid these inconveniences, the method of *intercalations* was devised, by which the civil year is made to consist of 365 days, care being taken to allow for the small annual error before it has accumulated beyond an entire day. On this account the corrections are frequent, but the civil year is made to vary between limits of small extent about the true mean year, and the influence of this error on the labors of society, is altogether imperceptible. It will be seen besides, that we need here concern ourselves only with the mean year; since, for civil purposes there is no necessity for taking into consideration the small periodical inequalities, which occasion a variation, sometimes greater and sometimes less, in the sun's return to the same equinox.

The most simple intercalation is that of one day every four years. It supposes the mean year to be $365^d\ 6^h$ or 365 days and a quarter, which differs but little from the truth. This intercalation was prescribed by Julius Cæsar, and thence derived the name of the *Julian correction*. According to this manner of reckoning, the *common years* are 365 days; they are divided into twelve months of thirty or thirty-one days, with the exception of February, which has only 28. The intercalary day is placed, every four years, at the end of February. The year has then 366 days, and takes the name of *bissextile*, so that there are always three common years between two bissextile. A hundred Julian years of $365^d\ 6^h$ forms the *century*, which is the longest period employed by society, and which has hitherto been sufficient for all the purposes of chronology.

182. The Julian intercalation has been transmitted to all the nations of Europe, but our era is different from that of the Romans, who reckoned from the foundation of Rome. In christian countries we reckon from the birth of Jesus Christ, or rather from a certain year fixed astronomically in regard to us, and to which we refer that event, the precise time of which is uncertain; chronologists not being agreed upon the subject. But this is not material as it respects the successive progression of years. It is sufficient that we have one of the years fixed by the observation of some one astronomical phenomenon. Now we know, that when the council of Nice was held, the equinox happened on the 21st of March, and, according to the calculations of chronologists, 325 years had elapsed since the commencement of the christian era. Nothing then is wanting to enable us to connect all chronology with this epoch, and to refer all eras to the christian era.

183. This manner of reckoning was continued till the year 1582; but as the year was supposed to be $365^d\ 6^h$, while in reality, it was only $365^d\ 5^h\ 48'\ 51'',61$, the small annual difference of $11'\ 8'',39$ had been accumulating, and had produced in 1257 years, $9^d\ 17^h\ 22'\ 46''$, that is, about ten days, by which the solar year had fallen back. The equinoxes had changed progressively from the time of the year, to which the council of Nice had fixed them, and the difference amounted to one day in 132 years nearly. Pope Gregory XIII. was hence led to make, a change in the calendar, which has been called the *Gregorian reformation*.

He began by correcting the error of ten days, ordering that the day following the 4th of October 1582 should be called not the 5th, but the 15th. He still retained the Julian intercalation of one day every four years; thus, *those years, of which the number is divisible by four, are bissextile.* But it was agreed to suppress this intercalary day in the centurial years 1700, 1800, and 1900, allowing it to remain at the same time in the years 2000, &c., so that *three common centurial years are always followed by one bissextile.* This intercalation is at the same time very simple and very near the truth; for the annual error of 11′ 8″,39, belonging to the Julian intercalation amounts after 400 years to $3^d\ 2^h\ 15'\ 56''$, that is, to three days nearly. The error of the centurial intercalation is then only $2^h\ 15'\ 56''$ in 400 years, or $22^h\ 39'\ 20''$ in 4000 years. It is accordingly more than sufficient for all the purposes of history, especially if one additional bissextile be suppressed every 4000 years.

The above manner of estimating the year, forms what is called the *Gregorian calendar,* according to which the vernal equinox always happens between the 19th and 21st of March. This calendar was immediately adopted only by the catholic states of Europe, which occasioned a difference in the manner of dating. Those, which observed the Julian calendar, reckoned ten days behind the others, from 1582 to 1700, 11 days behind from 1700 to 1800, &c. But, with the exception of Russia, which still retains the Julian mode of reckoning, the Gregorian calendar is now in use among all the states of christendom.†

184. The results at which we have arrived concerning the length of the year, will enable us to find the sun's mean motion

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† It was not till 1752 that the gregorian calendar was adopted in England and her colonies. At this time, moreover, the year was made to begin with the 1st of January, instead of the 25th March. With respect to events anterior to this time, the former, or the present mode of dating may be used; and they are distinguished by the phrases *old* and *new style.* The difference is now 12 days; also the number of the year will differ by one with respect to events occurring between the 1st of January and 25th of March. In this case, both years are sometimes written; thus, Washington was born 11th of February, old stile, or 22d, new stile, 1731–2.

in the ecliptic, that is, the mean arc which it describes in the interval of a day. For, its annual revolution comprehending 360°, its progress for a day would be, supposing it uniform,

$$\frac{360^{\circ}\,.\,1^{y}}{365^{d}\ 5^{h}\,48'\,51'',61} \text{ or } 59'\ 8'',33.$$

This result will be sometimes too great, sometimes too small, and sometimes exact; but at the end of the year, all inequalities will be compensated. It is very common for geometers and astronomers thus to accommodate what is irregular to exact rules, which, by continual approximation, are so modified as to adapt themselves more and more nearly to the truth.

The same method will make us acquainted with the sun's mean motion in a direction parallel to the equator, and since this luminary performs the circuit of the heavens in the interval of a year, its mean progress in right ascension for one day, will be 59′ 8″,33, as already stated.

185. Astronomers are accustomed to refer the motions of all the heavenly bodies, to that of a fictitious sun, which would have precisely the uniform motion, which we have just determined. The interval of time, which elapses between two successive returns of the fictitious sun to the meridian, is called a *mean solar day*; and in general, the instants of duration, marked by its successive positions, is denominated *mean time.*

From the results which we have just obtained, it is easy to find the relation of the mean solar day to the sidereal day, which we have hitherto adopted as the unit of time. For, the proper motion, 59′ 8″,33, being from west to east in a direction contrary to the diurnal motion of the heavens, this arc is to be added to the rotation of the celestial sphere; and the fictitious or mean sun does not return to the meridian from day to day till it has described, in a direction parallel to the equator, an arc of 360° 59′ 8″,33. If then the duration of a mean solar day is represented by 24^{h}, the time which this body employs in describing the diurnal arc 59′ 8″,33 will be $\frac{24^{h}\,.\,59'\ 8'',33}{360^{\circ}59'\ 8'',33}$, or 3′ 55″,91. This quantity subtracted from 24 mean hours, gives 23^{h} 56′ 4″,09 for the time of the diurnal rotation of the celestial sphere, expressed in mean solar time. The small difference 3′ 55″,91, or more accurately 3′ 55″,9091, is the excess of the mean solar day over the sidereal, expressed in mean time.

186. If we would employ the sidereal day for the unit, which is often done in astronomy, the mean solar day will be expressed by $\frac{1}{0{,}99726957}$ or $\overset{\text{sid. d.}}{1{,}00273791}$. The small difference

$$\overset{\text{sid. d.}}{0{,}00273791} \text{ or } 3' \; 56'',5554,$$

is the excess of the mean solar day over the sidereal, this excess being expressed in sidereal time. It is the quantity, by which a clock regulated according to the motion of the fixed stars, must gain from day to day, with respect to the sun, regard being had simply to its mean motion. This gain, being 3′ 56″,5554 sidereal time, in a sidereal day, is 9″,8565 for an hour, and for $\frac{1}{10}$ of an hour or six minutes, it is 0″,98565 or very nearly 1″ sidereal time.

Manner of Referring the Positions of the Heavenly Bodies to the Plane of the Ecliptic.

187. In order to determine the position of different points in the heavens, we first referred them to two planes, the horizon and meridian, which are fixed for each point of the earth's surface. But, that observations, made at different places, might be compared with each other, it was found necessary to have recourse to other planes independent of the position of the observer, and even of the figure of the earth. Accordingly, having ascertained the position of the celestial equator, and the method of estimating the angles of the meridians by corresponding portions of time, we made use of these data for the purpose of determining the position of the heavenly bodies, by means of their observed right ascension and declination.

But as most of the phenomena relating to our planetary system, take place in the plane of the ecliptic, or in planes very nearly parallel to it, we proceed to refer the heavenly bodies, in a similar manner, to this plane, such reference being of constant use in astronomy.

For this purpose we suppose great circles to pass through the several points of the heavens, perpendicular to the plane of the ecliptic; these are called *circles of latitude*. Then the position of a heavenly body is determined by two elements.

The first is the arc of a great circle comprehended between the ecliptic and the body. This arc is called the *latitude of the body*. The second is the arc of the ecliptic comprehended between the vernal equinox and the circle of latitude. This arc is reckoned, like the right ascension, from west to east, in the direction of the proper motion of the sun, and is called the *longitude of the body*, from its analogy to the longitude of the sun.

In figure 39, C represents the centre of the celestial sphere, ♈Q♎ the equator, ♈E♎ the ecliptic, ♈♎ the line of equinoxes, SA the declination of a star S, ♈A its right ascension, SL its latitude, and ♈L its longitude. When the star is situated north of the ecliptic the latitude is *north*; when it is situated south of the ecliptic the latitude is *south*. The angle PSP' is called the *angle of position* of the star.

188. We do not observe directly the latitude and longitude of the heavenly bodies, on account of the difficulty of verifying the proper instrument intended for this purpose; but we deduce them by calculation from the observed declination and right ascension. Top.105. To this end it is only necessary to resolve a spherical triangle.

The determination of the latitude and longitude of the heavenly bodies, supposes the observer placed at the centre of the earth. But as we are always confined to the earth's surface, it is necessary to apply a correction to the apparent latitude and longitude, precisely analogous to the parallax in declination and right ascension. Indeed the analogy is so complete that the formulas investigated for the latter will serve also for the former, as may be easily shown.

For this purpose we make use of a construction exactly similar 144. to that employed for the parallax in altitude. Fig. 40. Let C represent the centre of the earth, supposed to be a spheroid, COZ a radius of the earth produced, O the place of the observer, and Z the true zenith. Then P' being the pole of the ecliptic, let us suppose that the plane of the figure passes through the lines CP' and CZ. In this case, the plane of the ecliptic, which must be perpendicular to the line CP', will be projected in the figure, according to a straight line CE. The arc ZP' will be the distance of the zenith from the pole of the ecliptic, and the arc ZE, the complement of this distance, will be the *latitude of the zenith*. We here understand celestial latitude, referred to the plane of the ecliptic, and not terrestrial latitude.

If we now draw from the observer to the heavenly body *L*, a visual ray *OL*, the body will be seen at *S'* on the celestial sphere, and the arc *ZS'* will be the apparent distance of the body from the true zenith. The arc *P'S'* will be its distance from the pole of the ecliptic, or the complement of its apparent latitude; and lastly, the angle at the pole *S'P'Z*, contained between the circles of latitude drawn through the zenith and the body, will be the apparent difference of longitude of the body and the zenith. We here understand celestial longitude, reckoned on the ecliptic, and not terrestrial longitude.

But, if from the centre *C* of the earth, we draw to the same body the visual ray *CLS*, which will project it in *S* on the celestial sphere, *S* will be the true place of the body, *ZS* its true zenith distance, *P'S* its true distance from the pole of the ecliptic, or the complement of its true latitude; and the angle at the pole *SP'Z*, will be the true difference of longitude of the body and the zenith. The difference between these elements and those of the apparent place, will be the corresponding parallax. Thus the difference *SS'* between the zenith distances, will be the parallax in altitude, the difference between the polar distances *P'S*, *P'S'*, will be the *parallax in latitude*, and the angle *SP'S'* will be the *parallax in longitude.*

Thus the analogy above mentioned, is shown to be complete; for the figure here made use of, is the same as that employed for the right ascension and declination. The system of coordinates in latitude and longitude is of exactly the same nature; and therefore the consequences must be the same. Indeed, it will be readily seen that the parallax in latitude answers to the parallax in declination, and the parallax in longitude to the parallax of the horary angle or right ascension. In order then to apply the formulas already investigated, it is manifestly sufficient to give to the letters there used, the signification which we here intend they should have. We shall thus have, without any new calculation, expressions for the parallax in longitude and latitude.

Only as the angle *SP'Z*, which corresponds to the horary angle, is here the difference of longitude of the zenith and the star, it is necessary that we should be able to determine the longitude of the zenith. For this purpose we have only to ascertain the declination of the zenith, referred to the plane of the equator, and its right ascension referred to the equinoctial point; after which, we convert the coordinates into latitude and longitude, referred to the plane of the ecliptic, in the same manner as we should convert those of a star, by the formulas already given for this purpose.

The declination of the zenith is very easily obtained, it being the distance of the zenith from the equator or the latitude of the place of observation. As *CZ*, the radius of the earth, differs from the vertical, it is necessary to employ the declination of the true zenith; that is, to subtract from the terrestrial latitude the angle contained between the radius and the vertical.

As to the right ascension of the zenith, it is evidently measured by the variable horary angle contained between the equinoctial point ♈ and the fixed plane of the meridian of the place. It is the sidereal time converted into degrees on the equator. In order to obtain it, we pursue the method laid down in article 81. We ascertain the horary angle of a known star, by an observation of the altitude, this angle being reckoned from the superior meridian, and from 0° to 360°, in the direction of the diurnal motion. To this we add the right ascension of the star, and from the sum subtract the entire circumferences, if there be any, and the remainder will be the sidereal time, or the distance of the equinoctial point ♈ from the plane of the meridian.

This distance will be the right ascension of the zenith. It is what astronomers call *the right ascension of mid-heaven.* It might be obtained in a manner still more simple, by observing the transits of stars, or of the sun over the meridian, with a meridian telescope, and calculating the right ascension of the body in question for the moment of observation. For, since it is then in the meridian, its right ascension would be that of the meridian itself.

Knowing the declination and right ascension of the zenith, we can easily calculate its latitude and longitude; and as one of the two first coordinates, namely, the right ascension, varies at each instant in consequence of the diurnal motion, it will be perceived that the two others, both of which involve it, will be variable also. The latitude and longitude of the zenith change, therefore, at each instant. This might have been easily foreseen from the oblique direction of the plane of the ecliptic, with respect to the diurnal motion. But the preceding considerations furnish us with the means of obtaining for each instant the values of these variable coordinates.

Astronomers have given the name *nonagesimal* to the point E of the ecliptic, where this plane is intersected by the circle of latitude ZP', which passes through the zenith. The word *nonagesimal* signifies ninetieth. The denomination appears to have been selected because the point E is situated 90° from the point of the ecliptic which is in the horizon at the same moment. Indeed, if through the centre C of the earth we draw a plane perpendicular to the radius CZ, and consequently horizontal, this plane will be represented in the figure by the straight line CH, as the ecliptic is by the straight line CE. The intersection of these two planes will therefore be a straight line, drawn through the point C perpendicular to the plane of the figure. Consequently, the angles made by this straight line with CE and CH will be right angles.

This point of the ecliptic, thus situated in the horizon, 90° from the point E, or nonagesimal, was formerly termed the *horoscope;* and that species of superstition, called *judicial astrology*, consisted in believing that the point of the ecliptic which happened to be rising at the moment any event occurred, would have an influence upon it either favourable or unfavourable. Consequently, the position of this point in the ecliptic was carefully calculated, and according to the particular part in which it was found, good or bad omens were drawn from it. This was called *casting the horoscope.* The bare mention of this practical superstition is sufficient to show its absurdity.

189. According to the preceding definitions, the angular distance of the point E from the plane of the horizon, or the angle ECH, is called in astronomy the *altitude of the nonagesimal.* This distance is evidently equal to the angle PCZ, the distance from the pole of the ecliptic to the true zenith. These denominations being frequently employed, it is well to know what they signify. But it is evident that they are not very characteristic or necessary, and that the general expressions *latitude* and *longitude of the zenith* are much more convenient; we shall, therefore, employ them through the rest of this work, it being an evil to multiply technical terms without necessity.

Progressive Diminution of the Obliquity of the Ecliptic.—General Motion of the Stars parallel to the Ecliptic; whence results the Precession of the Equinoxes.

190. THE values assigned to the obliquity of the ecliptic by astronomers of different ages, do not agree with each other. They have been continually diminishing from the time of the earliest astronomers to the present. These differences cannot be ascribed wholly to the imperfection of instruments and observations; for had this been the case, the results obtained must have been sometimes too great and sometimes too small; and there is not the slightest probability that all would have agreed in indicating a progressive diminution of the obliquity of the ecliptic, if this diminution were not real. Indeed, the conclusion derived from observations, is completely confirmed by the theory of gravity. It is proved that the attraction of the different planets which compose our system, must necessarily change by degrees the position of the plane of the ecliptic in the heavens, and, according to the present disposition of the system, must diminish its inclination to the equator, by a quantity equal to 52″,109 in a century, or 0″,52109 in a year. In a memoir of Laplace, published in the *Connaissance des Tems*, for 1811, may be seen a comparison of the results of theory and observation.

Observations prior to the Christian Era.

Epochs of the Observations.	Names of the Observers.	Place of Observation.	Observed Obliquity.	Calculated Obliquity.	Excess of the observed Obliquity.
1100	Tcheou-Koung	China	23°54′ 02″	23°51′ 58″	+ 0°02′ 04″
350	Pytheas	Marseilles	23 49 20	23 46 07	+ 0 03 13
250	Eratosthenes	Alexandria	23 45 39	23 45 19	+ 0 00 20
50	Lieou-Hiang	China	23 45 39	23 44 03	+ 0 01 36
	Observations since the Christian Era.				
173	. . . . .	China	23°41′ 33″	23°42′ 17″	— 0°00′ 44″
461	Tsou-Choug	China	23 38 52	23 39 53	— 0 01 01
629	Litchou-Foung	China	23 40 04	23 38 17	+ 0 01 47
880	Albategnius	Arabia	23 35 41	23 35 13	+ 0 00 28
1000	Ebn-Jouni	Cairo	23 34 26	23 34 50	— 0 00 24
1279	Cocheou-King	Pekin	23 32 02	23 32 22	— 0 00 20
1437	Ulugh Begh	Samarcand	23 31 48	23 31 05	+ 0 00 43

These observations, taken together, put beyond all question the progressive diminution of the obliquity of the ecliptic. Their agreement with the results of theory, from which they vary by being sometimes greater and sometimes less, leaves no doubt that the diminution is owing solely, as the theory indicates, to the attraction exerted by the sun upon the planets, and by the different planets upon each other.

We perceive also the effects of this diminution, when we compare the positions of the same stars with respect to the ecliptic at very distant epochs. The effect is most remarkable in the stars situated near the summer and winter solstices. Those which were formerly situated north of the ecliptic, near the summer solstice, are now found to be still farther north and farther from this plane. On the contrary, those which, according to the testimony of the ancient astronomers, were situated south of the ecliptic, near the summer solstice, have approached this plane, and some are now either situated within it, or just on the north side of it. Analogous changes have taken place with respect to those situated in the neighborhood of the winter solstice. All the stars participate in this motion, but differently, and less according to their proximity to the line of the equinoxes; so that this line appears to perform the office of a hinge about which the rotation seems to take place. It is natural to conclude from these phenomena, that the plane of the ecliptic has an actual motion in the heavens, and that it thus produces, in a contrary direction with respect to the stars, the appearances we observe. For we can hardly believe that these motions really belong to the stars, since such a supposition would require among all the heavenly bodies a corresponding motion which no one would think of maintaining.

191. But it is important to observe, and in this particular we are confirmed by theoretical considerations, that the diminution of the obliquity of the ecliptic will not always continue. A time will arrive when this motion, growing less and less, will at length entirely cease, and the obliquity of the ecliptic will appear constant; after which, the displacement will commence in the opposite direction. The ecliptic will then gradually diverge from the equator by the same degrees according to which it before approached; and these alternate states will constitute an endless oscillation, comprehended within certain fixed limits. Theory has not yet enabled us to determine what these limits are, but it may

be demonstrated from the constitution of our globe, that such limits exist, and that they are very restricted. Thus, if we consider only the effect of those constant causes which act at present upon the system of the universe, it may be affirmed that the plane of the ecliptic never has coincided with the plane of the equator, and never will coincide with it; a circumstance, which, could it happen, would produce upon the earth a perpetual spring.

192. We have here spoken only of the slow and secular diminution of the obliquity of the ecliptic. This obliquity experiences, moreover, certain small oscillations, by which it varies from its mean value, in opposite directions alternately. Among these oscillations, that which is most considerable is completed in a period of 18 years; that is, at the end of 18 years, whatever depends upon this inequality is compensated, and there remains only the constant and general effect of the progressive diminution. Observation has made known the law of these small oscillations, and theory has made known their cause. They are produced by the action of the moon, and make a part of the phenomenon called *nutation.* But as their value and period depend upon the motions of this body, we shall not here undertake to explain the phenomenon; it is sufficient for the present to have noticed simply the effect thus produced upon the obliquity of the ecliptic. There is also a small effect of the same kind produced by the sun; but it is much less considerable, and takes place in the period of half a tropical year.

As we advance in the study of astronomy, we shall see that all the elements of our system are, like the obliquity of the ecliptic, subject to variations of two sorts. The first are so slow in their operation that their course appears to have been progressive from the time of the most ancient astronomers to the present. For this reason they are called *secular inequalities.* The other are more rapid, and accomplish their periods at intervals of time which are not very considerable. Several of their revolutions have already been observed by astronomers. These are called *periodical inequalities*, to distinguish them from the other, which are also periodical, but comprehended within limits very much more extended.

193. When we have determined by observation, for a certain epoch, the instant of the sun's passage through the vernal equi-

nox, it is easy to refer to this point the right ascensions of all the stars. For this purpose, it is sufficient to observe or calculate, the difference of right ascension between each star and the equinoctial point, according to the method explained in article 168. Now, if we repeat this observation, with respect to the same stars, at different epochs, comprehending intervals even of a small number of years, we shall find in them very sensible changes, the general result of which is, that the right ascensions of all the stars go on increasing, so that their meridians appear to depart continually from that of the equinox, in the direction of the proper motion of the sun. The declinations of the stars change also, at the same time; so that the equator ceases to pass through the same stars; and this change of place is so considerable that the situation of the different constellations with respect to the equator and the equinoxes is entirely different from that described by the ancient astronomers.

194. At first view, when we refer these motions to the plane of the equator, their law does not seem very manifest; but if we refer them to the plane of the ecliptic by means of latitude and longitude, and thus compare the present positions of the stars with those, observed and handed down by the ancient astronomers, we shall immediately perceive a remarkable agreement. It will be found that the latitudes of the stars have experienced no variations, or only such slight ones as would naturally result from the secular variation of the obliquity; whereas the longitudes of all the stars have been constantly increasing, and all by the same quantity. Thus it would seem as if they moved in the heavens in a direction parallel to the plane of the ecliptic; or, to express it in a rigorous and geometrical manner, the phenomena take place as they would do if the celestial sphere revolved about the axis of the ecliptic with a very slow motion, directed from west to east like that of the sun.

195. In order to measure this motion, it is sufficient to compare together the longitudes of the same star, observed at two different times. The effect being the same for all the stars, it is of no importance which we take.

For example, according to the observations of Dr Bradley, the longitude of the star called *Spica Virginis*, at the beginning of 1760, was 200° 29′ 39″.

According to the observations of Dr Maskelyne, the longitude of the same star was at the beginning of 1802, 201° 4′ 41″.

The difference is 35′ 2″ in 42 years; which gives for the yearly increase 50″,05. The comparison of a great number of observations gives for the yearly increase in longitude 50″,1. It will be perceived at once that an element so delicate cannot be established with certainty, except by taking the mean of a great number of observed results.

196. In consequence of this motion, the equinoctial points do not always correspond to the same points of the celestial sphere; and the sun employs a little less time in returning to the equator than in returning to the same fixed star. In this consists what is called *the precession of the equinoxes.*

In order to estimate this interval, it is necessary to remark, that when the mean sun has returned to the equinox after a tropical revolution of $365^d\ 5^h\ 48'\ 51'',6$, there still remains to be described upon the ecliptic, the small arc of 50″,1 before it reaches the point in the sphere of the stars which passed the equinox, the preceding year, at the same time. Indeed, this point is itself in motion on the ecliptic, in virtue of the precession; and it departs a little from the sun while that luminary is moving round to overtake it; but its annual motion being only 50″,1, the quantity by which it advances from the moment that the sun has reached the equinox until it reaches this point itself, is altogether insensible; so that we may suppose the point in question immoveable during this interval. Accordingly, in order to estimate the interval between the sun's passing the equinox and arriving at the above point, we have only to reduce to time this small arc of 50″,1, by the proportion

$$360^\circ : 365^d\ 5^h\ 48'\ 51'',6 :: 50'',1 : 0^\circ\ 20'\ 19'',9.$$

If we add this to the tropical year, we shall have the duration of a complete revolution of the sun with respect to the stars, expressed in mean solar time. This is called the *sidereal year*, and is equal to $365^d\ 6^h\ 09'\ 11'',5$.

197. From this calculation it is readily perceived that the line of the equinoxes has a retrograde motion in the ecliptic of one degree in 71,6 years.

It is likewise easy, on the supposition that this motion is uniform, to calculate the time required for the equinoxes to move entirely through the ecliptic; for, if it require a year to move through $50'',1$, the time necessary for moving 360°, must be equal to $\frac{360^{\circ}.1^{y}}{50'',1}$ or 25868 years. But the motion of precession is subject to inequalities in different centuries, which would probably affect in some degree the length of this period.

198. The discovery of the precession of the equinoxes, does not date farther back than the time of Hipparchus. Before this epoch, it was supposed that when the sun returned to the same equinox, it also returned to the same fixed stars; and as the presence of this luminary in the different parts of the heavens determined and regulated the labors of the husbandman, the ecliptic was divided, from the most ancient times, into twelve equal portions, beginning with the vernal equinox. These portions were denominated *signs*, undoubtedly on account of the labors belonging to the season when the sun was in them; for the names given to the different signs have a manifest reference to these labors. The passage of the sun through each of the signs was easily ascertained by means of the stars which are situated in the ecliptic, and which were therefore divided into twelve groups or constellations. But since this remote period the appearance of the heavens has considerably changed. The equinoxes have retrograded in the ecliptic, by the effect of the precession, and the same constellations no longer indicate the same labors. But the same division is still recognised in astronomy, and even the names of the 12 signs with the characters which represent them, are retained; but they are counted on the ecliptic from the vernal equinox, which is constantly changing its position.

These signs are, aries ♈, taurus ♉, gemini ♊, cancer ♋, leo ♌, virgo ♍, libra ♎, scorpio ♏, sagittarius ♐, capricornus ♑, aquarius ♒, pisces ♓.

They are arranged in order in the two following verses, which may be easily remembered;

Sunt aries, taurus, gemini, cancer, leo, virgo,
Libraque, scorpius, arcitenens, caper, amphora, pisces.

Each sign is a twelfth part of the circumference, and contains 30°. The assemblage of these signs forms what is termed the zodiac.

By the general consent of astronomers the first point of the sign aries always corresponds to the vernal equinox, the first of cancer to the summer solstice, the first of libra to the autumnal equinox, and the first of capricornus to the winter solstice.

199. About the time of Hipparchus, or a little before, the constellations aries, cancer, libra, and capricornus, were actually situated in the four points of the sun's orbit, above mentioned, but they have changed since that time, by the precession of the equinoxes, about 30°.

The vernal equinox now takes place in the constellation pisces, the summer solstice in gemini, the autumnal equinox in virgo, and the winter solstice in sagittarius. Each has retrograded one sign.

Hence we must carefully distinguish between the *signs of the zodiac*, which are fixed with respect to the equinoxes, and the *constellations*, which are moveable with respect to these points.

200. Hitherto we have ascribed to the stars this continued motion of precession, considering them as all moving together parallel to the ecliptic, and constantly changing their relative positions with respect to the equator. But, if we have considered these changes of place as real, it is only because we have found it convenient to regard the equator as a fixed and immoveable plane, to which we may refer the positions of all the stars, by means of their right ascensions and declinations. For, the appearances would be absolutely the same, if, instead of attributing these motions to the stars, we were to consider the earth as moving in a contrary direction. According to this supposition, it is the celestial equator which changes its position together with the earth, moving backward along the ecliptic in the inverse order of the signs, in such a manner that the common intersection of these planes, or the line of the equinoxes, shall describe annually upon the ecliptic an arc of 50″,1, in a direction contrary to the proper motion of the sun. In this case, the stars would be really stationary, and the earth only would be in motion; the celestial equator would change its place with respect to the stars, and not the stars with respect to it. According to this supposition, we must also consider the axis of the diurnal motion as changing its place at the same time with the plane of the equator, to which it always remains perpendicular; for the elevation of the pole at each place, remains invariably

the same, although it does not always correspond to the same stars; consequently the direction of the diurnal motion, being constantly the same with respect to each horizon, must change with it. But this displacement of the axis of rotation of the heavens, far from appearing extraordinary, is only an additional argument tending to confirm us in the supposition that the diurnal motion of the heavens is only apparent, and that the appearance is caused by the motion of the earth in a contrary direction. If we remember what has already been said respecting the illusions produced by the relative motions of a system of which the observer forms a part, the idea of giving to the earth a motion of rotation upon itself, about an axis fixed relatively to its surface, and to this axis a motion of precession in the heavens, offers nothing either impossible or strange. And after all, this mode of explaining the celestial phenomena is still more simple, than that of supposing throughout the whole system of the heavenly bodies, so many general and common motions of rotation and precession, which, considering the multitude of these bodies and the variety of their distances, would require in their particular motions, a connexion and correspondence which no one would think of maintaining. Besides, all the phenomena which occur as we advance in the study of astronomy, conspire to show that this latter supposition is not the true one. In the mean time, until their developement furnishes us with a series of inductions not to be resisted, it is well to be apprised of the possibility of this double mode of explanation.

201. The theory of universal attraction has taught us that the phenomenon of the precession of the equinoxes proceeds from the attraction exerted by the moon and sun upon the oblate spheroid of the earth. This attraction, being unequal upon the different portions of the spheroid, on account of the oblateness, tends continually to draw the plane of the equator out of its direction, and causes it to retrograde upon the ecliptic, conformably to the laws which we have deduced from observation. If the earth were exactly spherical, this effect would not be produced; no precession would take place, and the equinoxes would always correspond to the same points of the ecliptic, at least so far as the kind of action we have been considering, is concerned.

Since the attractions exerted by the sun and moon upon the terrestrial spheroid, vary with the positions of these bodies, there result certain small oscillations in the precession of the equinoxes, by which it sometimes increases and sometimes diminishes. The period of these oscillations is different in the case of the sun from what it is in that of the moon; it depends upon the time required in the body which produces them, after leaving a certain position in which it exerts a given influence, to arrive at a position in which it shall exert the same influence. The inequalities of this kind, produced by the action of the sun, have for their period half a tropical year; those which are produced by the moon, have a period of 18 years. Both are connected with corresponding oscillations in the obliquity of the ecliptic, as above mentioned; and they make a part of the phenomenon of nutation, the laws of which will be explained hereafter.

Moreover, the motion of the equinoxes is also modified by another cause. We have already mentioned that the attraction exerted by the planets upon each other and upon the sun, gradually changes the direction of the plane of the ecliptic in absolute space. This change causes a variation in the inclination of the ecliptic to the equator, as before remarked. At the same time it causes also a change of place in the points of intersection of these two planes, that is, in the line of the equinoxes. This foreign motion is combined with that which the attractions of the sun and moon would produce if they were the sole agents. In the present disposition of the planetary system, the displacement of the ecliptic, causes in the equinoxes an annual motion of 0″,31. This motion is *direct*, that is, in the direction of the sun's motion in longitude. It is, consequently, opposite to the *retrograde* motion caused by the sun and moon. Hence it appears that the annual precession which we observe is strictly the difference between these two contrary motions; and without the displacement of the ecliptic, the observed precession would be equal to 50″,1 + 0″,31 or 50″,41. These effects of the action of the planets are in themselves independent of the figure of the earth. But they have, nevertheless, an indirect influence, which is connected with this figure. For, as they displace the plane of the ecliptic which is the orbit of the sun, they bring this luminary into different positions with respect to the terrestrial spheroid, and thus modify the action which it exerts upon this spheroid, in virtue of the oblateness. The position of the orbit of the moon which always accompanies the ecliptic, is affected, in like manner, and by the same cause; so that the action of this body also varies. Hence arise new modifications in the motion of the equinoxes, and the obliquity of the ecliptic. These secondary effects modify considerably the results which would have been produced by the action merely of the sun and moon upon the terrestrial spheroid, on the supposition that the plane of the ecliptic is immoveable.

These variations which take place in the action of the sun, in consequence of the displacement of the ecliptic, are periodical like the displacement itself. The inequalities in the motion of the equinoxes and in the obliquity of the ecliptic, which result from these variations are, therefore, periodical also. These inequalities are analogous to the lunar nutation, but much slower. They may be termed the *secular nutation*. As they are not proportional to the time, they render the precession unequal in different centuries. But these variations are so slow, that observation alone would not be sufficient to detect them, especially on account of the imperfection of the ancient determinations, which it would be necessary to make use of in order to estimate and compare the precession of the equinoxes at different epochs. From the operation of this cause, the annual retrogradation of the equinoxes is now greater by about 0″,455 than it was in the time of Hipparchus. Consequently, the mean tropical year is shorter now by 11″,076 than it was then. For, since the retrogradation of the equinoctial points is greater now than formerly, the sun has, each year, a space of 0″,455 less to pass through in the ecliptic, in order to reach the plane of the equator. The mean tropical year, which is measured by the interval comprehended between two successive returns of the sun to the same equinox, is therefore less by the time required to describe on the ecliptic an arc of 0″,455 with its mean motion, which shortens the tropical year 11″,076, as may be proved by a simple proportion. In order that this diminution in the tropical year may be real, it is necessary to admit that the sidereal year with which we compare it, and the sidereal day by which we measure it, are constant periods. Indeed they are demonstrated by theory to be invariable, as has been before observed.

These results of theory, by giving to our observations and calculations an exactness which could not have been obtained without this aid, serve to throw light at the same time upon the true laws of the phenomena; it is for this reason that it was deemed proper to present them here by way of anticipation.

202. The progressive changes which take place in the longitude of the stars, in consequence of the precession of the equinoxes, extend to the declination and right ascension, and alter the value of these two elements. But we have already seen that the arcs which express them are connected with those of the latitude and longitude, by means of spherical triangles; we are thus enabled to ascertain their value. Hence we are able also to calculate the effect which a small change of longitude produces in the right ascension and declination. We accordingly obtain the *precession in right ascension and declination.*

203. If the plane of the ecliptic were immoveable in the heavens, and its inclination to the equator constant, the preceding considerations would be sufficient to enable us to determine, for any given epoch, the position of a star with respect to these two planes, provided this position had been once observed. It would only be necessary to make, in the longitude of the star, the change required by precession, and with this corrected longitude, the latitude remaining the same, we might calculate the right ascension and declination.

But the latitude of the stars cannot remain constant, because the ecliptic also varies its position in the heavens. The variations in latitude, therefore, combine with those in longitude, resulting from the precession of the equinoxes; and accordingly, in order to ascertain the place of a star at any given epoch, regard must be had to these simultaneous changes.

We have already remarked, that the theory of attraction explains, in the most perfect manner, the laws of these phenomena. Although we cannot here present the proofs, upon which these explanations rest; we shall, nevertheless, make known the principal results; for this affords the only means of forming clear and distinct ideas respecting the displacement of the ecliptic, and the motion of the equinoxes.

204. For this purpose, let E♈♈$'$ be the position of the ecliptic Fig. 41. at a given epoch, for instance, at the beginning of 1750, a year rendered memorable by the labors of Lacaille, and adopted as the origin of several astronomical calculations. Let ♈Q represent the equator at this same epoch, and ♈ the equinoctial point; so that the longitude shall be reckoned from ♈ towards

E, and the right ascensions from ♈ towards *Q*. Let us now suppose another epoch subsequent to the preceding. The equator will have retrograded in consequence of the precession. Let then, ♈′*Q*′ represent its new position at this epoch, and ♈′ its intersection with the ecliptic, considered as having the same position as in 1750; the spherical angle *E*♈′*Q*′ will be its inclination to the ecliptic. But then the true ecliptic will have varied its position in the heavens, and will have taken, for example, the direction *NE*′, making with the preceding a spherical angle *ENE*′. In consequence of this variation it will cut the new position of the equator in ♈″, and the point ♈″ will be the new position of the true equinoctial point; so that the longitude will be reckoned from ♈″ towards *E*′, and the right ascensions from ♈″ towards *Q*′.

Now, supposing the obliquity of the ecliptic to have been observed at the first epoch, the theory of attraction makes known for any other epoch; 1. the arc ♈♈′, or the precession of the equinoctial point upon the fixed ecliptic of 1750. 2. The angle *E*♈′*Q*′, or the obliquity of the equator to this same ecliptic. 3. The arc ♈′♈″ or *the motion of the equinoctial point in right ascension.* 4. The obliquity of the equator to the moveable ecliptic, or the angle *E*′♈″*Q*′. With these data it is easy to transfer the catalogues of the stars from one position to another, and to calculate what will be, or what must have been, the latitudes, longitudes, right ascensions and declinations. This process is, strictly speaking, nothing but a simple transformation of coordinates.

Indeed, knowing the longitudes l, and the latitudes λ, referred to the primitive position of 1750, we add to the longitudes the arc of precession ♈♈′ on the fixed ecliptic, which arc we shall call ψ; the longitudes $l + \psi$, and the latitudes λ, will determine the position of the stars with respect to the fixed ecliptic for the new epoch in question. Since we know also the angle *E*♈′*Q*′, the obliquity of the equator to this ecliptic at the same epoch, we can calculate, with these data, the right ascensions and declinations with reference to the new position ♈′*Q*′ of the equator. We shall call them a' and d'. The necessary formulas for obtaining them will be found in the Appendix.

But these arcs are reckoned from the equinoctial point ♈′. In order to reduce them to the true equinox ♈″, it is sufficient to subtract from all the right ascensions the arc ♈′♈″, which expresses the motion of the equinox in right ascension. Let this arc be α'. Then the right ascensions $a' - \alpha'$, and the declinations d', will determine the elements of the stars with reference to the new position of the equator and of the true equinoctial point ♈″; and as we know also the angle E'♈″Q', which expresses the apparent obliquity of the ecliptic to the equator for the new epoch, we can easily calculate the longitudes and latitudes for this same epoch, with reference to the moveable ecliptic. By this means we shall have the right ascensions, declinations, longitudes and latitudes of the stars, referred to the new position of the equator, ecliptic, and equinoctial point ♈″.

We have supposed the new epoch to be later than 1750. If it were earlier, the process would be the same, the signs only being changed. The precession ♈♈′, instead of being added to the longitudes, would require to be subtracted, and with respect to the motion of the equinoctial point, instead of subtracting it from the right ascension a', we should add it. The same formulas are applicable in all cases, the time being considered as positive for epochs subsequent to 1750, and negative for others.

205. We have said that the *sidereal day* had for its measure the interval of time comprehended between two consecutive returns of the same star to the meridian, this interval being supposed to be corrected for the effects of precession, aberration, and nutation. This, in other words, is equivalent to referring the star to the mean equinox ♈″, and considering it immoveable with respect to this point during the interval of a day. Consequently, the duration of a sidereal day is really measured by two consecutive returns of the mean equinox ♈″ to the meridian. The point ♈″ is here called the *mean equinox*, to distinguish it from the true equinox, which oscillates about it in virtue of the nutation.

But, according to what has been said respecting the secular displacement of the ecliptic, the equinoctial point ♈″, does not always correspond to the same physical point of the equator. It is, on the contrary, moveable upon this great circle, and its displacement, after a given interval of time, is measured by the arc ♈′♈″, as we have seen in figure 41. The small portion of this proper motion, which corresponds to one rotation of the heavens, makes, therefore, in reality a part of the sidereal day;

in the same manner as the diurnal arcs, described upon the equator by the sun, in virtue of its proper motion, make a part of the duration of a solar day. If the motion of the equinoctial point ♈″ were sensibly unequal, the sidereal days would themselves be unequal in duration, like the solar days. But this motion is so slow that the effect of its variations would scarcely amount to minutes in the total duration of several millions of years. The influence of these variations on the determination of absolute time, is therefore, altogether insensible, for the periods which astronomers have hitherto had occasion to consider. Accordingly, neglecting them, we may always consider the sidereal day as a constant period of time, the duration of which is measured by two consecutive returns to the meridian, of the equinox ♈″, moving uniformly.

A mere inspection of figure 41 will enable us to account for a phenomenon before mentioned; namely, that the displacement of the ecliptic, caused by the attraction of the planets, modifies in some degree the precession which we observe, and renders it somewhat smaller than that which takes place on the fixed ecliptic. To be convinced of this, let ♈, be the physical point of the moveable ecliptic, which, at the epoch of 1750, coincided with the point ♈ of the fixed ecliptic. When, after a certain lapse of time, these two planes become detached from each other, the true equinox comes to ♈″, and the arc ♈,♈″, reckoned on the moveable ecliptic, is the apparent precession which we observe. It remains to prove that this arc is less than ♈♈′, which represents the precession on the fixed ecliptic in the same interval. This is very easily done; for, since ♈ and ♈, corresponded originally to the same physical points on the two ecliptics, the arcs *N*♈, *N*♈,, reckoned from the point *N* where these two planes cut each other, are equal. Now the part *N*♈″ of the last, is evidently greater than the arc *N*♈′ of the other, since the acute angle *N*♈″♈′, expressing the obliquity of the moveable ecliptic to the equator, is less than the obtuse angle *N*♈′♈″, which expresses the supplement of the obliquity of the same equator to the fixed ecliptic. Consequently ♈,♈″ must be less than ♈′♈, that is, the apparent precession must be less than that which takes place on the fixed ecliptic. The point ♈″ is here regarded as the true equinox, since we leave the nutation out of consideration.

If from the true equinox $♈''$ we draw the arc of a great circle, $♈P''$ perpendicular to the fixed ecliptic; it follows from the smallness of the angle N that the arcs $♈P$, $♈,♈''$, will be very nearly equal to each other. Thus, calling ψ the precession upon the fixed ecliptic, and ψ' the precession on the moveable ecliptic, the arc $♈'P$ will be nearly equal to $\psi - \psi'$. Moreover, in this same triangle $♈'P♈''$, we know also the angle $P♈'♈''$, expressing the obliquity of the equator to the fixed ecliptic. With these data we can calculate the side $♈'♈''$, which expresses the *secular motion of the equinoctial point in right ascension*, and $P♈''$, which expresses its *secular motion in latitude.* But as these arcs are extremely small, even when we go back to the most ancient observations, it is sufficient to resolve the small triangle $♈'♈''P$, as if it were a plane triangle. We thus find $♈'♈'' = \frac{\psi - \psi'}{\cos V}$; and $♈''P = (\psi - \psi')$ tang V; designating by V the angle $♈♈'♈''$; that is, the obliquity of the equator to the fixed ecliptic. We hence perceive that these quantities are connected with the motion in longitude, and also how they depend upon this motion. It is to be observed that the motion of the equinoctial point in right ascension, is directed from east to west, and that its motion in latitude is from north to south of the fixed ecliptic.

206. In the Appendix may be found the exact formulas, furnished by theory, for calculating these values with reference to any epoch whatever, and for determining all the other elements of the apparent places of the stars. The operation reduces itself to a simple change of coordinates. But when the period in question embraces only a small number of centuries, before or after the epoch of 1750, which we have selected as the point of departure, the calculation is simplified by the consideration that the variations which take place in the coordinates are very inconsiderable, and we may make use of general and very simple formulas to express the slight corrections, necessary to be introduced into the primitive elements, in order to transfer them to the epoch fixed upon. In the Appendix we have proceeded in this manner in determining the effects of precession upon the right ascension and declination of stars for a short interval of time; and generally this mode of approximation is of constant use in astronomy, in reducing observations of the heavenly bodies, taken at

different periods, to the same position of the equator and equinox, for the purpose of rendering them comparable with each other.

This simplification is founded upon a principle of analysis, which may be advantageously employed in a variety of circumstances. When we examine the simultaneous effects of several causes of minute variations, this mode of approximation allows us to estimate each effect separately, and to consider the total effect as the sum of these partial results. It is thus that several simultaneous sounds may agitate the air at once and remain distinct. Thus too, on the surface of a tranquil lake, when small undulations are propagated circularly from any point, if we excite a new agitation in another point, the undulations here created, will spread over the surface, as if it had not been troubled, and at the points where they cross each other, the total elevation of the small wave, will not differ sensibly from the sum of the separate elevations. The same is true generally of very slight motions, and this is the principle which facilitates the approximation.

207. When we wish to transfer the positions of stars from one epoch to another little distant, the change in the position of the equator and ecliptic, and the change in the obliquity of the equator to the fixed and the moveable ecliptic, are all so small that the principle just laid down, may be employed in estimating them. Thus when we have calculated the absolute value of these changes for the period in question, it will be sufficient to estimate separately the effect which each is capable of producing upon the coordinates of the heavenly bodies. The sum of the partial effects will express the total variation which takes place in these coordinates. By this means the calculation is rendered much more short and simple, and yet furnishes a sufficiently close approximation, when the period does not embrace more than two or three centuries.

Nutation.

208. In the foregoing section we have examined all the slow and secular variations, which affect the obliquity of the ecliptic and the position of the equinoctial points. We have seen in

what manner the effects which these variations produce upon the right ascensions and declinations of the stars may be estimated. By taking account of these effects, and making allowance for them, in the results obtained for the right ascensions and declinations, observed at different epochs, every thing will be brought to the same condition as if the equator and ecliptic were immoveable. Consequently, if these planes and the heavenly bodies referred to them, have no other motions than those of which we have been speaking, it should follow that the right ascension and declination of any given body, when thus corrected, must always preserve precisely the same values, at whatever epoch they may be observed.

Now this is not exactly realized; there still remain some slight periodical variations, for which allowance must be made, in order to obtain constant coordinates. This discovery was made by Dr Bradley.

The first of these variations and that which was first discovered, is called *the aberration of light.* It is produced by the motion of the earth, which, by causing the luminous particles emanating from the heavenly bodies, to meet us in an opposite direction, gives us a sensation, compounded of this motion and the proper motion of light, which, although very rapid, is not instantaneous. We shall not now inquire into the laws of this phenomenon. It is sufficient for the present to know that the aberration prevents us from seeing the heavenly bodies in their true places; and that this illusion being subjected to fixed and known laws, may be corrected by calculation; and if allowance be made for it, the condition of things becomes the same as though it did not exist.

209. Still this correction is not sufficient to render the right ascensions and declinations constant. They experience also other changes. The simplest mode of proceding then, is to ascertain whether these changes can be ascribed to a small variation in the obliquity of the ecliptic, and in the position of the equinoctial points. Now we have examined, in the preceding section, the effects of indeterminate variations of these two elements, and in the Appendix have given formulas for calculating the changes they produce in the right ascension and declination of the stars. Since these changes are here given, we have only to introduce their values into our formulas, and take for the unknown quantities, the small variations of the ecliptic and equi-

noxes. Thus we arrive at the values of these unknown quantities, and if all the stars agree in assigning them the same value, we conclude that the small motions observed in the stars are owing to a similar cause. This was the course pursued by Dr Bradley, who has shown by a variety of examples, that the agreement is as exact as could be desired.

In this manner it was proved that the phenomenon was common to all the stars, and depended solely on the variations of the two elements which we have been considering. Still this was not sufficient for determining completely the law of the phenomenon. It remained to ascertain whether these variations of obliquity and precession, were indefinitely progressive, or whether they were periodical; and in the latter case, it was necessary to determine their period. This also Dr Bradley did. He observed that these variations had an intimate connexion with the positions *of the ascending node of the moon* in the ecliptic; he saw that they were subject to the same periods, and finally he ascertained how they depended on the nodes of the moon.

To understand this dependence it is necessary to know that the moon, as will be shown hereafter, moves like the sun in an orbit, the plane of which passes through the centre of the earth. This plane, or rather the great circle of the celestial sphere which represents it, cuts the ecliptic, which is also a great circle, in two opposite points, called the *moon's nodes.* They are to the orbit of this body what the equinoxes are to the plane of the equator. These nodes do not always correspond to the same points of the ecliptic. They have, like the equinoxes, a retrograde motion upon this great circle, but it is much more rapid; for they make the circuit of the ecliptic in 18 years and 214 days nearly; while the equinoxes require about 26000 years. That node through which the moon passes in coming from the south to the north side of the ecliptic, is called the *ascending node,* and the other the *descending node.* The first is analogous to the vernal equinox, and the second to the autumnal.

The theory of universal attraction has made known the reason why the periodical variations, observed by Dr Bradley, in the obliquity of the ecliptic and the position of the equinoxes, are connected with the position of the moon's nodes. We have already stated that they are produced by the attraction of this body, which causes the equator of the earth to oscillate. Hence we have been enabled to understand also why the two motions of obliquity and of the equinoxes, are connected together, a connexion already known as a fact from observation, but which could only be established experimentally, without our knowing whether the relations which it indicated were rigorous or simply the result of approximation. It is to D'Alembert that we owe this important confirmation of the theory of universal attraction. It has been also found that the attraction of the sun produces a similar effect but much more feeble.

The phenomenon of nutation being thus reduced to a small change in the obliquity of the ecliptic and the position of the equinoxes, caused by a derangement of the equator, nothing can be easier than to calculate the effects which it produces upon the declinations and right ascensions of the stars.

210. But why has it been called nutation? This can be explained by a geometrical construction, in which its effects may be represented. Let *C* be the centre of the earth and of the celestial sphere, ♈*Q*♎ the equator, ♈*E*♎ the ecliptic, *P* the north pole of the first of these two circles, and *P′* the north pole of the second; the point ♈ will be the vernal equinox, ♎ the autumnal equinox, and the angle *PCP′* will be the obliquity of the ecliptic to the equator. This being supposed, if, without changing this angle, we cause the axis *CP* to describe a conical surface about the axis *CP′*, so that the pole *P* shall describe the circumference of a circle perpendicular to this axis, this motion will transfer the intersection ♈ of these two planes through the whole extent of the ecliptic without changing their mutual inclination. Consequently, if this rotation be made in the direction ♈♓♒, contrary to the order of the signs, and if we give to the pole *P*, upon its circle, the annual motion of the equinoxes, the intersection ♈ will retrograde on the ecliptic in the same manner, and this construction will exactly represent the *mean precession.* Fig. 42.

But since the obliquity of the equator to the ecliptic, is not perfectly constant in different centuries, the angles *PCP′* at the centre of the cone must be made to vary in such a manner as to follow and represent these changes; or, more simply, the points *P* and *P′*, must be considered as the *mean poles* of the equator and ecliptic, about which the true poles move, approaching and receding alternately, according to the inequalities to which they are subject.

Fig. 43. Accordingly, let P, P', represent the poles of the equator and ecliptic, determined for a certain epoch, as we have described, regard being had to the secular inequalities only. In order to represent the periodical inequalities of nutation, the *true pole* ϖ of the equator must be supposed to move about the pole P, conformably to the laws deduced from observation. According to these laws, when the ascending node of the moon is in ♈ on the ecliptic, that is, when it corresponds to the vernal equinox, the apparent pole ϖ, is found to correspond to the summer solstice, 90° in arrear. In proportion as the node retrogrades on the ecliptic, the apparent pole ϖ follows its motion, and thus turns about the true pole P while the node makes the circuit of the ecliptic. Its orbit is a small ellipse, whose transverse axis $\varpi\varpi''$, always remains a tangent to the circle of latitude PP', drawn through the poles of the equator and ecliptic, and occupies on this circle an arc of 19″,26. The conjugate axis of this ellipse, being perpendicular to the transverse, is a tangent to the circle $PP_{/}P_{//}$ in which the pole of the equator moves parallel to the ecliptic; and since it is very small, it may even be considered as making part of this parallel, on which it occupies an arc

36″,01 of ~~100° 01′ 88″~~,6. To obtain for any instant the position of the apparent pole ϖ in this ellipse, we make the following construc-

Fig. 44. tion, the elements of which are determined by calculation. Suppose a circumference of a circle $\varpi K \varpi''$, concentric with the ellipse of nutation, and having its transverse axis for a diameter; in this circumference suppose a radius ϖP, first situated in ϖ at the extremity of the transverse axis nearest the ecliptic, when the ascending node of the moon is in the vernal equinox. Then cause this radius to pass round the circle with a retrograde motion equal to that of the node in the ecliptic. If in each position of this radius, we draw through the extremity K an ordinate KR to the circle, the point ϖ'', where this ordinate meets the ellipse of nutation, will be, for that instant, the place of the apparent pole. This construction is only the enunciation of the results furnished by calculation and subjoined in the Appendix.

This oscillatory motion of the pole is very well designated by the word *nutation*. We have said that the sun also produces a similar oscillation but much smaller; it may, therefore, be represented geometrically in the same manner. The combination of these two makes what is called the *luni-solar nutation*. Lastly, the attraction of the moon produces also another inequality of the same kind, the period of which is half a month; but it is so small that it is unnecessary to take account of it. Now, that we know the laws of these oscillations, we can suppose them to have been all along employed in reducing the positions of the stars to comparable terms; and, if we have sometimes, by way of anticipation, made use of this knowledge, for the purpose of obtaining, in the shortest manner, the definite results of observation, it is evident that we have only anticipated what we should in the end have been obliged to do by a circuitous process, when we had made ourselves acquainted with these phenomena.

We shall close this section by remarking that the phenomenon of nutation is not regulated by the motion of the moon's *apparent node*, which is variable in the ecliptic, but by the motion of the *mean node*, that is, the node considered apart from its periodical inequalities. This is a theoretical result which cannot here be explained. Some astronomers, from inattention to this point, have thought they had found an error in the formulas hitherto used for calculating the nutation, and which are regulated by the mean node; but no such error exists.

Second Approximation to the Motions of the Sun.—Theory of its Elliptical Motion.

211. In what precedes, we have considered the sun as passing each year through a great circle of the celestial sphere. We have determined the position of this circle, and furnished the means of ascertaining its variations. We are now to examine more critically the motions of the sun in this plane. We have already seen the method of determining by observation, the extent of the arcs described each day, as well as of collecting these results together and forming tables, by means of which the successive positions of this luminary on the celestial sphere, may be predicted. But results thus insulated, would never be free from error and uncertainty; and their changes, if they experience any, could only be corrected by continually comparing

them with new observations. In order to make the requisite corrections with greater security and facility, it becomes necessary to investigate rigorously the law which governs them; or at least to approximate as near to it as possible, by some simple geometrical construction; for the mutual dependence of the facts being once known, a sufficient degree of accuracy will be attained by the precise determination of a small number of elements. This dependence is evidently founded upon the motion of the sun in space. Let us, therefore, endeavour to ascertain the laws of this motion.

212. For this purpose two kinds of data are necessary; the angular motion of the sun, deduced from the measurement of its altitudes; and the variations of its distance, computed from observations of its apparent diameter. Let us combine the consequences which are hence derived.

In the first place the angular motion of the sun in its orbit, the ecliptic, is not uniform, but sometimes slower, and sometimes faster. This is made evident, by calculating from day to day its longitude, for the moment of its passage over the meridian, by means of its observed right ascension and declination. For these meridian longitudes do not increase uniformly, and their differences from one day to another, are not proportional to the intervals of time, which elapse between the consecutive passages of the sun over the meridian. Repeated observations prove that the greatest differences take place in two points of the ecliptic, situated the one toward the winter, the other toward the summer solstice.

In the first, the sun describes an arc of the ecliptic equal to $1^\circ\ 01'\ 09'',95$ during a mean day; its velocity is then the greatest. This happens at present about the 31st of December. In the second, it describes only $57'\ 11'',48$ in a mean day; then its velocity is least; this takes place at the present time about the first of July. Its mean velocity is equal to

$$\frac{1^\circ\ 01'\ 09,''95 + 57'11'',48}{2}, \text{ or } 59'\ 10'',72.$$

The above two points correspond to equal declinations on opposite sides of the equator, and their right ascensions differ 180°. They are, therefore, situated in the same straight line, drawn through the centre of the earth and the celestial sphere. Their longitude varies a little from time to time, which shows that they are in motion with respect to the equinoxes. We shall soon see how to make allowance for this change.

213. We now come to the apparent diameter of the sun. This observes the same periods as the angular velocity; that is, it increases and diminishes with the velocity, but in a less ratio. It attains its *maximum* at the point where the velocity is greatest, and is then 32′ 35″,56. It reaches its *minimum* at the point where the velocity is least, and is then equal to 31′ 30″,9. The mean is 32′ 03″,23. These estimates should be diminished by some seconds in consequence of *irradiation*, which dilates a little, the apparent diameters of objects; but the exact quantity of this dilatation is not yet well ascertained.

214. By means of these observations we can determine the greatest and smallest distances of the sun from the earth, or at least their ratio; for we know that the distances of the same object are reciprocally proportional to its apparent diameters. Accordingly, if we compare the greatest and least apparent diameters of the sun, their ratio will be $\frac{32' \, 35'',56}{31' \, 30'',9}$ or 1,03416; that is, if 1 represent the smallest distance, the greatest will be 1,03416. The arithmetical mean between these distances is 1,01708. If, according to the practice of astronomers, we consider it as unity, the smallest will have for its expression

$$\frac{1}{1,01708}, \text{ or } 1 - \frac{0,01708}{1,01708};$$

and the greatest

$$\frac{1,03416}{1,01708}, \text{ or } 1 + \frac{0,01708}{1,01708};$$

which are reduced to 1 — 0,01679 or 0,98321 and 1,01679. *The distance of the sun from the earth therefore varies annually by a quantity nearly equal to the one hundred and sixty-eight ten thousandth part of its mean value.*

215. The points of the sun's orbit which correspond to its greatest and least distances from the earth, are called the *apogee* and *perigee*, two Greek words, which signify *from the earth* and *about the earth.* They are designated by the common name of *apsides.*

216. From these results it appears that when the sun approaches the earth, its velocity increases, and when it recedes from the earth, the velocity diminishes. But we have not yet

determined whether these variations are real or only apparent, for from the very circumstance that it approaches, the arcs it describes in its orbit must appear greater, and when it recedes, they must appear smaller. In order to compare these arcs with accuracy, we must refer them to the same distance. The arc which has for its measure 0° 57′ 11″,48, and which the sun describes in its apogee, would have appeared greater if it had been observed at the perigee. In order to refer it to this point, it is sufficient to multiply it by the inverse ratio of the radii at these two points, that is, by $\frac{1,01679}{0,98321}$ or 1,03416; for its apparent magnitude must be reciprocally proportional to its distance. We thus obtain 0° 59′ 08″,77 for the measure of this arc, as seen at the distance of the perigee. Now the arc actually described at this point by the sun, is equal to 1° 01′ 09″,95, as we have seen above, and consequently exceeding the former by 0° 02′ 01″,18; thus the apparent diminution of the sun's motion at the apogee, is not to be ascribed to its increased distance merely; but arises also from an actual retardation in its motion, as it recedes from the earth.

217. If, in like manner, we compare the angular motions and apparent diameters of the sun for other epochs in the year, we shall find this general law; *the angle described each day, being multiplied by the square of the distance, gives a product very nearly constant.* Consequently the angular motion diminishes very nearly as the square of the distance increases.

218. When we have thus obtained a simple law, which extends with very inconsiderable and irregular errors, to a great variety of observations, independent of each other and separated by long periods of time, we may regard it as more exact than the observations themselves; for, if it were not really a law of nature, it is incredible that it should represent the observations so accurately; indeed it is in the highest degree probable that it would go on varying from them more and more; and this probability amounts to absolute certainty, when the number of observations is greatly multiplied, as is the case in astronomy.

219. With the results now obtained, we can calculate the ratios of the distances of the sun from the earth, in any two points of its orbit, without having recourse to observations of its apparent diameter; for these distances will be reciprocally pro-

portional to the square roots of the daily angles described on the ecliptic. For example, if the perigee distance be supposed equal to 1, the apogee distance will be

$$\frac{\sqrt{1^\circ\ 01'\ 09'',95}}{\sqrt{0^\circ\ 57'\ 11'',48}}, \text{ or } 1,0341\ ;$$

as deduced above from the values 1,01679 and 0,98321, which represent these distances.

This result, being expressed geometrically, makes known a very remarkable law respecting the motion of the sun. We draw from the centre of the earth to the sun a straight line, called the *radius vector*. This straight line will describe each day in the solar orbit a small sector, the area of which will be nearly equal to half the product of the daily arc by the square of the radius vector. This area is therefore constant; and the total area thus described, beginning from a fixed point, increases in proportion to the number of days elapsed. This is found to be confirmed in the most rigorous manner, when we calculate exactly by analysis any elliptical sectors, and compare them with the times employed in describing them. Hence we derive this conclusion, *the areas described by the radius vector of the sun, are proportional to the times.* This is one of the great laws discovered by Kepler, and serves as the basis of the theory of the sun and planets.

220. By following this law and combining with it the observation of the daily angles, we may trace the curve described by the sun in the plane of the ecliptic. For this purpose, from a given point representing the centre of the earth and the celestial sphere, we draw upon a plane straight lines, whose angular distance is equal to the angular motion of the sun for the interval of a day. These lines will represent the visual rays drawn to the sun each day. We set off on their direction from the fixed point, the corresponding distances of the sun from the earth, deduced from the daily motion, taking one of them for unity. The points determined in this manner, will indicate the place of the sun for each day, and the curve which unites them, will represent the sun's orbit.

221. Let us take, as an example, the 24 following observations made at Greenwich, by Dr Maskelyne. They are a part of that series employed by M. Delambre, in constructing his first tables of the sun.

Dates of the Observations, 1775.		Sidereal Time at Greenwich.	Observed Longitudes.	Difference of Times.	Difference of Longitudes.
		h ' ''	° ' ''	h ' ''	° ' ''
January	12	19 36 02,8	292 13 36,7	24 04 19,2	10 1 06,7
	13	19 40 22,0	293 14 43,4		
February	17	22 03 30,8	328 43 47,2	24 03 51,4	1 00 25,8
	18	22 07 22,2	329 44 13,1		
March	14	23 37 06,2	353 45 49,1	24 03 39,1	0 59 37,0
	15	23 40 45,3	354 45 26,0		
April	28	2 22 19,9	37 57 13,4	24 03 47,7	0 58 14,3
	29	2 26 07,6	38 55 27,7		
May	15	3 28 03,7	54 23 10,9	24 03 55,8	0 57 27,4
	16	3 31 59,5	55 20 38,3		
June	17	5 42 29,8	85 59 05,1	24 04 09,6	0 57 16,7
	18	5 46 39,4	86 56 21,7		
July	18	7 50 07,3	115 33 12,0	24 04 00,7	0 57 16,0
	19	7 54 08,0	116 30 28,0		
August	20	10 19 24,2	152 53 48,0	24 03 39,7	0 57 57,2
	21	10 23 03,9	153 51 45,2		
September	22	11 57 14,7	179 14 56,9	24 03 35,7	0 58 49,3
	23	12 00 50,4	180 13 46,2		
October	24	13 58 55,6	211 54 24,2	24 03 50,7	1 00 00,6
	25	14 02 46,3	212 54 24,8		
November	18	15 38 57,4	237 02 04,3	48 08 22,8	2 01 24,2
	20	15 47 20,2	239 03 28,5		
December	17	17 40 15,4	265 28 14,0	24 04 26,3	1 01 07,4
	18	17 44 41,7	266 29 21,4		

222. These longitudes are all reckoned on the ecliptic from the same straight line, drawn from the centre of the earth to the vernal equinox, conformably to the general usage of astronomers. The difference between the observations made on consecutive days, gives the daily angles, and the square roots of the ratios of these angles, make known the distances of the sun from the earth at the different times. By referring these distances to the intermediate instant between the two observations of each month, we form the following table, in which 1 represents the distances corresponding to the mean angular velocity 59′ 10″,72. The observations are taken exactly as they were made, without any modification, that the results might be independent of any anticipated knowledge.

Dates of the Observations.		Longitudes of the Sun.			Distances from the Earth.
		°	′	″	
January	12 to 13	292	44	10,0	0,98448
February	17 to 18	329	14	00,1	0,98950
March	14 to 15	354	15	37,5	0,99622
April	28 to 29	38	26	20,6	1,00800
May	15 to 16	54	51	54,6	1,01234
June	17 to 18	86	27	43,4	1,01654
July	18 to 19	116	01	50,0	1,01658
August	26 to 27	153	22	46,6	1,01042
September	22 to 23	179	44	21,5	1,00283
October	24 to 25	212	24	24,5	0,99303
November	18 to 20	238	02	46,4	0,98746
December	17 to 18	265	58	47,7	0,98415

By means of these data we construct figure 46, which represents the annual orbit of the sun.

223. This curve is a little elongated in the direction of the line which joins the observations of December and June. It is consequently toward these points that we are to look for the perigee and apogee. The resemblance which this orbit bears to an ellipse, suggested the idea of a careful comparison, by which it was found that the *solar orbit is actually an ellipse, having the earth in one of its foci.*

224. The ellipse, it will be recollected, is a plane curve of such a nature that the sum of the distances from any point, situated in it, to two fixed points, called *foci*, is always the same. In order to describe it, we attach to the foci the extremities of a thread; we then extend this thread by means of a stile round in such a manner as to keep the parts of the thread always extended. The curve is completed by an entire revolution of the stile. This curve is represented in figure 47. The right line drawn through the foci and terminating each way in the curve, is called the *transverse or major axis* of the ellipse. The middle of this axis is called the *centre;* it is equally distant from the two foci, and its distance from either of them is called the *eccentricity;* because, if the two foci were united at the centre, the ellipse would be changed into a circle, from which it differs only by reason of its eccentricity. Lastly, a right line drawn through the centre perpendicularly to the transverse axis, is denominated the *conjugate or minor axis;* and the distance of its extremities from either *focus* forms the *mean* distance, because it is in fact the arith-

metical mean between the distances of the perigee and apogee.† In the figure, the points *F* and *F'* are the foci, *C* the centre, *AP* the transverse axis, *BF* or *BF'* the mean distance, and *CF* or *CF'* the eccentricity.

225. Now in order to ascertain whether the curve indicated by the above observations of the sun, is strictly an ellipse, we must take the indeterminate equation of some ellipse, and subject it to the test of satisfying some of these observations; and when its elements have been determined by this condition, we shall see whether it likewise represents the other observations, that is, whether it gives, for the distance of the sun from the earth, at the different longitudes, values equal to those found in the preceding table.

This process may be facilitated by making use of the indications furnished by the observations themselves, with respect to the position of the apogee and perigee. It is manifest that the apogee must be between the observation of June and July, which in the table of article 40, gave the greatest distances of the sun from the earth. Moreover, as these values are nearly equal, the apogee must be nearly midway between the longitudes answering to those distances. Upon this supposition, the longitude of the apogee is 101° 14′ 46″; and consequently that of the perigee 281° 14′ 46″, for these two points are opposite to each other in the ellipse. Indeed, if we consider the observations of December and January which give the smallest values for the distance, and which, consequently, must contain the perigee, the mean of the corresponding longitudes is 279° 21′ 29″, differing but little from the one obtained above.

It is obvious, therefore, that if we had observations made when the sun was precisely in these two longitudes, his distances from the earth, thence deduced, would be exactly those of the apogee and perigee. Now, if we consult with this view, the catalogue of observations published by Dr Maskelyne, which we have already employed, we shall find the following exactly suited to our purpose.

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† It is indeed the mean of all the distances; for, being equal to the semitranverse, it is equal to half the sum of all the distances to the two foci, from the nature of the curve; and the two foci, being in all respects similarly situated with regard to the curve, the semitransverse, or half the sum of all the distances to the two foci, must be equal to the sum of all the distances to one focus.

Dates of the Observa. 1775.	Sidereal Time at Greenwich.	Observed Longitudes.	Difference of the Times.	Difference of the Longitudes.	Intermediate Longitudes.	Distance from the Earth.
	h ′ ″	° ′ ″	h ′ ″	° ′ ″	° ′ ″	
June 30	6 36 31,4	98 23 15,2				
July 1	6 40 39,6	99 20 24,1	24 04 08,2	0 57 08,9	98 51 49,6	1,01767
Dec. 30	18 37 59,7.	278 43 33,7				
31	18 42 25,1	279 44 48,9	24 04 25,4	1 01 15,2	279 14 11,3	0,98309

The arc of longitude contained between these observations, is not exactly equal to a semicircumference, as it would be if the points to which they are referred, were situated exactly opposite in the same straight line; but the difference 22′ 21″,4, being very small, may be neglected in the first approximation. Then the distances belonging to these observations may be considered as the distances of the apogee and perigee. Their half sum, equal 1,00038, will be the mean distance or semitransverse of the ellipse. Their half difference, equal to 0,01729, will be the eccentricity; and if we divide this last by the other, we shall have 0,01728 for the ratio of the eccentricity to the semitransverse axis. Lastly, if we take the arithmetical mean between the observations of the longitude, we shall have 99° 03′ for the longitude of the apogee in 1775, and consequently 279° 03′ for the longitude of the perigee.

226. We have here employed only four observations to determine these elements. Moreover, these observations were not exactly opposite to each other in the orbit. Besides, the apogee and perigee which we have here supposed to be fixed, are actually in motion in the heavens, as we shall soon see, and strict accuracy requires that we should have regard to this motion in our comparison of observations. Finally, the method here employed to determine the distances of the apogee and perigee, is by no means the most accurate that can be made use of, and we shall soon give another much preferable to it. The result we have obtained, therefore, is to be regarded only as an approximation, and it will be easily conceived that, by employing more precise methods and combining a greater number of observations, we might arrive at results entitled to greater confidence. Indeed, by taking all possible precautions, M. Delambre found the semitransverse of the sun's elliptical orbit, at the close of 1775,

equal to 1,00041; the ratio of the eccentricity to the semitransverse 0,0168, and the longitude of the perigee 275° 03′ 17″. These values do not differ essentially from those deduced above; but still they ought to be employed as more exact.

227. Now in order to be assured that the solar orbit is really an ellipse, and such as our computation has made it, we have only to calculate, according to the above ellipse, the values of the distances of the sun from the earth for each of the longitudes given in the table, and then see if these values agree with those obtained from observation. We have made this comparison, and the results are contained in the following table.

Observed Longitudes.	Distances deduced from the Obser.	Calculated Distances.	Excess of the observed Distances.
° ′ ″			
292 44 10,0	0,98448	0,98406	+ 0,00042
329 14 00,1	0,98950	0,98949	+ 0,00001
354 15 37,5	0,99622	0,99585	+ 0,00037
38 26 20,6	1,00800	1,00845	— 0,00045
54 51 54,6	1,01234	1,01233	+ 0,00001
86 27 43,4	1,01654	1,01680	— 0,00026
98 51 49,6	1,01767	1,01722	+ 0,00045
116 01 50,0	1,01658	1,01647	+ 0,00011
153 22 46,6	1,01042	1,01003	+ 0,00039
179 44 21,5	1,00283	1,00286	— 0,00003
212 24 24,5	0,99303	0,99352	— 0,00049
238 02 46,4	0,98746	0,98762	— 0,00016
265 58 47,9	0,98415	0,98403	+ 0,00012
279 14 11,3	0,98309	0,98349	— 0,00040

The differences between the results of observation and those of calculation, are so small and irregular, that we may, without fear of mistake, ascribe them partly to errors in the observations; though some portion of this difference must undoubtedly be considered as proceeding from the elliptical hypothesis, and the manner in which it has been employed. This will be illustrated hereafter. In the mean time this correspondence is sufficiently exact to justify the conclusion that the sun's orbit is an ellipse having the earth in one of its foci.

228. We have seen that, the mean distance of the sun from the earth being 1, the eccentricity is 0,0168. This eccentricity would, therefore be less than 17 lines in an ellipse whose semitransverse is only 1000 lines. Hence, we see why it is scarcely

sensible in the figure. But in the heavens the semitransverse being nearly 100000000 of miles, this eccentricity must amount to about 1700000 miles.

Having now arrived at a near approximation to the elements of the sun's orbit, we next proceed to a consideration of the methods employed by astronomers for ascertaining the exact values of these elements.

Method of Determining exactly the Position of the Solar Ellipse in the Plane of the Ecliptic.—Origin of Mean Time.

229. In order to determine exactly the position of the solar ellipse, it is necessary to fix with precision the places of the apogee and perigee in the ecliptic. The circumstance that the angular velocity is least in the one and greatest in the other of these points, has already afforded us the means of a close approximation, making use of our daily observations. But we are still unable to determine within half a day, the exact instant when the sun passes either of them. In order to obtain them with greater precision, it is to be remarked, that the sun must employ half a year in passing from one of these points to the other, and that the difference of longitude at these points is 180°. The union of these two conditions belongs exclusively to the transverse axis of the ellipse; for, if we draw through the centre of the earth any other straight line cutting the ellipse in two points, though the difference of longitude will still be 180°, yet the time employed by the sun in passing from one to the other, will differ from it half a tropical revolution. It will be greater if the arc described, contains the apogee, and less if it contains the perigee, the motion being slower in the former, and more rapid in the latter, than in any other part of its orbit.

It would be very extraordinary if we should happen to fall upon observations differing in longitude exactly 180°, and the interval of which should be precisely half a year. But when we have hit upon the observations which best satisfy these conditions, the known laws of the sun's motion will enable us to make the small corrections that are necessary in order to arrive at the true times and longitudes. Formulas for this purpose will be given hereafter.

By applying these considerations to the observations of Dr Maskelyne, M. Delambre found the longitude of the perigee, as before stated, to be 275° 03′ 17″ for the end of the year 1775.

230. Let us compare this result with more ancient observations. In 1690, Flamstead found the longitude of the perigee to be 277° 35′ 31″. This point of the sun's orbit therefore advanced 1° 27′ 46″ in 85 years, which makes the annual advancement 61″,952. The theory of attraction makes the advancement 61″,905, and this is to be regarded as more accurate than observation wherever such small quantities are concerned.

The transverse axis of the solar orbit is not, therefore, fixed in the heavens, but advances annually 61″,905 in the direction of the sun's motion.

When the sun has returned to the point of the ecliptic where the perigee was situated the preceding year, the perigee will have removed from it, and the sun must, therefore, describe an arc of 61″,905, before overtaking it. The time necessary for this is the fourth term of the following proportion

$$360^\circ : 365^d\ 5^h\ 48'\ 51'',6 :: 61'',905 : 0^h\ 25'\ 07'',3.$$

In reality the solar ellipse does not remain perfectly stationary, while the sun is describing this small arc; but its motion is so slow, that for so short a time it may be neglected; for in $0^h\ 25'\ 07'',3$ it would only describe an arc equal to

$$\frac{0^h\ 25'\ 27'',3\ .\ 61'',905}{365^d\ 5^h\ 48'\ 51'',6},$$

a quantity which is less than the three-hundredth part of a second. If we would perform the calculation rigorously, we have only to employ the same process that we used in article 55, for ascertaining the exact value of the excess of the sidereal year over the tropical.

231. It will be seen, as a necessary consequence of these phenomena, that the sun employs a little more than a tropical year in returning to the apogee or perigee of his orbit. The difference is equal to $0^h\ 25'\ 07'',3$, and the duration of a revolution with respect to the apsides is $365^d\ 5^h\ 48'\ 51'',6 + 0^h\ 25'\ 07'',3$ or $365^d\ 6^h\ 13'\ 58'',91$. This is denominated the *anomalistic* revolution, the angular distance of the sun from the perigee of his orbit being called the *anomaly*.

232. This period differs sensibly from the tropical revolution. It is therefore to be employed, instead of the tropical, for determining the opposition of observations made at the apogee and perigee. Indeed, it is evident that regard should be had to the displacement of the perigee during the interval of the observations which we compare. These conditions form the basis of the method by which we reduce to the apogee and perigee the observations which are made very near these points.

The motion which we have been considering, is that of the perigee with respect to the vernal equinox; and we find that its longitude increases continually. But we have before seen that the equinox itself, has a retrograde motion in the ecliptic which increases yearly the longitude of all the points of the heavens 50″,1. The absolute displacement of the perigee with respect to a fixed point of the ecliptic, or with respect to the stars, considered as immoveable, is therefore, equal to the excess of its apparent motion above that of the equinoxes, that is,

$$61,''905 - 50'',1 \text{ or } 11'',806.$$

This is the *sidereal motion* of the sun's perigee. We observe that the tropical motion of the perigee exceeds the retrograde motion of the equinoxes; accordingly the sidereal motion of the perigee is direct. By the theory of gravity we learn that it arises from the attraction exerted by the sun and planets upon the earth. It is consequently subject to some inequality proceeding from the displacement of the orbits of these bodies. But these variations being very slight, we may represent it with sufficient accuracy by an expression compounded of two terms, the one proportional to the time, and the other much smaller, being proportional to the square of the time. We here speak only of the sidereal motion. When we refer this motion to the varying equinox, the inequalities of precession enter also into the expression.

233. We have mentioned that the sun experiences small periodical perturbations, in consequence of the attraction of the planets, which cause it to oscillate about its mean place. Accordingly, if we calculate directly the longitude of the perigee, from the longitudes of the true sun as given by observation, the result will be affected by all these small perturbations; these we correct before completing the calculation. Moreover, we reduce the observations to the mean equinox by making in the obser-

ved longitudes the corrections required by nutation. Then the longitude of the perigee, deduced from observations thus corrected, is exempt from these small inequalities, and reduced to its mean position.

We have also applied the term *anomaly* to the angle contained between the radius vector and the line joining the centre of the earth and the perigee. It should be observed that astronomers have for a long time reckoned the anomaly from the apogee. This is attended with no advantage or disadvantage, as it respects the sun or the planets; it becomes impracticable in the case of comets, which, as we shall see hereafter, have their orbits so exceedingly elongated, that they can, for the most part, be observed only in that portion which is nearest the perihelion. This consideration induced the author of the *Mécanique Céleste* to reckon the anomaly from the perigee, and the Board of Longitude have followed his example in their new tables, which renders them uniform for all the heavenly bodies. The same will be done in this work. But since, notwithstanding the advantages of this mode of reckoning, it has not yet been generally adopted, the learner should be apprized that, before employing the anomalies laid down in other works, it is necessary to observe from which point they are reckoned, and to make his calculations accordingly; if this precaution be taken, the difference will be attended with no difficulty.

234. The moment the sun passes the perigee of its orbit, is that which astronomers have agreed to adopt as the arbitrary origin of absolute mean time; we say agreed, because they might have fixed this origin in any other way. When they have ascertained by observation the mean longitude of the perigee, making all the corrections above described, they reduce it to the plane of the mean equator, reckoning from the vernal equinox, and proceeding from west to east, as if it were right ascension.* This longitude thus reduced, they regard as the right ascension of the mean sun, referred to the mean equinox, for the instant when the true sun is in the perigee of its

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* By *mean equator* we here understand the equator which corresponds to the *mean pole* of the earth. This pole remains stationary at the centre of the small ellipse of nutation, while the true pole is constantly moving in the circumference of the ellipse, and drawing the true equator with it.

orbit. The position of the mean sun is completely determined by this construction, for the instant of its passing the perigee; and hence it is determined once for all, since we know that the motion is uniform.

Suppose we wish to calculate for this instant, the mean horary angle of the mean sun for a determinate place, sa Paris, for example. We have only to determine for that instant the distance of the mean equinox from the mean meridian of Paris.* This distance is simply the sidereal time converted into degrees. If from this we subtract the right ascension of the mean sun, determined as above, it is evident that the remainder will be, for the same instant, the distance of the mean sun from the mean meridian of Paris, or its horary angle reckoned from east to west, like that of the true sun. If we convert this arc into mean time by the proportion of 15° to an hour, we shall have the mean hour, or absolute mean time, for the instant of the true passage of the sun through the perigee, and consequently for any other time, the motion being uniform.

This direct and rigorous method may be much simplified in the application, by considering that the true equator and equinox always differ very little from the mean equator and equinox.

In order to illustrate this simplification clearly, let ♈′♈E, Fig. 49.
represent the ecliptic, ♈Q the mean equator, and ♈ the mean equinox; also let ♈′Q' be the true equator, and ♈′ the true equinoctial point; so that the two equators being produced, shall intersect each other in I. Then the angle at I will represent the variation of the obliquity, and the arc ♈♈′, the variation of longitude, occasioned by the nutation. Let us now suppose that at the instant for which we wish to calculate the mean time, the arc ♈′M' is the true right ascension of the zenith of the place of observation, that is, the apparent sidereal time converted into degrees. Also let ♈M be the mean right ascension of the mean zenith of the same place, on the mean equator, so that ♈M shall be the mean sidereal time converted into degrees. All the questions which can arise respecting mean time, are reduced to calculating ♈M, ♈′M' being given, or reciprocally, to calculating

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* By *mean meridian* we here understand that which passes through the mean zenith of Paris and the mean pole of the earth, perpendicular to the mean equator.

♈′*M*′, ♈*M* being known. In performing this calculation, it is to be remembered that in nutation the poles and the equator vary their positions at the same time, so that the points and the circles which make part of this system, are displaced together without altering their relative positions. From this self-evident fact, it follows that the two points *M*, *M*′, always correspond to the same physical point of the equator, whether true or mean; for, being the intersection of this plane by the meridian of the place of observation, they move with the whole system of terrestrial circles, and consequently do not change their relative positions with respect to each other. This amounts to the same as saying that the points *M*, *M*′, always preserve the same geographical longitude on the globe. Accordingly the distances *MI*, *M*′*I*, of these points from the intersection of the two equators are equal; that is, the true right ascension ♈′*M*′, increased by the arc ♈′*I*, is equal to the mean right ascension ♈*M*, increased by the arc ♈*I*, or in other words, to obtain the true right ascension of the zenith of a place, when we know its mean right ascension, we must add to this last the excess of the arc ♈*I* above the arc ♈′*I*.

Now if from the mean equinox we draw an arc of a great circle ♈♈″, perpendicular to the true equator, and which will be the meridian of ♈, it is easy to see that on account of the extreme smallness of the angle *I* and the arc ♈♈′, which expresses the nutation in longitude, the difference between the arc ♈*I* and ♈′*I*, is nearly equal to the arc ♈′♈″, contained between the true equinox and the meridian of ♈. This distance ♈′♈″ is really the apparent right ascension of the true equinox. Thus, if we stop at this approximation, which is always sufficient, it appears that *the mean right ascension of the zenith is equal to its apparent right ascension minus that of the mean equinox.* This last may be easily calculated; for in the right-angled triangle ♈♈′♈″ we know the angle ♈′, equal to the apparent obliquity of the ecliptic, and the side ♈♈′, equal to the nutation in longitude. It is easy to see that ♈′♈″, the arc sought, is equal to the nutation in longitude multiplied by the cosine of the apparent obliquity of the ecliptic. In all these calculations we can resolve the triangles *I*♈♈′, ♈♈′♈″, as if they were plane triangles, on account of the smallness of the arcs by which they are formed.

Reciprocally, to obtain the apparent right ascension of the zenith of a place, when the mean right ascension is known, we add to the latter the apparent right ascension of the mean equinox.

By these very simple methods it is easy to determine for any instant whatever, the absolute mean time, when we know, for the same place and instant, the mean right ascension of the mean sun upon its equator and the apparent sidereal time. We have only to apply the rules just given.

For example, by means of a variety of observations of the sun, made by Dr Maskelyne, M. Delambre ascertained that the true sun, corrected for the small perturbations which affect it, and thus reduced to the elliptical motion, passed the perigee of its ellipse the 30th of December 1780, at $10^h\ 04'\ 00'',4$ sidereal time at the meridian of Paris,† this time being reckoned from the superior meridian and from the true equinox. The longitude of the perigee, referred to the mean equinox, was at this instant $279^\circ\ 09'\ 26'',7$; this therefore was the mean right ascension of the mean sun, according to the preceding definition. If we reduce it to time, it becomes $18^h\ 36'\ 37'',8$. The nutation of the mean equinoctial point in right ascension, or its apparent right ascension, was $-\ 0^\circ\ 10'',7$ or in time $-\ 0'',7$. It was negative, because the node of the moon was then in the ascending signs of the ecliptic, between ♈ and ♋. Thus, by subtracting it from the sidereal time $10^h\ 04'\ 00'',4$ it becomes additive, and we shall have $10^h\ 04'\ 01'',1$ for the mean right ascension of the zenith referred to the mean equinox. If from this we subtract the mean right ascension of the sun $18^h\ 36'\ 37'',8$, adding 24^h if necessary, to render the subtraction possible, the difference $15^h\ 27'\ 23'',3$ will be the horary angle of the mean sun at this epoch. Thus we say that the true sun, corrected for perturbations, and reduced to the elliptical motion, passed the perigee of its orbit the 29th of December 1780 at $15^h\ 27'\ 23'',3$ mean time at the meridian of Paris. We have taken 1 from the number of days, because one day was converted into hours to render the subtraction possible.

235. Now that the origin from which we reckon mean time is well known, we return to the phenomena presented by the mobility of the solar ellipse. The major axis of this ellipse having a progressive motion in the plane of ecliptic, it must at cer-

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† The time is readily changed from the meridian of Paris to that of Greenwich, by subtracting $0^h\ 9'\ 20''$, and *vice versâ*, by adding the same quantity.

tain epochs, coincide with the line of the equinoxes, and at others be perpendicular to it. These epochs are easily determined, because we know by observation the present position of this axis and its motion.

According to the observations of Lacaille, the longitude of the perigee in 1750 was 278° 37′ 27″,95. When the major axis was perpendicular to the line of the equinoxes, this longitude was 270°. The difference is 8° 37′ 27″,95, which, at the rate of 61″,905 for a year, gives a number of years equal to

$$\frac{8^\circ\ 37'\ 27'',95}{61'',905},$$

or about 500. This phenomena therefore took place in 1250. Then the perigee of the sun coincided with the winter solstice, and the apogee with the summer solstice.

In like manner, when the major axis coincided with the line of the equinoxes, the longitude of the perigee was 180°. From this period to 1750, it advanced 98° 37′ 27″,95. The number of years necessary for this advancement is equal to

$$\frac{98^\circ 37'\ 27'',95}{61'',905}$$

or about 5735; which refers this phenomenon to about 4000 years before the christian era. By a singular coincidence, this is nearly the time when most chronologists fix the first residence of man upon the earth; although a variety of arguments, drawn from natural science, tend to show that the existence of the earth itself is to be traced to a period far more remote.

236. It is evident that the same phenomenon will again take place when the sun's perigee shall have attained a longitude of 360°; that is, when it shall have described 90° — 8° 37′ 27″,95, from its place in 1750; and from what has already been shown, it will be seen that the number of years necessary for this, is equal to 5735 — 2 × 500 or 4735, which refers the phenomenon onward to the year 6485. Then the perigee will coincide with the vernal equinox, whereas in the opposite position it coincided with the autumnal. In these two cases, the line of the solstices which is always perpendicular to the line of the equinoxes, coincides with the minor axis of the solar ellipse.

In general, it is obvious that this line of the solstices never corresponds during two consecutive years to the same points of the solar orbit. The preceding results are evidently only approximations; for it is plain that we should obtain values somewhat different, if account were taken of the variations of the perigee and equinoxes; which might easily be done, by employing the exact expressions which we have given for these motions. But such precision could not be of any great utility.

237. When the major axis AP, is perpendicular to the line of the equinoxes, the equator Ee divides this ellipse into two unequal portions, the smallest of which is situated next to the perigee. This part must, then, be described in less time than the other; since the extent is less, and the surfaces described are proportional to the times. This circumstance, added to the motion of the major axis, renders the duration of the four seasons unequal, and different in different centuries. Fig. 50.

At the time of this position of the sun in 1250, the perigee coincided with the winter solstice. Then the interval between the vernal equinox E and the summer solstice, was equal to that between this solstice and the autumnal equinox. Spring was therefore equal to summer, and autumn to winter.

At the time of Hipparchus, or 128 years B. C. the apogee was less advanced than in 1750, by a quantity equal to

$$61'',905 \times 1878 \text{ or } 32^\circ\ 17'\ 37'',56.$$

The longitude of the perigee, at this epoch was therefore equal to 246° 19′ 49″,39. The ellipse was situated very nearly as is represented in figure 51, and the angle PTS was equal to 23° 40′ 10″,7. The interval EAS' between the vernal equinox and the summer solstice, was then 94½ days, and the interval $S'e$ between this solstice and the autumnal equinox, was only 92½ according to the observations of this great astronomer. Spring was therefore longer than summer, and winter longer than autumn.

238. At present the position of the ellipse, is such as is represented in figure 52. At the beginning of 1800, the angle PTS was 9° 43′ 52″; and the lengths of the different seasons were as follows:

	d	h	′	″
From the vernal equinox to the summer solstice	92	21	44	28,0
From the summer solstice to the autumnal equinox	93	13	34	48,6
From the autumnal equinox to the winter solstice	89	16	47	20,3
From the winter solstice to the vernal equinox	89	01	42	23,0

At present, therefore, spring is shorter than summer, and autumn longer than winter.

As long as the solar perigee remains on the side of the equator where it now is, spring and summer, taken together, will be longer than autumn and winter. In this century the difference is about 7 days, as may be seen by the preceding statements. These periods will become equal, about the year 6485, when the perigee will have reached the vernal equinox. It will then pass to the other side, and spring and summer will together be shorter than autumn and winter.

239. These phenomena would not take place, if the motion of the sun were circular and uniform. All the seasons would be equal to each other, and we should never perceive any difference. The eccentricity of the orbit, though very small, has, therefore, a sensible effect upon the duration of these periods. The motion of the major axis, although very slow, produces the variations observed in different centuries; variations which are imperceptible during the short period of a single life, but whose effects will be actually felt by the successive generations of men. This is the second time we have had occasion to point out the application of astronomy to the past and future condition of our system; a consideration which has induced us to dwell so long upon this subject.

Exact Determination of the Eccentricity, from Observations of the Equation of the Centre.

240. The position and eccentricity of the solar ellipse are determined by the preceding calculations. If these were rigorously exact we might thence deduce the course of the sun.

But there still remains much uncertainty with respect to the eccentricity. We first deduced it from a comparison of the apparent diameters of the sun at the apogee and perigee. But the observations in this case are liable to inaccuracy from irradiation, of which we have no means of divesting them. Besides, this method is impracticable in the case of the planets, on account of the smallness of their apparent diameters. Accordingly, we have had recourse to this method only to demonstrate the ellipticity of the sun's motion; and, this ellipticity being once known, we have deduced the eccentricity from the observed angular velocities at the perigee and apogee. But in order to obtain this element with all the necessary precision, we must seek some result in which its influence shall be more continued and more sensible. Such is the variation in the angular motion of the sun.

The fundamental law of this motion is that the areas described by the radius vector are proportional to the times. If the sun's orbit were circular and without eccentricity, equal areas would correspond to equal angles, and the motion would be uniform. The periodical inequalities which we observe are, therefore, the necessary effect of the eccentricity of the orbit. They are connected with this eccentricity, and their extent depends upon its magnitude. Now these inequalities may be observed with great precision, by means of the excellent clocks which are now in use; so that we may hope to determine the eccentricity by this means, with much greater precision, than by direct observation.

241. In order to have a clear idea of these inequalities, we must consider the motion of the sun as composed of a uniform circular motion, and a correction depending on the eccentricity of the ellipse, by which the first value is modified.

Or, if we would express these ideas geometrically, we must suppose a fictitious sun *s* moving uniformly about the earth in a circumference whose radius is equal to the distance of the perigee. Fig. 53. To this fictitious sun we ascribe a mean motion equal to that of the true sun; so that departing together from the perigee *P*, they return to it together after an entire revolution. Let us follow their course from this point. While the fictitious sun moves in a uniform manner, the true sun moves unequally, forming, with the distance of the perigee, elliptical sectors proportional to the times. As the velocity of the true sun is greatest at its departure, it at first precedes the sun *s*; but its velocity diminishes as it removes from the perigee; and there is an instant when its motion is the same as that of the other; after which this last has the greater velocity, and overtakes the first at the apogee *A*, at which they arrive together. The contrary takes place in passing from thence to the perigee. In this case, the fictitious sun *s* at first precedes the true one. Soon the velocity of the latter increases; and there is a moment when the velocities of the two are equal; after which that of the true sun predominates; it approaches the other, and the two arrive together at the perigee, performing their mean tropical revolutions in the same time.

Consequently, if we conceive two radius-vectors, drawn at any instant from the centre of the earth to the two suns, the angle formed by these lines, will first be nothing at the perigee; it will then increase to a certain limit where it will have attained its maximum; after which it will diminish toward the apogee where it will again become nothing. From thence to the perigee it will vary, in a contrary direction, by the same degrees. This angle, therefore, is the correction to be made in the circular motion, in order to obtain the elliptical motion. It is called the *equation of the orbit* or the *equation of the centre*, since it is the custom of astronomers to apply the term *equation* to those quantities which require to be added to the mean results, or subtracted from them, in order to make them *equal* to the true results.

242. From the perigee to the apogee, the equation of the centre is to be added to the mean motion of the sun, to obtain the true motion; from the apogee to the perigee, it is to be subtracted. Moreover, there are two points in the orbit where it is at its maximum. The calculus shows that if this value is known, the eccentricity may be deduced directly from it. Now it is easy to determine this maximum by observation, as we shall see.

In these two points, the two suns move for some instants with the same velocity, and the motion of the real sun, is therefore equal to that of the mean, or $0^\circ\ 51'\ 23'',2974$. This is the condition by which we know the epoch of the greatest equation.

Now there are two such points in the orbit; and since the figure of the orbit is symmetrical, these points must necessarily be situated symmetrically on the two sides of the major axis, as in figure 53, where they are denoted by S and S'. Moreover, before the sun passes the apogee, the equation is additive, and after the passage, it becomes subtractive; whence it follows that in both cases, the true sun is nearer the apogee than the other, as appears in the figure.

It is obvious, therefore, from a mere inspection of the figure, that if we could calculate the angles STS', $s\,T\,s'$, their difference would be equal to the sum of the angles $ST\,s$, $S'T\,s'$, or double the greatest equation of the centre; for, on account of the symmetry of the orbit, these two angles must be equal.

Now, if we represent by $E e$ the line of the equinoxes, the angle STS' is easily calculated. It is the difference between the true longitudes of the sun ETS, ETS', observed at the points of the greatest equation.

The angle $s T s'$ is also readily found. It is the angle which the sun would describe in virtue of its mean tropical motion, during the same interval of time. It is equal to the difference between the *mean longitudes*, this expression being used to denote the longitude which the sun would have had at these epochs, on the suppositon of a uniform motion from the perigee.

Hence we have the following rule. *Having observed the true longitude of the sun at the epoch of the greatest equation on each side of the line of the apsides, we obtain the equation of the centre by taking half the difference between the mean and true motions in longitude for the same interval.*

243. Hitherto we have paid no regard to the progressive displacement of the solar orbit. Yet it has an influence on the positions of the sun with respect to the major axis.

In consequence of this displacement, the longitude observed at the first epoch, becomes too small; and in order to reduce it to the same point of the ellipse, to which it first corresponded, we must add to it the arc described by the perigee, during the interval between the first and second observation. Thus the angle STS', deduced from the difference between the true longitudes, is too great by the same quantity. But the difference of the mean longitudes, which is estimated by the time elapsed between the two observations, is augmented in the same manner and by an exactly equal quantity. The error, therefore, disappears in the final result, which is the difference between the preceding, and the whole is reduced to the simple rule above given.

244. For example, among the observations of Dr Maskelyne for the year 1775, we find the longitude of the sun on the 2d of April at 0^h 03′ 35″,01, mean time at Greenwich, 12° 33′ 39″,06. The sun was at this epoch very near its greatest equation, a circumstance indicated by its diurnal motion.

On the 30th of the following September at 23^h 49′ 35″,2 the longitude of the sun was 188° 05′ 44″,45. At this epoch also, it was near its greatest equation.

The difference between the two longitudes is 175° 32′ 05″,39. To obtain the difference of time between the two epochs, we must consider that from the 2d of April at 0^h to the 30th of September at 24^h is 182 days. The first observation is later by $0^h\ 03'\ 35'',01$; the second earlier by $1^h\ 10'\ 24'',8$, which makes in all $0^h\ 13',59'',81$ to be subtracted from 182 days. The remainder, $181^d\ 23^h\ 46'\ 00'',19$, is the interval required.

	°	′	″
The mean tropical motion corresponding to this interval is $\dfrac{360° \times 181^d\ 23^h\ 46'\ 00'',19}{365^d\ 5^h\ 48'\ 51'',61} =$	179	22	41,48
The true motion is	175	32	05,39
The difference is	3	50	36,09
Which gives for the greatest equation of the centre	1	55	18,04

To have this result rigorously exact, the longitudes should have been observed precisely at the epochs of the greatest equation, which is hardly to be expected. But the error is always very small; since about this epoch, the true and mean suns move on with nearly the same velocity for several days, and the equation of the centre varies very little. Yet, if greater accuracy be desired, we may proceed as in the observations for the solstice. We ascertain from the tables how much the observed longitude wants of being that of the greatest equation, and this may be done with great accuracy from the approximate value of the eccentricity. We find also how much the equation of the centre wants, at the epoch of observation, of being the greatest in the orbit, and this also may be done with great precision, although we have only the approximate value of the eccentricity. Then, by adding to the observed longitude the reduction of the longitude, and also the reduction for the equation of the centre, the whole is brought to the same condition as if the observation had been made at the point of the greatest equation; and the result obtained from observations, thus reduced, becomes rigorously exact.

We may even dispense with the correction of the longitude; for if we transfer the true longitude to the epoch of the greatest equation, we must transfer the mean longitude also; and for this purpose, we must perform a reduction similar to that for the true longitude. Now these corrections are not only similar but exactly equal; for, about the epoch of the greatest equation, the true and mean motions are equal. Since, then, the mean interval is to be subtracted from the true, it is obvious that this common correction cannot affect the result, and it may accordingly be dispensed with.

As we have the means of reducing observations to the epoch of the greatest equation, it is manifestly not necessary to confine ourselves to a single one; but the greatest possible number should be made to concur in the determination of this important element. Accordingly, we reduce to this epoch several observations made a few days prior and subsequent to it. Each of these will give the equation of the centre for the moment it is made, and the mean of the whole will express its value very exactly. It was nearly in this manner that M. Delambre found the greatest equation of the centre, for the year 1775, to be 1° 55′ 31″,66. From this we deduce immediately the eccentricity for the same epoch, namely, 0,016803.

245. The observations we have been considering suggest an interesting remark. The epochs which separate them are about 182 days distant from each other, that is, half a year. The corresponding radius vectors must therefore be nearly in a straight line; and as we know, besides, that they make equal angles with the major axis of the orbit, they must also be nearly perpendicular to this axis. Thus the anomaly which corresponds to the greatest equation differs little from a right angle. It appears by calculation that this circumstance is owing to the smallness of the eccentricity.

246. It is proved by a comparison of observations, that the equation of the centre diminishes nearly uniformly. This diminution is confirmed by the theory of attraction, and its value is thus made known more exactly than it could be by observation. It is 0° 0′ 17″,17 in a century for the greatest equation; and from this we can easily deduce the corresponding variation for each point of the orbit.

247. This phenomenon supposes a corresponding diminution in the eccentricity of the solar orbit, for these two quantities are connected, and must increase and decrease together; since, if the eccentricity were nothing, the equation of the centre would also be nothing. The theory of attraction establishes this dependence, and makes known the corresponding diminution of the eccentricity. It is 0,0000416612 a century, the mean distance being unity. If we call the mean distance of

the sun from the earth 95000000 miles, the diminution is about 3958 miles in a century or 39½ miles in a year. Thus we see that fractions which appear almost insensible in the heavens, become very considerable quantities when we refer them to our ordinary measures.

248. If this diminution were always progressive, the solar ellipse would at length become a circle; and consequently the perpetual decrease of eccentricity would cause the earth after a long series of ages to be precipitated upon the sun. But it is shown, by the theory of attraction, that the variations of the eccentricity and the equation of the centre, are periodical; so that after decreasing for a certain period, the eccentricity will begin to increase, and pass through the same gradations in the inverse order. Although the limits within which this oscillation is confined are not yet accurately fixed, still we know that they are very restricted. Thus it appears that the order of our system will be perpetual, unless some foreign and unknown cause shall operate to modify the laws by which it is at present governed.

Inequality of the Solar Days.—Equation of Time.—Conversion of True Time into Mean or Sidereal Time and the Reverse.

249. Having ascertained with precision the sun's motion and the dimensions of its orbit, we must be able to calculate the arc which it describes parallel to the equator, during each day of the year. If these daily arcs were equal to each other, the duration of the solar days would be constantly the same. But they are unequal, and hence the solar days are unequal.

In order to understand clearly the principle and the laws of these inequalities, let us leave nutation out of the question, and suppose the equator and ecliptic immoveable. Through the pole of the equator, let us suppose two meridians to be drawn to the extremities of the arc of the ecliptic, described by the sun, in a sidereal day. The arc of the equator, intercepted by these meridians, will represent for this epoch, the difference between the sidereal and solar day; for, when the rotation of the celestial sphere is completed, this small arc of the equator must still traverse the plane of the meridian before the sun passes it. Now it is easy to see that these daily arcs are not of the same length in the different parts of the year.

In the first place, they must evidently vary from the mere effect of the inequalities of the sun's proper motion in its orbit. But this is not the only cause of their inequality. They would still be unequal, though the sun were to pass through the ecliptic with a uniform motion, describing equal arcs each day. Indeed, on account of the obliquity of the ecliptic, these arcs take successively different inclinations, with respect to the equator. At the time of the equinoxes, they cut this plane under an angle equal to the obliquity of the ecliptic; because their direction is then perpendicular to the line of the equinoxes, which is the common section of the ecliptic and the equator. On the contrary, at the time of the solstices, they became parallel to this same line. The meridians drawn through the extremities of the daily arcs, near the solstice, are therefore, farther apart than they would be if they embraced the same arc of the ecliptic near the equinoxes; and as the divergence of these meridians, measures, on the equator, the successive retardations of the sun in its return to the meridian, compared with that of a star, these retardations are unequal and smaller at the equinoxes than at the solstices.

The daily variations of the sun's motion in its orbit, is another cause of inequality in the lengths of the solar days; for it augments or diminishes the daily arcs which the sun describes on the ecliptic; and this affects the divergence of the meridians, and the length of the arcs of the equator comprehended between them.

250. It appears then that the inequality of the solar days, is owing to two distinct causes, namely, the obliquity of the ecliptic, and the inequality of the sun's proper motion. These two causes have each their separate effects, so that both must be destroyed in order to render the solar days equal; that is, the ecliptic must coincide with the equator, and the motion in the orbit become uniform. Now it is demonstrated, by the theory of attraction, that this can never be the case, at least, under the present system of things.

251. Hence in order to determine the inequality of the solar days, it is necessary to calculate the arcs which the sun de-

scribes each day in the ecliptic, by virtue of its true motion in longitude, to project these arcs on the equator by means of meridians, and to take the successive differences of the horary angles comprehended between them.

252. It is the inevitable inequalities in the sun's motion in right ascension, which have prevented astronomers from making use of this motion in their tables and observations as a measure of time. For these perpetual variations, which must be taken into consideration, would have rendered the calculations exceedingly complex without being attended with any advantage. This inconvenience is not felt in civil reckoning, where such extreme accuracy is not necessary; and accordingly, for civil purposes, we measure time by the true motion of the sun, considered as uniform, its inequalities being of no importance in this point of view. This mode of reckoning is the more judicious, since all the labors of society are distributed with reference to the true and not the mean motion of the sun.

But with astronomers uniformity is indispensably requisite in order to attain to any thing like simplicity in their calculations, and the civil mode of reckoning would be inadmissible. On the other hand, the necessity of dating their observations, and of finding without difficulty the approximate epochs, admonished them not to depart too far from the common usage of society which perfectly fulfilled the conditions in question. These considerations induced them to adopt as the measure of time, the mean motion of the sun, and they fixed the origin of their reckoning in such a manner that the mean solar time should only oscillate about the true, without ever varying much from it. This is indeed a necessary consequence resulting from the mode of estimating mean time as already explained; but we shall comprehend it better by means of a geometrical construction, representing the conditions there established. For this purpose, we must first abstract from the true sun, the perturbations which affect it, in order to reduce it to the elliptical motion, without which it would be impossible to represent its irregularities. We then suppose a fictitious sun to describe the great circle of the ecliptic with a uniform motion, and to pass the perigee and apogee at the same time with the true sun. This fictitious sun is already exempt from the inequalities of the elliptical motion. We next suppose a third sun to pass through the mean equinoxes at the same time with the second, and to move in the mean equator with a uniform velocity, in such a manner that the angular distances of these two fictitious suns from the mean vernal equinox, shall be constantly equal to each other. The effect of the obliquity of the ecliptic would then disappear with respect to this third sun; and, as we have corrected the inequality of the proper motion, by the first supposition, the motion here supposed is subject to no irregularity whatever. It is this which measures the *mean time* of astronomers. The *mean day* is the interval comprehended between two consecutive passages of this fictitious sun over the mean meridian; and *mean midnight* and *mean noon* are the instants when it is in this plane.

253. It is obvious that this construction completely satisfies the conditions already made known, respecting the origin of mean time. Indeed, according to the definition we have given, the third sun, whose motion in right ascension is uniform, passes the mean equinox at the same time with the second. Accordingly it then coincides with the second. Moreover, departing from this point, the two have a perfectly equal motion, the second in the ecliptic, and the third in the equator. When, therefore, the second has arrived at the apogee or perigee of its orbit, the third will have its right ascension equal to the longitude of the apogee or perigee. This is precisely the condition by which we have determined the origin of mean time in the article above referred to.

Now since we know the right ascension of the mean sun at the instant of passing the perigee, if we take this right ascension from 360°, we shall have the arc which remains to be described by the third sun before arriving at the mean vernal equinox. Since we know also its proper motion, which is $0^\circ\ 59'\ 08'',3297$ in a mean day, we can easily calculate the time required to reach this equinox. This interval, added to the time of passing the perigee or apogee, will give the instant of the mean sun's passing the equinox. This instant is called the *mean equinox*.

254. The interval between two consecutive returns of the mean sun to the same mean equinox, forms the *mean tropical year*, which, as we have before seen, consists of $365^d\ 5^h\ 48'\ 51'',61$. It differs a little from the civil year which is measured by the returns of the true sun to the true equinox. This last is subject to periodical oscillations, caused by nutation, which

sometimes increase and sometimes diminish the year, by causing the equinoctial point by turns to advance and recede. The progressive displacement of the solar orbit produces also slight variations in the civil year, since the velocity of the sun is unequal in the different points of the ellipse which arrive successively at the equinox.

255. In general it is easy to find the place of the mean sun in the plane of the mean equator for any given instant; for we have only to determine, by the tables for this instant, the value of the mean longitude of the sun reckoned from the mean equinox; and this longitude, referred to the plane of the mean equator, is the right ascension of the mean sun, or the *mean right ascension* of the sun.

If there were no nutation, the true pole would coincide with the mean, and the true right ascension be measured on the same equator as the mean. These two right ascensions would generally differ from each other, because the motion of the true sun in right ascension is unequal while the other is uniform. Their difference reduced to time, would give the interval between the true and mean sun for the instant under consideration. This interval is called *the equation of time;* since, when its value is known, by adding it to the true time we obtain the mean time, and by subtracting it from the mean time we have the true time.

This may be easily demonstrated if we suppose no nutation; for, calling A the right ascension of the true sun, M that of the mean sun, and a that of the zenith, it is evident that $\frac{a - A}{15}$ will be the true time, and $\frac{a - M}{15}$ the mean time. The difference between the two will then be $\frac{a - M}{15} - \frac{a - A}{15}$,

or simply

$$\frac{A - M}{15};$$

so that, a disappearing, there remains only the difference between the right ascensions.

Nutation modifies the above result a little, since, in consequence of this, the two right ascensions A, M, are reckoned neither on the same equator nor from the same equinox. Still we may, by an analogous process, find the difference between the true and mean time. For this purpose, let T be the mean time, and T' the

the true time, reckoned for the same instant, in a given place, as Paris, for example. If we convert these times into degrees, 15 T will be the horary angle made by the mean sun with the mean meridian of Paris, and 15 T' will be the horary angle made by the true sun with the true meridian of the same place.* Now, since we know the mean time T, we calculate for this same instant, by means of the tables, the mean right ascension M, of the mean sun, reckoned from the mean equinox; and adding this to 15 T, the arc $M + 15\,T$, will be, for this instant, the right ascension of the mean zenith, reckoned from the same equinox. To convert this into the true right ascension, we have only to add to it the true right ascension of the mean equinox, that is, $\psi \cos \omega$, ψ representing the nutation in longitude. This mode of reduction will be explained hereafter. It is founded upon the smallness of the nutation, and the slight obliquity of the true equator to the mean. $M + 15\,T + \psi \cos \omega$ will therefore be the true right ascension of the true zenith, reckoned from the true equinox. Dividing it by 15, in order to reduce it to time, we have the true sidereal time reckoned from the same equinox.

But this right ascension may likewise be obtained by another process, that is, by adding to the true horary angle 15 T', the true right ascension A' of the sun, reckoned from the true equinox on the true equator. The arc $A' + 15\,T'$ must then be equal to $M + 15\,T + \psi \cos \omega$; and from this equation we obtain $T - T' = \frac{A' - \psi \cos \omega - M}{15}$;

the quantity

$$\frac{A' - \psi \cos \omega - M}{15}$$

is, therefore, properly the *equation of the time;* for, by adding it to the true time T', we obtain the mean time T; and by subtracting it from the mean time T, we obtain T'.

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* The mean meridian is that which is drawn through the mean zenith, perpendicular to the mean equator; the true meridian is that which is drawn through the true zenith, perpendicular to the true equator. It is the mean meridian which is displaced by nutation, together with the rest of the earth, including the equator itself which it always cuts in the same physical point. In general, we must consider the true meridians and true equator as forming with the earth an invariable system, which oscillates about its mean position conformably to the laws of nutation.

When we have formed tables of the sun for the meridian of a given place, it is obvious that with these tables, we can calculate for any given instant of mean time, the mean right ascension M, the true right ascension A', and the nutation $\psi \cos \omega$. In this way the preceding formula may be reduced to numbers, and it is thus that *tables of the equation of time* are constructed.

These tables give the true time when we know the mean time; but, by a very simple process, we may reverse the problem, and obtain the mean time by means of the true. The difference between the two being always very small, since it never exceeds $0^h\ 16'\ 20''$, we first employ the given true time as if it were the mean time; from this we deduce a value for the equation of time, which, being added to the true time, gives an approximate value for the mean time; and by means of this we obtain a second equation of time which is sufficiently exact.

256. The equation of time is sometimes additive, sometimes subtractive. In the first case, the true sun precedes the mean in right ascension; in the second, the true sun is behind the mean. In passing from one of these states to the other, the two suns must obviously come to the same meridian together; then the equation of time is nothing. Indeed, by a very remarkable property, it becomes nothing four times a year.

To show clearly the reason of this phenomenon, we must at first put nutation out of the question; and afterwards consider what modifications are required on account of it. The fundamental principle on which we proceed, is, that if we take on the ecliptic two equal arcs, the first near the equinox, and the second near the solstice, and refer them to the equator by means of meridians drawn through their extremities, the arcs intercepted on the equator by these meridians, will be unequal; the first will be less, and the second greater, than the given arc, as
249. already shown.

Now let us suppose two suns S'', S''', to set out together from the mean vernal equinox, and to move with an equal and uniform motion, S'' in the ecliptic, and S''' in the equator; and let us suppose their motions referred to the equator. S'' will at first
Fig. 54. precede the meridian of S'''; but the latter will afterwards overtake the former, and they will arrive together at the solstice.

From this point S'' will at first precede S''', after which S''' will come up with S'', and both will arrive together at the second equinox. The same will take place in the remaining half of the orbit. Hence the two suns will coincide four times a year, namely at the two equinoxes and at the two solstices.

257. In reality the mean sun does not set out from the mean equinox at the same time with the true sun; they will not therefore coincide exactly at the above points. The inequality of the sun's proper motion in the ecliptic will also affect the points of coincidence. But these causes only change the epochs of the coincidences, the number remaining always the same. This we proceed to show, putting nutation for the moment out of the question.

In the present position of the solar orbit, let us suppose the two suns S'', S''', to depart together from the autumnal equinox towards the perigee. The true sun S', moving in the ecliptic, being referred by its meridian to the equator, is then behind them; for it is preceded by S'' up to the perigee, as we have shown. Let us now consider the order in which these three bodies move. Up to the winter solstice, S''' precedes S'', and S'' precedes S'; their order, therefore, in winter, is S', S'', S'''. At the solstice S'' coincides with S''', and afterwards passes it, which leads to the order S', S''', S''; but at the perigee S' coincides with S'', and then precedes it; whence, in the interval, S' and S''' must have coincided. The order then becomes S''', S'', S'. Thus, between the winter solstice and the perigee, the meridian of the true sun S' comes up with the mean sun S''', and the equation of time becomes nothing.

From the perigee to the vernal equinox, S' precedes S'', and S'' precedes S'''; no coincidence, therefore, takes place in this interval. At the vernal equinox, S''' overtakes S'', then passes it, and the order becomes S'', S''', S'; but S''' cannot long remain between the two others; for, in each quadrant of the circle, the distance between S''' and S'', amounts to 2° 28′ 24″,49, as may be seen by calculation. The distance between S' and S'', on the contrary, never exceeds, in the ecliptic, 1° 55′ 27″,12. This is at present the greatest equation of the centre; and as it takes place near the equinoxes, in the present position of the solar orbit, the arc which corresponds to it on the equator, must be still smaller. It is evident then that between the vernal equinox and the summer solstice, there must be an instant when the mean sun S''' overtakes the meridian of S', when the equation of time becomes nothing a second time."

Proceeding thence toward the summer solstice, the order is S'', S', S'''. At the solstice S' rejoins S''', and then passes it. In the mean time S' precedes S'' up to the apogee; consequently before the summer solstice, the meridian of S' must coincide with S''', and the equation of time become nothing a third time.

From the summer solstice to the apogee, the order is S''', S'', S'. At the apogee S'' coincides with S', and then passes it toward the perigee, which leads to the order S'', S', S'. Now at the autumnal equinox S''' rejoins S''; consequently, in the interval, it must coincide with the meridian of S', and the equation of time is equal to nothing a fourth time. Then the order becomes S', S'', S'''; and the same phenomena are repeated each revolution.

258. From this simple exposition, it is evident that in consequence of the obliquity of the ecliptic, combined with the unequal motion of the sun, the equation of time becomes nothing four times a year; namely, once between the winter solstice and the perigee, twice between the vernal equinox and summer solstice, and once between the apogee and the autumnal equinox. It is evident also that the epochs of these phenomena vary with the position of the line of the apsides. At present they take place about the 25th of December, 16th of April, 16th of June, and 1st of September. In general, we perceive that the progressive motion of the line of the apsides must gradually change the absolute values of the equation of time belonging to each longitude of the sun. Accordingly, these secular variations have been calculated by theory, and annexed to the solar tables.

We perceive also that the causes which render the equation of time nothing, must continue to operate, notwithstanding the slight variations occasioned by nutation, in the movement of the three suns. These variations, which never amount to more than a few seconds, will change a little the epochs of the conjunctions, but can neither prevent them, nor carry them beyond the limits we have assigned.

259. If the obliquity of the ecliptic were nothing, that part of the equation of time which depends upon this obliquity would be nothing also. Then the mean and true motions would differ only by the equation of the centre. The true and mean sun would coincide only at the perigee and apogee; and true and mean time would agree only twice, namely, at the apsides.

260. From what we have said, it follows, that the instant of true noon, as indicated on a meridian by the shadow of a style, differs in general from that of mean noon. But, as we know for each day, the equation of time, we can mark for each day, on the two sides of the meridian, the direction and limit of the shadow, at the instant of mean noon. We shall thus have a series of points which mark, about the true meridian, the successive meridians of the mean sun. The curve which unites their extremities, must evidently meet the true meridian in four points corresponding to the four epochs when the equation of time becomes nothing. Moreover, it must return into itself so as to enclose a space, since the equation of time takes the same value after each revolution. Accordingly, this curve will nearly resemble the form of the figure 8. But it is not symmetrical, because the epochs at which the equation of time becomes nothing are unequally distant from one another. It is called the *meridian of mean time*, and when we trace it about the true meridian on sun-dials, we mark on its contour, signs corresponding to the different seasons, that we may easily determine the portion of the curve which belongs to each part of the year.

The Sun's Spots, Form, and Rotation.

261. HAVING calculated with great exactness the motions of the sun, we come now to consider its form and constitution.

When we reflect how far we are from the sun, it would seem that our knowledge of these two points must be very uncertain. But if we consider the results already obtained, there is reason to hope, that however minute the differences of aspect which this luminary may exhibit, at the vast distance at which it is placed, they will be detected by careful and repeated observations.

The sun presents the appearance of a round luminous disc; but we must not conclude that its surface is really flat; for all globular bodies, when viewed from a great distance, must necessarily have this appearance. The telescope has enabled us to decide the question, since, by means of it, we find that the sun revolves upon its axis, in a period of about 25¼ days; and none but a globular body, presenting all its surfaces to us, could always appear under the form of a circle.

The observations thus made, have moreover taught us, that a violent ebullition and effervescence takes place on the surface of the sun. Every thing indeed concurs to show that it is an immense globe surrounded with flame, and emitting torrents of light from about its surface, of which we feel the effects at the distance of nearly a hundred millions of miles.

As these phenomena are both curious in themselves, and important in acquiring a knowledge of the system of the world, we shall here detail them at considerable length.

262. We often observe on the sun's disc, black spots of an irregular form, which traverse its surface in the space of a few days. Their number, position, and magnitude, are extremely variable. Some have been seen, which, judging from the space they occupy on the apparent diameter of the sun, must be five or six times as large as the whole earth. The dazzling light of the sun does not often admit of our observing them with the naked eye; but we can see them very distinctly with a telescope, in which the intense brightness of the sun is diminished, but not extinguished, by means of colored glasses placed before the eye lens. Each black spot is usually surrounded by a penumbra about which we distinguish a border of light more brilliant than the rest of the sun. Sometimes we observe at first luminous clouds on the edge of the disc, without perceiving spots at their centre; but in proportion as they advance the spots begin to be formed, or at least to become visible; and this phenomenon is so constant, that we can thence predict their appearance. When the spots first present themselves on the edge of the disc, they resemble a narrow dark streak. By degrees their apparent magnitude increases, as they advance towards the middle of the disc; they then diminish by the same gradations, and at length entirely disappear. But the luminous border which surrounds them, is visible some time after. These augmentations and diminutions are easily explained, if we suppose the spots to adhere to the globular surface of the sun; for then the mere motion of rotation would cause them to be perceived under different degrees of obliquity and magnitude.

263. For the purpose of observing the spots which appear on the sun's disc, and determining exactly their successive positions on this disc, we make use of an instrument adapted to measure small differences of right ascension and declination. By this

means we are able to trace the apparent path of a spot over the disc, with great accuracy, either by a graphic design, or by calculation which is much more precise.

When we have carefully observed the same spot during the whole time employed in traversing the sun's disc, which is about 14 days, we see it again, after nearly the same interval of time, but on the limb of the sun opposite to that where it disappeared. This revolving course is common to all the spots. But it generally happens that after several similar returns, we at length cease to see them. Yet, as we know that they sometimes grow less and less and disappear on the disc of the sun, we are led to believe that those which do not return, are in like manner dissipated on the opposite surface. Indeed their number and appearance are extremely variable and irregular. There are some years when none are to be seen, and others when they are very frequent.

264. These phenomena led Sir W. Herschel to the supposition that the body of the sun is solid and opake, surrounded by an extensive atmosphere, and that the atmosphere is almost constantly filled with luminous clouds, which, floating about at random, sometimes separate and discover to us the dark body or nucleus of the sun; in the same manner as when elevated by means of a mountain or balloon, far above the surface of the earth, we can perceive the lower regions through the openings in the clouds which float beneath. Sir W. Herschel believes also that there are very high mountains on the surface of the sun, whose summits, appearing at intervals above the luminous matter, exhibit the appearance of black spots. These explanations are, it is true, sufficiently conformable to the facts observed. But the phenomena may be accounted for equally well, by supposing with Laplace, that the body of the sun is itself inflamed. For the developement of the elastic fluids which would be formed in the interior of this mass, would occasion terrible convulsions there; and on this supposition, the spots might be deep cavities, whence issue at intervals, vast eruptions of fire, feebly represented by terrestrial volcanoes.

265. Having discovered the spots on the sun, we endeavour to follow them and ascertain their motions. The most simple method of doing this, is, as we have already said, to note for several days the differences of right ascension and declination between

the spot and the sun's centre; and thence deduce the differences of latitude and longitude. Top.105.

Fig. 55. Let us suppose that T represents the centre of the earth, Ee the ecliptic or great circle described by the sun, BSA the sun's apparent disc, C its centre, and M the apparent position of the spot, which we suppose to be south of the plane of the ecliptic. The angle CTP will be the difference of longitude, and the angle PTM will be the difference of latitude, that of the centre being nothing. These two angles will be known from observation, that is, they can be deduced from the differences of observed right ascension and declination.

These angles being very small, we may, in our first approximation, consider the arcs which measure them, as so many straight lines; and, if we suppose them, as is generally the case, measured on the celestial sphere whose radius is CT, the angle CTP, or the arc which measures it, will be represented by the line CP; the angle PTM by the line PM, and the sun's apparent diameter by BA. Consequently, in order to obtain a precise idea of the position of the spot, we may describe Fig. 56. at pleasure, a circle ASB, the diameter of which may be supposed to contain as many parts of unity, as the apparent diameter of the sun contains seconds. We take the line CP equal to the number of seconds in the difference of longitude, and the perpendicular PM, equal to the number of seconds in the latitude of the spot; and we shall thus have the position of the spot represented on the apparent disc of the sun. If we repeat the same Fig. 55. operation for several successive days, we shall obtain a series of points as M, M', M'', M''', representing the apparent path of the spot across the sun's disc; or rather, we shall have the projection of the curve which it describes, on a plane perpendicular to the visual ray drawn from the earth to the centre of the sun. This projection is generally an oval curve much resembling an ellipse, and all the spots which are observed at the same time, describe similar and parallel curves. The duration of their revolution is also the same. They all employ about $27^d\ 7^h\ 12'$ in returning to the same apparent position on the disc.

266. The form, curvature, and inclination of these ovals, are subject to very great variations. At the end of November and Fig. 57. the beginning of December, they are simply straight lines, as Mm, $M'm'$, $M''m''$. Then the spots move from M to m, that is, from the south to the north of the ecliptic. The points M,

M', ... where they begin to appear, and which may be called their rising, are less elevated than the points m, m', m'' . . . . where they disappear, and which may be called their setting. By degrees these straight lines become curved and form ovals, as in figure 58. During the winter and spring, the convexity of these ovals is turned towards the north pole of the ecliptic; but at the same time, their inclination changes; and about the first of March, the points where the spots begin to appear, are as much elevated in the ecliptic as those where they disappear. See figure 59. From this moment, the change of inclination continuing in the same direction, the curvature of the ovals diminishes. They gradually contract until, by the last of May or the first of June, they again appear under the form of straight lines. But Fig. 60. their inclination to the ecliptic is precisely the opposite to what it was six months before. From this time they again become curved as in figure 61, and their convexity is directed toward the south pole of the ecliptic; at the same time their inclination changes. The first of September, they appear under the form represented in figure 62. The points where the spots appear are as high as those where they disappear. Having reached this limit the ovals contract and become again inclined to the ecliptic; till in the month of December we see them under the form of straight lines, as we did the year before.

267. These phenomena are repeated every year, observing the same periods of increase and diminution. Hence we infer that the cause which produces them is equally uniform and regular. It must also be common to all the spots, since it makes them describe orbits exactly parallel, and subject to the same variations in their appearance. The most simple supposition is, that these spots adhere to the surface of the sun, and revolve with it in the course of a certain number of days.

Moreover the different and successive inflexions of the lines, described by the spots, indicate an axis of rotation which is not perpendicular to the ecliptic; for if it were, all the spots would describe circles parallel to this plane. These circles, seen from the earth at a great distance, would appear like so many straight lines parallel to the ecliptic; and these appearances would remain constantly the same, which is not conformable to observation.

On the contrary, all the phenomena are explained in the most simple manner by supposing the sun to revolve about an axis inclined to the ecliptic, and remaining constantly parallel to itself during its annual revolution. This axis, carried by the sun, takes successively different positions with respect to the earth, and exhibits the circles described by the spots under equally variable inclinations. Hence the changes we observe in their apparent curvature. Among all these positions, there are two which ought to present straight lines; these occur when the plane drawn through the axis of rotation, perpendicular to the ecliptic, becomes also perpendicular to the visual ray drawn from the earth to the centre of the sun. This can take place only twice a year, namely, in two opposite points of the ecliptic, and with an interval of six months between them. Then we perceive the *two poles of rotation of the sun*, that is, the two points in which the axis of rotation meets the surface. In every other position, the course of the spots must appear oval; but when we perceive the superior or north pole, the convexity of the ovals is turned towards the southern part of the ecliptic; and when we perceive the inferior pole, the convexity is turned in an opposite direction. The course of the spots being always in the same direction, from left to right, or from east to west, the points of the disc where they rise must, during six months, be more elevated than the points where they set. The contrary takes place during the other six months; and there are two epochs in the year when these points are equally elevated above the ecliptic. From the moment of this equality the inclination of the ovals must increase from day to day, and attain their maximum in three months; after which it must begin to diminish until a second equality takes place; hence it must happen that the epoch of greatest obliquity is that when the path of the spots appears to be a straight line; and, on the contrary, the moment of equal elevation must be that at which the curvature appears greatest. At all other times when the inclination decreases, the curvature must increase, and *vice versâ*.

We have endeavoured to represent these appearances in figure 63, where A, B, C, D, E, are the successive positions of the sun with respect to the earth, situated in T. Pp indicates throughout, the axis of rotation, and Mm, $M'm'$, $M''m''$, are the paths of the spots.

It being impossible to represent, in the figure, the extreme distance of the sun, we must in imagination supply the imperfection. *B* is the place where the axis of rotation becomes parallel to the plane of the solar disc, and consequently perpendicular to the visual ray *TB*. *A*, *C*, are the points where the apparent opening is the greatest. As to the opposite points, we cannot represent the body of the sun as seen from the earth; but it is evident that the appearances would be the reverse of those at the points *A*, *B*, *C*. For example, the sun having arrived at *C*, will present to a spectator, placed at *T*, the same appearance that it would have presented in *A* six months before, to a spectator placed at *A''* on the opposite side of the earth; that is, the path of the spots then appears in the form of ovals, whose extremities are equally elevated above the ecliptic, but present themselves in the opposite direction. A similar opposition takes place in all the situations of the sun, which are diametrically opposite in the orbit.

268. All these consequences being exactly conformable to observation, we may conclude with certainty, *that the sun actually revolves from east to west about an axis inclined to the ecliptic.*

The plane drawn through the centre of the sun, perpendicular to this axis, is called *the sun's equator*. It cuts the plane of the ecliptic in a straight line called the *line of the nodes* of this equator. The *nodes* themselves are the points where this straight line, produced indefinitely in both directions, meets the celestial sphere.

269. In order to ascertain the position of the axis of rotation in space, we must determine the inclination of the solar equator to the ecliptic, and the angle which the line of the nodes makes with a fixed straight line drawn in the plane of the ecliptic; for instance, with the line of the equinoxes. This angle is called *the longitude of the node.* The following is the method which appears the most simple and exact for determining these elements.

When we have observed the position of a spot on the sun's disc, and calculated its latitude and longitude, we know the direction of the visual ray drawn from the earth to this spot, at the moment of observation. We know also, for the same instant, the longitude, distance, and apparent diameter of the sun. It then becomes a simple problem in geometry, to find the intersections of its supposed spherical surface, by the visual ray drawn to the spot. Three similar observations of the same spot determine three points on the surface of the sun, and these are situated in the circumference of the same circle, parallel to the solar equator. Now, in general, the position of a plane is fixed, when we know that it passes through three known points; the plane of the circle described by the spot, will therefore be fixed by these three observations, and we can thence deduce the position of the solar equator which is parallel to it.

270. To fix the successive positions of spots on the surface of the sun, supposed to be spherical, we conceive to be drawn through the centre of the sun, in known directions, three rectangular axes, remaining constantly parallel to themselves, during its annual revolution. The first is perpendicular to the ecliptic; the two others are situated in this plane. One is parallel to the line of the equinoxes, and the other perpendicular to it. They are represented in *S*. By *heliocentric latitudes* and *longitudes*, we understand those which are reckoned from the centre of the sun, about the three preceding axes. They are evidently determined when we have their analogous values, measured from the centre of the earth, which are called, by way of distinction, *geocentric latitudes* and *longitudes*. These values are easily ascertained by spherical trigonometry. Fig. 64.

By means of this method, we find that the solar equator is inclined to the ecliptic at an angle of 6° 21′ 53″. It appears to remain constantly parallel to itself. The points of this equator, rising by their motion of rotation above the ecliptic towards the north pole, traverse this plane in a point which, as seen from the centre of the sun, is found to be 70° 44′ 44″ from the vernal equinox; this, therefore, is the *heliocentric longitude of the ascending node of the solar equator.* It does not appear to experience any sensible variations, unless it be those which result from the precession of the equinoxes, a general effect already considered.

271. We have said that the mean period of the sun's rotation with respect to the same point of the earth is $27^d\ 7^h\ 26'\ 24''$. This is the time after which a given point of the sun's surface returns to the same distance from the apparent centre; but it is not the time of an absolute rotation. During this interval, the sun describes in the ecliptic an arc equal to

$$0°\ 59'\ 08'' \times 27^d\ 7^h\ 26'\ 24'' \text{ or } 26°\ 55'\ 08''.$$

In consequence of this motion alone, it discovers to us each day new points of its surface, of which it must, in the course of a year, present successively all the parts. Hence results an apparent rotation, which appears to take place annually about an axis perpendicular to the ecliptic. The effect of this optical deception is combined with that of the real rotation, in the results of observation; and in order to distinguish between the effects of these two causes, they must be considered separately.

First, let us leave out of the question the real rotation. Then let us suppose a visual ray drawn from the earth to the centre of the sun, and a plane passing through the centre perpendicularly to this ray. The sun's disc is only the projection of all the points of its surface on this plane. The intersection of the plane and the visual ray, forms what we call the centre of the disc. It is to this centre that we refer the point of the surface of the sun, which is in the direction of the visual ray. Now it is not difficult to see that this point varies on the surface of the sun, when the sun changes its position in the plane of the ecliptic. If it is S, for example, T being the centre of the earth, and Fig. 65. TS the visual ray drawn to the centre of the sun, C will be the point of the surface referred to the centre of the disc. But when the sun is in S', the point C', which will then be referred to the centre of the apparent disc, will differ from the point C; and to determine the arc described by this last, we must draw $S'T'$ through the point S parallel to ST; the point C will then be in c. This point is, therefore, removed from the apparent centre of the disc, by the single effect of the sun's annual motion. The quantity of the removal is the angle $TS'T'$, which is equal to $S'TS$, or the motion of the sun in its orbit, during the interval of the observations.

The effect of the real rotation alters this apparent rotation, since it is in a contrary direction. If it took place about an axis perpendicular to the ecliptic, it would be easy to estimate it. For when the same point of the sun's surface returned to the same distance from its apparent centre, which it would do in $27^{d}\ 7^{h}\ 26'\ 24''$, it would in reality have described $386^{\circ}\ 55'\ 08''$, that is, an entire circumference plus the angular motion of the sun during this interval; whence, by proportion, we easily perceive that the time of a complete rotation would be $25^{d}\ 9^{h}\ 36'$.

The real axis of rotation being oblique to the ecliptic, this result is not strictly exact, and the arc of $26^{\circ}\ 55'\ 08''$, described by the points of the surface parallel to the ecliptic, do not produce exactly the same angle on the solar equator. But, as the inclination of these two planes is inconsiderable, the error is much less than those to which this kind of observations is liable. Finally, the duration of the real rotation is naturally determined when we calculate the successive positions of the same spot, with reference to three fixed axes drawn through the centre of the sun, as we have described above; for by this method we ascertain the arc actually described by the spot on its parallel, between the observations, and by comparing this arc with the time employed in describing it, we deduce the duration of the total rotation.

272. By a very careful comparison of the intensity of the light which comes from the sun's limb, with that which comes from the centre of its disc, Bouguer assured himself that the latter is the greatest. But as the sun is a round body, its borders are presented to us more obliquely, offering a greater surface under the same angle. The light from this part ought, therefore, to appear more intense. If the fact were otherwise, it must be because there is at the borders some more active cause to diminish the intensity of the light, than at the centre. Such would be the effect of a thick atmosphere; for, if there exist a medium of this kind about the sun, the luminous rays coming from the borders of the disc, must traverse it for a much greater extent than those which come from the centre. It is thus that our atmosphere weakens the light of the sun and moon much more in the horizon than on the meridian. This phenomenon, therefore, furnishes a strong probability of the existence of an atmosphere about the sun.

273. Another remarkable phenomenon to be attributed undoubtedly to the actual state and nature of the sun, is that luminous appearance accompanying it, to which has been given the name of *zodiacal light.* We observe it at evening after sunset, and at the very place where the sun has left the horizon. Its form is that of a very flat lens seen edgwise, and placed obliquely with respect to the horizon, its tranverse axis being extended far into the heavens, as in figure 66. The light is whitish like that of the *milky way;* we have sufficient evidence that it always accompanies the sun, and in total eclipses of this luminary, it may be seen about the disc like a luminous

atmosphere. It is always directed in the plane of the solar equator, and it is on this account that we do not see it, in the evening, equally well at all seasons. For this equator being differently inclined to the horizon, according to the different positions of the sun in the ecliptic, the zodiacal light is inclined also, and for the most part concealed below the horizon; or at least its brightness is greatly weakened by the vapours which rise near the surface of the earth. The most favourable time for observing it in our climate is about the vernal equinox, in February and March. Then the line of the equinoxes is in the horizon at evening. The arc of the ecliptic SS' in which the sun is, rises higher above the horizon than the equator SEZ. The difference is equal to the obliquity of the ecliptic. Thus the zodiacal light, always directed in the plane of the solar equator, which is nearly coincident with that of the ecliptic, is then higher than the equator, by the whole of this quantity. In our climate, no other position of the sun is so favorable. For example, at the summer solstice the arc $S''T$ of the ecliptic, is parallel to the arc EQ of the celestial equator. The luminous pyramid, is at evening, parallel to this equator, that is, much more inclined than at the vernal equinox. The same may be said of all other positions.

274. There are several hypotheses respecting the nature and cause of this light. It was first supposed that it emanated from the atmosphere of the sun; but the author of the *Mécanique Céleste* has shown from its form and magnitude, that this is impossible. Some have thought they could perceive a diminution when the sun had fewer spots, and an increase when the number was greater. But whatever be the cause of this light, it is certain that the matter which it sends to us, is extremely rare, since the smallest stars may be seen through it.

275. With respect to the name zodiacal light, it is derived from a zone called the *zodiac*, extending 8° on each side of the ecliptic, and within which it was formerly believed that the orbits of all the planets were contained. The luminous appearance in question being always comprehended within this zone, it has hence received its denomination.

276. We shall not here speak of the magnitude of the sun, nor of the means employed to estimate its density. These subjects cannot be discussed till we know exactly the distance of the sun

from the earth, and that of the other heavenly bodies. We must, therefore, defer the consideration of them until we have explained the method of determining the sun's parallax, and the true dimensions of the planetary orbits.

Inequality of Days and Seasons in Different Parts of the Earth.

277. We now return to the earth, and since the presence of the sun has so much influence on its productions, we avail ourselves of the knowledge we have just acquired, for the purpose of considering the earth in this new relation. By comparing the different aspects under which it presents itself to the sun in its different positions, we shall be led to some curious conclusions.

We remark, first, that we cannot trace the plane of the ecliptic on the surface of the earth, as we already have that of the equator. For the equator is perpendicular to the axis of rotation of the celestial sphere; and in turning with it, it does not change its position with respect to the earth, which it always cuts in the same points. But the ecliptic is oblique to the axis of the equator; it is fixed in the heavens, but moveable with respect to the earth. In turning with the celestial sphere, it necessarily cuts the earth in different points, and the line of its intersection is perpetually varying.*

But we can fix the portion of this intersection, and determine the part of the earth in which it is always comprehended during the rotation. It is bounded on the north and south by two terrestrial parallels, corresponding to the tropics of cancer and capricorn. If we suppose a straight line drawn from the centre of the earth to two opposite points of the celestial tropics, and cause the earth to turn upon its axis, this straight line remaining fixed, will trace on the surface of the earth the two parallels in question, which receive the same names as the corresponding lines on the celestial sphere. All the places which are situated in them, have one of the points of the celestial tropics for their zenith, and their latitude is equal to the obliquity of the ecliptic or 23° 27′ 47″. These conditions are sufficient to enable us to determine them.

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* We generally trace the ecliptic on globes intended for the study of geography, and represent it as passing through the points where the equator cuts the first meridian. But this is attended with a great inconvenience; for the learner may be led to believe that the intersection of the ecliptic is actually fixed by the points marked on the globe; while it is in reality variable.

278. The *tropic of cancer*, or *northern tropic*, traverses the northern part of Africa, passing to the south of Mount Atlas, through Sienna in Ethiopia, across the Red Sea, and to the north of Mecca. It then enters India south of the Persian Gulf, and passes out through the coast of China; thence over a part of the South Sea to North America, at the southern extremity of California. Finally, it extends through the Gulf of Mexico, and crossing the Atlantic Ocean returns by the western coast of Africa not far from the Canaries.

279. The *tropic of capricorn*, or *southern tropic*, cuts the southern extremity of Africa and the Island of Madagascar; thence it passes through the Indian ocean to New Holland; crosses the whole extent of the South Sea, traverses Paraguay in South America, and returns to Africa through the Atlantic ocean. A great portion of this tropic lies along the surface of water. In general, there is much less land in the southern hemisphere than in the northern.

280. It is well, moreover, in physical geography to distinguish, on the surface of the earth, the two small circles corresponding to the celestial polar circles. If we cause the earth to rotate in the direction of the diurnal motion, the axis of the ecliptic, remaining fixed, will trace on the terrestrial surface the parallels in question. The places which are situated on them have some point of the polar circles for their zenith. Their latitude is therefore equal to the declination of these circles or 66° 32′ 13″. This is the complement of the obliquity of the ecliptic.

281. The *north polar* or *arctic circle* crosses Iceland in the Northern ocean, enters Norway, and cutting the northern extremity of the gulf of Bothnia, traverses Asiatic Russia, the Northern Strait, and having passed through unknown countries in North America, extends through Davis' Straits, and a part of Greenland, and thence returns to Iceland.

282. The *south polar* or *antarctic circle*, seems to be bound on all sides by perpetual ice.

283. In general, the southern hemisphere appears to be colder than the northern. The region of ice surrounding the north pole, scarcely extends to the distance of 9°, while that surrounding the south pole, extends more than 18°, and the enormous masses which detach themselves from it, float 25° or 30° from the pole. The same holds true with respect to the lands abandoned by the waters; and countries like Terra del Fuego, situated in the southern hemisphere, a latitude not exceeding that of France, are covered with perpetual snow.

The two polar circles and the two tropics divide the surface of the earth into five *zones*, which are as much distinguished from each other by their position with respect to the sun, as by the variety of their productions and temperature.

284. To understand these varieties, let us follow the course of the sun from one tropic to the other, and as the heat which it diffuses, depends simply on its elevation and the duration of its presence, we need only notice its motion in declination.

The sun, by reason of its magnitude, illuminates more than half the earth at once; for it is proved by the eclipses of the moon, that the earth's shadow has the form of a very elongated cone. But the rays of this cone cross each other under so small an angle, that, in these general views we may regard them as absolutely parallel. Then, if we suppose a straight line drawn from the centre of the earth to the centre of the sun, the plane perpendicular to this line, will separate the illuminated part of the earth's surface from the dark. The circle forming this limit, is called the *circle of illumination.*

Let us now suppose the sun to be in S at the southern tropic. Fig. 69. Then the ray CS makes with the equator CE an angle equal to the obliquity of the ecliptic. The circle of illumination II' makes with the same plane an angle ICE of 90° — 23° 27′ 47″ or 66° 32′ 13″, equal to the complement of the obliquity. Thus the parallel IP is the arctic polar circle, and $I'P'$ is the antarctic polar circle. Now if we cause the earth to turn about the axis AB of the equator, to represent the effect of the diurnal motion, the following phenomena will present themselves:

The equator EQ will be half in the illuminated and half in the dark part; the night will therefore be equal to the day.

All the points situated between the north pole B and the arctic circle IP, will have no day.

All the points situated between the south pole *A* and the antarctic circle *I'P'*, will have no night.

The parallels intermediate between these two extremes, will have the night or day the longer, according as they are situated north or south of the equator.

Lastly, this position of the sun is that of the winter solstice for the northern parallels, situated beyond the tropics, and of the summer solstice for those to the south.

Fig. 70. As the sun approaches the equator, the circle of illumination *II'* approaches the poles; it leaves a portion of the south polar circle which is thrown into the shade, and embraces a portion of the north polar circle, which begins to be illuminated by the sun. By the effect of this change, the days increase in length to the people of the north, and decrease to those of the south.

In general, the northern parallel which begins to enjoy daylight, has its polar distance equal to the declination of the upper limb of the sun, that is, equal to the declination of the centre minus the apparent semidiameter; and the southern parallel, which begins to be exposed to night, has its polar distance equal to the declination of the sun's centre plus the apparent semidiameter. It is evident that we must here employ the apparent declination, that is, the declination affected by refraction minus the horizontal parallax.

Fig. 71. Having reached the plane of the equator the sun illuminates the earth from one pole to the other, and the nights and days are equal; this takes place at the instant of the equinox.

Soon after, advancing toward the northern tropic, the sun abandons entirely the south pole, which is exposed to continued night; and illuminates entirely the north pole, which is favoured with uninterrupted daylight. At the equator the equality remains constant. Fig. 72.

Fig. 73. This progressive state continues till the sun reaches the northern tropic.

Then all the points situated between the north pole and the arctic circle have continued daylight.

All the points situated between the south pole and the antarctic circle, are exposed to night without interruption. The intermediate parallels have the day or the night the longer, according as they are situated north or south of the equator, where the days and nights are always equal.

This is the instant of the summer solstice for the northern parallels situated beyond the tropic, and of the winter solstice for those to the south.

These phenomena are the reverse of those which take place when the sun is in the southern tropic. In returning toward this tropic the same phenomena are presented in the inverse order.

The zone comprehended between the two tropics always has the sun nearly vertical. The heat is therefore great, and hence it is called the *torrid* or *burning zone.* It is there that nature has developed all her resources. Animals, plants, and even inorganic substances, are there endowed with the liveliest colours; and there are found the most delicious fruits.

On the contrary, the regions extending from the poles to the polar circles, never have the sun, except under a very great obliquity. They have long intervals of day and night; and directly at the poles, there is but one day and one night in the year, each of six months. These regions are exposed to extreme cold; they are consequently barren and almost uninhabitable. They are called *frigid* or *frozen zones.*

The countries, which, like Europe, lie between the tropics and polar circles, never receive the sun's rays under a very great or a very small obliquity, and being never subject to long periods of day and night, they preserve a mean temperature, on which account they are called *temperate zones.*

285. There are, however, several causes which tend to lessen the long continued darkness of the polar regions. In the first place, the smallest visible portion of the sun's disc is sufficient to afford a considerable light. Accordingly the day commences when the centre of the disc is below the horizon 0° 15′ 32″, that is, by a quantity a little less than its semidiameter. This circumstance adds several days to the time the sun is visible, under the polar circles. Refraction also increases this effect, and the more so as it is more considerable in these frozen regions, where the air is always rendered very dense by cold. Another cause which tends to increase the refraction still more, is the almost constant congelation of the surface of the ground, which must render the decrease of the density of the air very rapid, for small heights. These circumstances combined, must often produce extraordinary refractions which render the sun visible much

sooner than it would otherwise be. A phenomenon of this kind was observed in 1597, by three Hollanders, who having advanced to the parallel 75½° north latitude, were overtaken by ice, and obliged to pass the winter at Nova Zembla. After a night of three months duration, the cold having become excessive, the sun suddenly appeared in the horizon at the south 14 days sooner than they expected it in this latitude, and continued from this time to rise higher and higher. If what is here alleged be true, there must have been a refraction of nearly 3½°.

Moreover, twilight which continues longer in these countries than in ours, affords a degree of light sufficient for many important occupations. To understand the reason of this it must be remembered that twilight does not cease till the sun has descended 18° below the horizon. This limit is determined by noting the time which elapses between the setting of the sun and the instant when the small stars in the direction of the twilight become visible to the naked eye. If then we draw a plane parallel to the horizon and passing 18° below the centre of the earth, the twilight will be visible till the upper limb of the sun has reached this plane. Let us suppose ourselves placed at the north pole *B*. We have the equator in the horizon; the sun Fig. 74. will remain visible as long as his upper limb, elevated by the refraction minus the parallax, is above this plane, and will disappear when it passes below; but the twilight will continue till it has descended 18° below. Now the greatest depression of the sun takes place when it is in the southern tropic 23° 27′ 47″ distant from the equator. Thus, if we call the semidiameter of the sun, plus the refraction, minus the horizontal parallax, only 0° 59′ 24″, the time of complete obscurity at the pole, will only be the time employed by the sun in describing

$$23^\circ\ 27'\ 47'' - 18^\circ - 59'\ 24'' \text{ or } 4^\circ\ 28'\ 23'',$$

before and after the solstice, which amounts to only 70 days of night instead of 6 months. This duration would be still less, if instead of supposing ourselves at the poles, we were situated at any point nearer the polar circle, where the means of subsistence are afforded.

286. Moreover, when the moon passes to the north of the equator, it revolves constantly about the pole, and the inhabitants of the polar regions see it continually near the horizon, as they do the sun at the time of its northern declination.

Lastly, the long nights of these regions are alleviated by a variety of meteoric lights, as the aurora borealis, and fire balls which are very frequent, and serve to diminish the dreariness of a polar winter.

287. As we have undertaken to speak of twilight, it may be worth while to give some further details on this subject. Accordingly, let O represent the place of an observer on the surface of the earth, and let him be supposed, at the same time, at the centre of the celestial sphere; let ABH represent the horizon, EQ the equator, HEH' the meridian, and $A'B'C'$ the *crepusculum* circle 18° below the horizon. The twilight will not cease till the sun has reached this limit; but the sun will not always reach this limit in the same time. If we leave out of account the inequalities of its proper motion, which are of little importance in the view we are now taking, the sun will describe the same number of degrees in the same time in every parallel of latitude. Now the arcs AA', BB', CC', are unequal in two respects. 1. They do not contain the same number of degrees. 2. The degrees are not of the same length. These two causes are opposed to each other. If we consider, for instance, a southern parallel, situated near the equator, it will be readily seen that the increase of southern declination tends at first to diminish the length of the arc AA'. For, the parallel becoming south, the plane of the horizon and that of the crepusculum circle, cut it nearer its centre than if it were north. The arc AA', intercepted between these planes, becomes less oblique to them, and as their direct distance asunder is always the same, the length of the arcs intercepted, diminishes with their obliquity. But, on the other hand, as the declination increases, the radius of the parallel diminishes, and the number of degrees belonging to the same length, becomes greater. Whence it is obvious that there must be one parallel where the compensation takes place in the most perfect manner; and it is this which gives the shortest twilight. For the latitude 48° 51′ N. it is found by calculation, that this parallel is situated about 6° 50′ 56″ south of the equator. When, therefore, for the above latitude, the sun's southern declination is 6° 50′ 56″, that is, about the 3d of March and the 10th of October, the twilight is the shortest, being $1^h\ 47'$. The duration, as well as the time of the shortest twilight, is different in different latitudes. But the shortest possible twilight takes place at the equator at the time of the equinoxes. Its du-

Fig. 75.

ration is 1^h 12′. The longest twilight takes place at the time of the summer solstice, with respect to all countries which are in an *oblique sphere*, that is, with respect to those which have the axis of the equator inclined to the horizon. In latitude 42° 23′ 28″ it is 2^h 20′ 31″.

288. By means of the twilight we are able to calculate the height of the atmosphere, or rather of that part of the atmosphere whose density is sufficient to reflect to the observer a sensible light. Let *C* be the centre of the earth, *O* the observer placed Fig. 76. at the surface, *SH* the direction of the solar rays at the end of twilight, that is, when the angle *S′CH′* which they form with the horizon, is 18°. Then the last particles of the atmosphere which give us twilight, are in the horizon at *H*; and the ray *SH*, by which they are illuminated, is a tangent to the earth. Accordingly, if we draw the radius *CO*, and the secant *CH*, the angle *O′CO*, equal to *H′CS′*, will be 18°, and *HCO* will be 9°. Now the tables give, for the secant of 9°, 1,01, radius being unity. *GH* is therefore equal to the hundreth part of the radius of the earth, that is, 39,56 miles. 105.

Thus the superior strata of the atmosphere are elevated at least 39,56 miles. As this result is taken from observations which may not have been very exact, it is not to be regarded as the precise limit. For this reason we have neglected the fractions in the calculation, and paid no regard to atmospherical refraction, although this must modify, in some degree, the duration of twilight.

289. What we have said respecting the duration of twilight, is to be understood as applying only to the general level of the earth's surface. It appears that on the tops of high mountains the light reflected by the atmosphere is visible much longer. This is not to be ascribed to the effect of the atmospheric strata where the observer is placed, for the density of the air grows less and less as we ascend; but the light is reflected by the thick and extensive volume of air, which borders the horizon on all sides. During several nights which Saussure passed on one of the summits of the Alps, about 11000 feet high, called *Col du Géant*, he saw the whole extent of the horizon bordered by a pale but distinct light, which continued from sunset to sunrise; although at midnight the sun descended much below the ordinary limit of twilight, since it was no less than 45° below the

horizon. M. de Humboldt observed a similar light from the summit of the volcanic mountain Antisana. It has been supposed that this phenomenon was occasioned by phosphoric vapours spread through the atmosphere; but it has been proved that a decrease of density in the atmosphere, a little more rapid than that which ordinarily takes place in the lower strata, is sufficient to produce this effect, and might even bring to an observer placed on the summit of a mountain, the light from the opposite hemisphere. We shall hereafter notice a similar phenomenon arising from the same cause, to be observed in eclipses of the moon.

Hypothesis of the Annual Motion of the Earth.

290. Hitherto we have supposed the earth to remain stationary in space, and the sun to move about it in the plane of the ecliptic. But this may be only an appearance; it is possible that the sun is in reality the fixed body, and the earth the moveable one. According to this supposition, the earth must perform about the sun, all the motions that the sun appears to perform about the earth. The annual revolution must take place in an ellipse, the stationary body occupying one of the foci; and all that we have said with respect to the solar ellipse, must be understood of the terrestrial ellipse.

Not only is this supposition possible, but analogy renders it highly probable; for we shall see hereafter that all the planets and comets move thus about the sun. We shall state in succession the arguments we have to offer in favour of this hypothesis; and, in order that their force and connexion may be the better perceived, we shall present the phenomena of the annual revolution, in the same manner as we have done those of the diurnal motion.

291. Let us suppose, then, that the earth has two motions, one of rotation on its axis, and the other of revolution round the sun.

The axis of rotation of the earth which is perpendicular to the terrestrial equator, remains very nearly parallel to itself during each revolution; for the changes which nutation and the attraction of the planets, produce in the inclination of the equator to the ecliptic, amount only to a few seconds in the interval of a year. The earth, therefore, on the supposition that it revolves in an ellipse about the sun, will describe areas proportional to the times.

292. This double motion composed of rotation and revolution, has, therefore, in itself nothing impossible or contrary to the laws of mechanics. We have a striking example in the tops with which children amuse themselves. The motion is produced and supported by lateral impulses, which cause them to turn upon an axis, and at the same time, to describe with their point very various curves. Indeed, it is demonstrated in mechanics that such a motion may result from a single lateral impulse; and this applies at the same time to the top and to the earth, with this difference, that the earth not appearing to experience any resistance in space, there is no need of renewing the force; whereas that of the top is incessantly diminished, in consequence of the friction of its point and the resistance of the air.

293. To show in what manner the phenomena presented to us are explained upon this hypothesis, let us apply it to the difference and inequality of the seasons.

When we suppose the earth stationary, we explain the difference of the seasons by making the sun to ascend and descend from one tropic to the other. When we suppose the earth in motion, the same phenomena are explained by the different aspects under which it is presented to the sun in different parts of its orbit.

At the summer solstice, about the 22d of June, the earth is near its *aphelion*, or point of greatest distance from the sun; then its position is that which is represented in figure 77. The sun is vertical at the northern tropic $T\,t$; the north pole is entirely illuminated, and it is summer in the northern hemisphere.

At the winter solstice, about the 22d of December, the earth takes a position directly the reverse, as represented in figure 78. It is now near its *perihelion*, or point of least distance from the sun; the rays of the sun fall vertically upon the southern tropic $T'\,t'$, and the south pole is completely illuminated. It is winter in the northern, and summer in the southern hemisphere.

Between these positions, there are also two others opposite to each other, in which the plane of the terrestrial equator passes through the centre of the sun. These are the equinoxes which happen about the 21st of March and the 24th of September.

This brief statement is sufficient to show how the explanations already given from observation, apply to the motion of the earth.

Precession of the Equinoxes considered as the Effect of the Displacement of the Terrestrial Equator.

294. We have seen that the annual motion of the sun in the ecliptic may be an appearance caused by the real motion of the earth in a contrary direction. The same is true of the precession of the equinoxes, and those small motions of the stars referred to under the name of nutation. In general, we may observe that this must be the case with respect to all those general motions which appear to belong to the heavenly bodies. There is no more reason for attributing them to the heavenly bodies in common, than to the earth simply, in a contrary direction. Indeed, if we were not influenced by the involuntary habit of considering our position as fixed, it might be said that the idea of referring these motions to the earth is infinitely more simple than the other. But conjectures alone are not sufficient to enable us to decide such a question. We must enter into details, and find arguments on which to rest our opinions. Let us then inquire whether the phenomena of precession can be represented by each of the two hypotheses, and by which they can be explained with the greatest simplicity.

295. When we consider the motion of the sun as apparent, and that of the earth as real, the signs of the ecliptic correspond to the different points of the earth's orbit. Fig. 79.

Then a spectator on the earth sees the sun always in the part of the ecliptic, opposite to that which he occupies. When he is in the sign aries, he sees the sun in libra; when in taurus he sees the sun in scorpio, and so on. As the earth moves in the order of the signs, the sun appears to move in the same direction in the opposite part of the orbit.

We proceed to show how the precession of the equinoxes may be explained according to this hypothesis, without supposing the whole celestial sphere in motion.

296. Let *TT′ t t′*, be the annual orbit of the earth, having the sun in one of the foci. Let *ETQ* be the plane of the equator, and *TP* the axis which is always perpendicular to it; the equinox will take place when the line *TQ*, the intersection of the equator and ecliptic, passes through the centre of the sun; for then the sun will be in the plane *ETQ* of the equator. Fig. 80.

The equator being supposed to remain parallel to itself during each revolution, there will be two opposite positions in the orbit, *T* and *t*, when an equinox will take place; one will be the vernal equinox on the 21st of March; the other the autumnal on the 24th of September. The passage of the earth through these points does not then divide the year into two equal portions. This is owing to the nature of the elliptical motion, and to the present position of the perihelion, as we have already seen. According to the values above obtained, we find that, at the present time, the earth employs $186^d\ 11^h\ 19'\ 16'',61$ in passing from the vernal to the autumnal equinox, and only $178^d\ 18^h\ 29'\ 43'',30$ in returning from the autumnal to the vernal.

If the line *TQ* remained always parallel to itself, the equinox would always take place in the same points. But while the earth moves in the direction *TO*, this line itself has a slight motion; and when the earth arrives at *T′*, it no longer has the direction *T′S′*, parallel to *TS*, but that of *TQ*, which makes with *T′S′*, a small angle *S′T′Q′*, equal to 50″,1. Then the line *T′Q′* passes through the sun's centre before the earth arrives at *T*; and the equinox takes place sooner than it otherwise would do. In this consists the *precession of the equinoxes*.

The line *T′Q′* produced appears to correspond to *t′* in the ecliptic. It seems, therefore, to fall back each year by the quantity *t t′*, contrary to the order of the signs, or to the annual motion of the earth. For this reason, the motion of the equinoxes is said to be *retrograde*.

To complete the explanation of the motions of the terrestrial equator, we have also represented in the figure the circle parallel to the ecliptic, described by the mean pole of the equator, in virtue of the motion of precession; and the small *ellipse of nutation*, in which the *apparent pole* oscillates, according to the laws laid down in a preceding section.

THEORY OF THE MOON.

General Phenomena attending the Motions of the Moon.

297. We have made known the phenomena attending the proper motion of the sun, the laws of this motion, and the applications of these laws to practical purposes. We have thus fulfilled the design of astronomy with respect to this luminary. We shall proceed in the same manner with the motions of the other heavenly bodies, advancing by observation and reasoning from the most simple appearances to the most complicated laws. But we shall find the inquiry greatly facilitated by what we have already done; for, the end being the same, the same methods may be almost always employed; and if diversity of circumstances obliges us to modify them, the changes necessary to be made will be indicated by the nature of the case.

298. We shall begin with the moon whose appearance is so remarkable, and follow out the series of phenomena which it presents. Its light is paler than that of the sun, and it produces no sensible heat. It is subject to periodical variations in its degree of illumination, to which we give the name of *phases*. If we observe it when it passes the meridian at midnight, its disc will be found to be completely illuminated, and of a circular figure; it now rises when the sun sets, and sets when the sun rises. If we continue to observe it for several days, we shall perceive that it gradually loses its light. The illuminated part of its disc grows less and less; at the same time it rises later; and when its disc is reduced to a semicircle, it is visible only during the last half of the night. Some days after, this figure is reduced to a *crescent*, the extremities of which are turned towards the west, that is, towards the part of its disc most distant from the sun. It then rises only a short time before the sun. At length, by the daily diminution of the crescent, the moon be-

comes entirely obscured; it now rises with the sun and ceases to be visible.

After having disappeared for three or four days, it reappears at evening in the west a little after sunset. At first it exhibits only a thread of light which gradually increases in width. For several days it presents the form of a crescent, the extremities of which are turned towards the east, that is, to the part of the heavens opposite to the sun. It continues to recede from the sun, its disc being gradually enlarged, until at length it regains that full, round, bright figure, we first observed. The period of these phases is about 29½ days.

Such are the phenomena presented by the moon. In order to attain to a sure knowledge of the causes of these appearances, we must, in the first place, endeavour to acquire some precise notions respecting the lunar motions.

First Approximation to the Moon's Motions.—Theory of its Circular Motion.

299. We have seen that the moon, after passing the meridian at the same time with the sun, arrives at it later and later each succeeding day, receding in a direction opposite to that of the apparent diurnal motion. It has, therefore, like the sun, a proper motion, but more rapid, and directed the same way, namely, from west to east. This is fully confirmed by the manner in which it recedes from the stars which it meets with in its course.

To ascertain the precise law of these motions, we employ the general method laid down in the first section of the preceding part as applied to the sun. We observe for several days the meridian altitude and right ascension of the moon's centre, and thence deduce by calculation its longitude and latitude. The The results of these calculations determine the velocity and direction of its angular motion.

300. In this manner we find that the moon describes on the celestial sphere a great circle, of which the earth is the centre. This great circle is oblique both to the equator and the ecliptic. Its inclination to the latter varies within moderate limits which we shall soon determine. But its mean value is 5° 08′ 59″,93. The variable values which it has each revolution, may be de-

duced immediately from the results of observations employed in determining the moon's greatest latitude; in the same manner as the obliquity of the ecliptic is deduced from the greatest declination of the sun.

301. *The position of the moon's nodes*, or the intersection of the moon's orbit with the ecliptic, is ascertained by observing when the latitude is nothing; for then it is obviously in the plane of the ecliptic, and consequently its longitude is the same as that of its node.

302. The *ascending node* is that through which the moon passes when moving from the south side of the ecliptic to the north; and the *descending* is that through which the moon passes in returning from the north side to the south. The longitude of the ascending node, or its distance from the vernal equinox, on the first of January 1800, was 15° 55′ 26″,29, which determines for this epoch the position of the lunar orbit.

303. These nodes are not fixed in the heavens, but have a retrograde motion in the ecliptic. The mean arc described by the moon parallel to the ecliptic in a hundred Julian years, forms what is called its *secular tropical motion;* it is the difference between the mean longitudes for this interval. Its value in 1800, was 481267° 52′ 45″,4764 or simply 307° 52′ 45″,4764, taking out 1336 entire circumferences. This arc, divided by 365,25 days, gives the *daily tropical motion* equal to 13° 10′ 35″, about 13 times as great as that of the sun.

304. With these data we determine by a simple proportion the time which the moon employs in returning to the same longitude at any given epoch, or its *periodical revolution.* It is equal to $\frac{360^\circ \times 365{,}25}{481267^\circ\ 52'\ 45'',4764}$ or $27^{d}\ 7^{h}\ 43'\ 04'',7$. This is called a *periodical lunar month.*

305. If from the secular tropical motion we subtract the precession of the equinoxes for a century, that is, 1° 23′ 30″, 01, we shall have the *secular sidereal motion*, or the moon's motion with respect to the stars; for the motion of the stars and moon with reference to the equinoxes being in the same direction, their relative motion is equal to the difference of the arcs which they describe. We thus obtain 481266° 29′ 15″,46 or simply 306° 29′ 15″,46 for the secular sidereal motion of the moon; and by a simple proportion we obtain $27^{d}\ 7^{h}\ 43'\ 11'',5$ for its sidereal revolution.

These two revolutions of the moon are therefore connected together by the precession of the equinoxes, and may be deduced the one from the other; they gradually diminish from century to century, since the motion of the moon is accelerated; and these variations are perceptible in observation made after inconsiderable intervals.

306. In general the motions of the moon, like those of the sun, are composed of a small number of elements, whose mean value remains the same; of several *secular* inequalities, which change these values very slowly; and lastly, of a great number of periodical inequalities which go through their successive values at short intervals. These last sometimes accelerate and at others retard the motion of the moon; whereas the secular inequalities observe a constant course of increase or diminution; at least they have done so from the time of the most ancient observations to the present; and many centuries must yet elapse before their periods will be completed. The effects of these different causes will hereafter be examined in detail. For the present we barely mention them.

307. In these motions, as in those of the sun, we cannot arrive at once at accurate results. We begin with approximations, and arrange the results in tables. By comparing these tables with the observations of several years, we can detect the errors which exist in the mean motion; for these continually increase with the time, while those arising from the periodical inequalities, only oscillate within certain limits. It is obvious then, that in the latter, as in case of the sun, the effect is weakened when we compare observations made at very long intervals, for they must have been compensated and reduced to nothing a great many times, thus leaving only mean results. Accordingly, by such observations the mean duration of the moon's revolution may be obtained with a great degree of accuracy for such intervals, and may even be employed without great error for several years before and after. But so long as the secular inequalities which affect the moon's motion, are not all known, their variable effect being included in the estimate of the mean motion, must necessarily affect its value, and affect it unequally according to the epoch of the observations. It is necessary, therefore to know all these inequalities and to make allowance for them in the results of observation, before we can ascertain the true mean

motion. Then only can we foresee the changes which must be made in the tables; and in this way we can extend them with confidence to distant centuries before or after the time of the observations which have served to form them. It is the theory of attraction which has furnished us with the knowledge and measure of these phenomena, and enabled us to determine their laws exactly. We can here only indicate these laws.

On the first of January 1801, at mean midnight for the meridian of Paris, the *mean longitude* of the moon was 111° 36′ 42″,088. This, together with the mean motion, enables us to calculate the mean longitude for any other epoch anterior or posterior. This primitive value of the mean longitude is deduced from observation, in the same manner as the *epoch* of the solar tables, or the instant of the mean equinox. From a great number of observations, made at short intervals, we calculate the value of the true longitude; we then correct these values for the periodical inequalities laid down in the tables already existing; the result is the mean longitude for the moment of observation. We reduce all the values thus obtained to the same epoch, by knowing simply the mean motion; for even when there remains some little uncertainty with respect to this element, the influence of it is not sensible in the reduction of observations made near to each other. This being done, we take the arithmetical mean of all the results to prevent any small errors in the observations; and we thus obtain, with precision, the value of the mean longitude for the epoch in question. By performing the same reductions and calculations upon observations separated from the first by a long interval, we obtain another value of the mean longitude reduced to another epoch very distant from the first; the difference between these two longitudes will verify the mean motion which we employ, and if it involve any small error, will enable us to correct it.

308. Hitherto we have spoken only of the proper motion of the moon; that of its nodes is measured in the same manner by the difference of their longitudes at different epochs; and the time employed in returning to the same longitude, gives the *tropical revolution of the nodes.* The mean secular and tropical motion of the nodes in 1800 was 1936° 56′ 26″,639, which gives 6788^{d} 12^{h} 57′ 52″,42 for their tropical revolution, and 6793^{d} 10^{h} 06′ 29″,95 for their sidereal revolution.

If this motion were uniform, it would be sufficient to know, by observation, the longitude of the nodes for a certain epoch, in order to deduce by a simple proportion, their longitude for any other epoch. But their motion is subject to several inequalities, and is moreover diminishing from century to century, as we shall see hereafter.

Phases of the Moon.

309. HAVING ascertained the circular motion of the moon, we now proceed to investigate the cause of those periodical variations which we witness in its light. We cannot reasonably suppose that this body successively acquires and loses the capacity of shining; for, in this case, we could not attribute to any probable cause the regularity of these changes. Neither can we suppose that it has a luminous and an obscure face which it shows us alternately; for it is ascertained that it always presents to us the same face; indeed when its disc is only partly illuminated, its obscure part is not altogether invisible. It still presents a faint light by means of which the same spots and sinuosities may be observed, as when the whole disc is illuminated. These phenomena can only be explained, in a reasonable manner, by supposing the moon to be a round body, in itself obscure, and shining with a borrowed light; in which case, the variations in its phases will be the simple effect of its different positions.

But if the moon is an opaque body, whence does it receive its light. To determine this, let us observe the course of its phases. These are connected in the most obvious manner with its position in reference to the sun. When the moon is entirely illuminated and passes the meridian at midnight, the sun is below the horizon; the face which it presents to us is therefore enlightened by this luminary, and its light ought to be reflected to the earth, provided it be so far removed as not to be enveloped in the earth's shadow. On the contrary, when the moon and sun appear in the same part of the horizon, at the same time, the enlightened face being always turned towards the source of its light, we do not perceive it, or perceive only a small part of it;

and that face which is turned towards us is in the shade. Generally when any part of the moon's disc appears illuminated, that part is turned towards the sun, and so situated that its light may be reflected to the earth. It is moreover proved that the moon is an opaque body. From all these considerations then, we must conclude that the light which we receive from the moon comes originally from the sun.

This opaque body must appear as a dark spot when it crosses the disc of the sun in passing between the sun and the earth. It must also be obscured when, being opposite to the sun, it passes into the conical shadow which the earth projects to a distance behind it in space. Hence the phenomena of eclipses.

Reciprocally, the earth must perform the office of a moon to this opaque body whenever it is properly placed, that is, it must reflect to it the rays it receives from the sun. Accordingly, when the face which the moon presents to the earth is not wholly illuminated by the sun, the portion which does not receive the direct light, is not entirely obscure. If we look at it through a telescope, we shall perceive a feeble light which renders it sensible, and serves to define its outline in the heavens.

310. With the knowledge of these truths we are led to the discovery of others. The moon presents in its phases exactly the same appearances that would be presented by a round body, always seen from the same point, and illuminated successively on different sides. The light does not spread over it suddenly, as it would do if its disc were a plane surface without thickness; but advances in a slow and progressive manner. The convexity of the surface of the moon is therefore rendered evident by these regular increments in the portion illuminated, and we may hence infer that it is very nearly spherical.

.311. This appears with still stronger evidence, when we consider the order of the phases. Figure 82 represents all the positions of the lunar spheroid with respect to the earth and sun. These positions are projected on the plane of the ecliptic. *C* is the centre of the earth, and *CS* the direction of the solar rays, which, for greater simplicity, are supposed to be parallel to each other. The face which the moon continually presents to the earth, is successively obscure, or illuminated to a greater or less extent; and the curve which forms, on this face, the separation between the light and shade, being seen in perspective from the earth, takes successively different figures according to the positions of the spheroid. In the first place, when the moon is at L on the same side of the earth with the sun, we do not see it. It is then called *new moon.* When it passes out of the line of the rays falling directly upon the earth, as to L', the luminous part appears under the form of a crescent whose points are turned to the part of the disc most distant from the sun. In L'' the limit of the shadow is a straight line, and we say that the moon is in its *first quarter.* In L''', the luminous part approaches more nearly to the appearance of a circle; and finally when it arrives at l, it becomes a complete circle, and is then called *full moon.* The same appearances are presented in an inverse order, in passing from l to L. In l'' it has the form of a semicircle, which is called its *last quarter;* and after completing its revolution, it again returns to the state of new moon in L.

These positions do not in reality take place in the ecliptic, but in the orbit of the moon which is inclined to this plane. We have supposed them to coincide for the sake of simplicity; but if we would see the phenomena as they actually present themselves, we must have recourse to figure 83, where C is the centre of the earth, $LL'L''L'''$ the moon's orbit, $l\,l'\,l''\,l'''$ the projection of this orbit on the plane of the ecliptic, and CS the direction of the radius-vector, drawn from the earth to the sun. The different positions which the lunar spheroid takes with respect to this radius-vector, above and below the plane of the ecliptic, give rise to the different phases of the moon in the order in which we have considered them.

312. We have also, in this first view, supposed the sun immovable in the plane of the ecliptic; but in reality, this body and its radius-vector appear in motion in the ecliptic. This circumstance, combined with the proper motion of the moon, determines the period of the return of the different phases. If the moon's revolving motion were exactly the same as that of the sun, so that the same interval of time would bring the two to the same point of their orbits, the two radius-vectors, drawn to them from the earth, would always preserve the same angular distance, and the same relative position with respect to each other. We should then see the moon always under the same aspect with respect to the sun, and the enlightened part of its disc would always preserve the same magnitude. For example, if it had been placed

originally at the distance of 90 degrees from the sun, we should always see it in its first quarter; and if it had been placed in opposition, we should always see it a full moon. Hence, in order that the moon may have the different appearances which it actually presents, it is necessary that its revolving motion be not the same as that of the sun.

313. This naturally brings us to consider the period of the different phases, or the interval between two consecutive returns of the moon to the point of conjunction. If we take into view only the mean motions, this inquiry is very simple. The circle of latitude which passes through the moon only removes from the sun by the excess of the moon's motion in longitude. Now the mean secular motion of the sun, is 36000° 45′ 44″,9993. That of the moon is now 481267° 52′ 45″,4764. Their difference, equal to 445267° 07′ 00″,4771, is therefore the quantity by which the moon has removed from the sun in the space of 100 Julian years, by virtue of its mean motion. Hence we see by a simple proportion, that it removes from it 360° in the number of days expressed by

$$\frac{360^\circ \times \overset{\text{days}}{365{,}25}}{445267^\circ\ 07'\ 00'',4771} \text{ or } 29^d\ 12^h\ 44'\ 03''.$$

This interval is denominated a *synodic revolution* of the moon, from two Greek words which signify *to move together*. It is also called a *lunar month*.

314. We can also determine the time which elapses between two consecutive returns of the sun to the node of the lunar orbit. The secular motion of the node is 1934° 11′ 14″,928; that of the sun is 36000° 45′ 44″,9993. As they move in contrary directions, the relative motion of the sun and the node is equal to their sum or 37934° 56′ 59″,9273. This is the quantity by which the sun and the node depart from each other during 100 Julian years. Hence, by proportion, the time required for a departure of 360°, is expressed by $\frac{360^\circ \times 365{,}25}{37934^\circ\ 56'\ 59'',9273}$ or is equal to $364^d\ 14^h\ 52'\ 16''$. This period is denominated the *synodic revolution of the nodes*. It is less than the tropical year, for the obvious reason that the node moves round to meet the sun, and the junction consequently takes place before the sun has made a complete circuit of the heavens. As we have made use of only mean motions, it is evident that our results will not always con-

form exactly to the true motion of the moon; but they will not vary much. By neglecting the periodical inequalities, which alone affect them, they will become exact.

315. Several other points of the lunar orbit deserve attention on account of their relation to the different phases; such, for example, as those in which the radius-vector of the moon coincides with that of the sun. Then these two bodies, together with the earth, are situated in the same plane, perpendicular to the ecliptic, and the longitude of the moon is the same with that of the sun, or exceeds it by 180°. These points are denominated *syzygies*. When the moon is thus situated, it is either between the sun and the earth, or the earth is between the sun and the moon. In the first case, the moon is said to be in *conjunction*; in the second, in *opposition*.

316. New moons and solar eclipses take place when the moon is in conjunction. In order that the latter phenomenon may happen, it is not absolutely necessary that the moon be in the plane of the ecliptic; it is sufficient that the distance of its inferior limb from this plane, be less than the sun's apparent semidiameter.

317. On the other hand, full moons and lunar eclipses take place when the moon is in opposition. It is necessary for a lunar eclipse, that the distance of the moon's nearest limb from the plane of the ecliptic, be less than the semidiameter of the shadow projected by the earth at this distance. We shall again recur to these phenomena.

318. The points in which the moon attains its first and last quarters, are denominated *quadratures*. The angular distance of the moon from the sun is then 90°; that is, the projection of the moon's radius vector on the plane of the ecliptic makes an angle of 90° with the direction of the solar rays. The points intermediate between the syzygies and quadratures, which are 45° from the sun, are denominated *octants*.

Moon's Apparent Diameter and Parallax.

319. HITHERTO we have considered only the moon's motion about the earth, and its successive positions on the celestial sphere. But its distance from the earth is subject to variations

so considerable as to deserve attention. They are indicated by observations of its apparent diameters. These diameters are capable of being directly measured by means of micrometers. They may also be deduced from occultations of the stars, by observing the time which elapses between immersion and emersion. By means of these different methods, we find that the greatest apparent diameter of the moon is 2011″,07, or 33′ 31″,07, and the least 1761″,91 or 29′ 21″,91. The corresponding values for the sun are 1955″,56 or 32′ 35″,56, and 1890″,96 or 31′ 30″,96. It follows then that the variations of the moon's distance from the earth must be considerable, since they are sufficient to render its apparent diameter greater than the greatest, and less than the least, apparent diameter of the sun.

320. This conclusion is confirmed by observation of the parallaxes. That of the moon is, as we have before seen, so great that it may be obtained by methods incapable of being applied with complete success to the sun, on account of its distance. By comparing the parallaxes of the moon, observed at different epochs, in the same place, we find very great variations. The greatest horizontal parallax of the moon at Paris, is 1° 1′ 29″,32, and the least 0° 53′ 51″. It has successively all possible values between these two extremes.

321. The greatest and least distances of the moon from the earth, obtained from these parallaxes, are 63,8419 and 55,9164. The arithmetical mean of these two is 59,879118, nearly 60 times the radius of the earth for the latitude of Paris. The distance of the sun is incomparably greater, as we have before seen.

322. If we divide the greatest distance by the least, their ratio 1,1417, will express the greatest variation in the moon's distances from the earth. The same result may be obtained by dividing the greatest parallax by the least, since these are inversely as the distances. For the sun, this value was only 1,034. Combining the above results, we arrive at two important conclusions. The moon is incomparably nearer the earth than the sun, and is subject to variations in the ratio of its distances, which are accordingly much greater.

323. The moon's parallax not only varies with different epochs, in consequence of these inequalities, but its value is not the same at the same epoch, when it is observed at different places on the earth's surface. Modern observations give very

perceptible differences in this respect. In order to explain this phenomenon, or rather to understand the consequences to be thence deduced, it must be remembered that the horizontal parallax of a heavenly body is the angle under which we should see, from this body, a radius drawn from the centre of the earth to the observer. This parallax, not being the same at the same instant in different places, is a proof that all the radii of the earth are not equal, and that the surface is not perfectly spherical. Thus, by the variations of its parallax, the moon makes known the inequalities of the terrestrial surface, as by its eclipses it discovered to us the earth's globular form.

The inequality of the terrestrial radii, therefore, has an influence upon the apparent positions of the moon as observed in different latitudes. It affects them also in the same place, according to the vertical planes in which the moon is situated at the moment of observation. It is very necessary to pay attention to these differences, when we wish to fix with precision the apparent positions of the moon; for the supposition of the earth's perfect sphericity might occasion sensible errors in the epoch of a star's occultation, or of any other phenomenon where precision is important. In determining what corrections are necessary, we are obliged to adopt an hypothesis respecting the figure of the earth, and we usually suppose it elliptical, which leads to no material error.

324. If we divide the apparent semidiameter of the moon reduced to the centre of the earth, by the value of its horizontal parallax, for a given place, we find between these quantities a constant ratio. From what has been demonstrated, we see that 143. this ratio is that of the radii of the moon and earth, supposed to be spherical. The value of this ratio for the equator, is determined by the most exact observations to be 0,27293056, supposing the semidiameter reduced to the centre of the earth, to be 0° 16′ 22″,5494 when the parallax is 1°. This result being once obtained, we make use of it to calculate the apparent diameter of the moon when its horizontal parallax is known; and *vice versâ.*

325. As the moon describes its diurnal circle about the earth, each observer, placed on the earth's surface, sees it nearer when in the zenith, than when in the horizon. This difference, as before stated, produces a sensible effect upon its apparent diameter,

which increases as it rises. This increase is easily calculated, and its total value from the horizon to the zenith is found to be about $\frac{1}{60}$, because, during this interval, the distance of the moon from the observer diminishes by a quantity equal to the radius of the earth, which is nearly a sixtieth part of this distance.

Second Approximation to the Motion of the Moon.—Theory of its Elliptical Motion.

326. We have seen that the moon's distance from the earth is subject to considerable variations. Its motion therefore is not circular, as it appears to be when projected on the celestial sphere. By measuring a great number of these distances and comparing them with the angular motion, we ascertain the nature of its orbit. The method is the same as in the case of the sun. In this manner we have obtained the following results which may be regarded as certain.

The moon's orbit is an ellipse having the centre of the earth for one of its foci. The radius vector describes areas nearly proportional to the times. The mean distance being taken for unity, the eccentricity of the orbit was in 1800, 0,0548553, which gives 6° 17′ 54″,49 for the greatest equation of the centre. The methods to be pursued in obtaining these results have been explained in the elliptical theory of the sun.

327. The moon's perigee has a direct motion, which is measured by determining, from observation, the longitude of the perigee at different epochs. In the beginning of 1800 this longitude was 266° 6′ 5″,098. The motion of the perigee with respect to the equinoxes, was then 4069° 2′ 22″,2 in a century, which gives for its tropical revolution 3231^d 11^h 24′ 8″,6, and for its sidereal, 3232^d 13^h 56′ 12″,5. These periods change with the time, since the motion of the perigee is retarded, while that of the moon is accelerated. But their variations are very small, like those of the mean motion, with which they are found to be connected by the laws of attraction.

328. These results being obtained, we can easily calculate the anomalistic revolution of the moon, that is, its revolution with respect to the apsides. For, its proper motion being in the same

direction with that of the perigee, it moves toward this point by the excess of its velocity over that of the perigee. This excess is 477198° 41′ 30″,62 in a century, as may be easily ascertained from what precedes; and this gives for the anomalistic revolution $\frac{360° \times 365,25}{477198°\ 41'\ 30'',620}$ or $27^d\ 13^h\ 18'\ 37'',4$.

329. Such are the present elements of the lunar ellipse. We have already said that they change gradually with the time. But even when we take into account these variations, the elliptical motion is still very far from representing exactly the course of the moon. This body is subject to periodical perturbations, the effect of which is very considerable, and to which it is absolutely necessary to have regard, even in the least accurate observations. In order then, to attain to precise ideas of its motions, we must know the value of these inequalities. We shall proceed to explain the principal ones, beginning with the secular changes of the elliptical elements. But this course, which is the most methodical and the most simple, is not according to the order in which they were actually discovered. The results which we shall first consider were not obtained independently of those which follow. They were not fixed, often not suspected, till others which here follow them, were recognised and determined with considerable accuracy, by observation, the errors of the first supposition being gradually corrected as they were developed by time.

Secular Equation of the Moon's Mean Motion.

330. As the motion of the moon and the elements of its orbit do not remain constantly the same, the progressive variations which they experience, must soon render the tables inaccurate, and make it necessary to correct them by new observations. The only means of avoiding this inconvenience, is to calculate these elements for various distant epochs, and thence to deduce the variations which they may have undergone, seeking afterwards the laws most proper to satisfy them.

Let us apply these considerations to the mean secular motion of the moon, which has been determined with great accuracy by modern observations, and which is made the basis of all the other results. Let us compare this with the result of ancient observations and see whether it has always been the same, or whether it has undergone any change.

The process by which this is done, is the same as that employed for correcting the tables of the sun. The ancient observations are regarded as so many observed longitudes. We calculate the place of the moon by the tables for the same epoch, and see whether the results agree or not with the observations.

331. Let us take as an example the ancient lunar eclipse, observed by the Chaldeans 721 years before the Christian era, and recorded by Ptolemy.* From observing the phases of this eclipse, we can determine the instant when the longitude of the moon was in opposition to that of the sun. This last is easily calculated by means of the tables of the sun, since Ptolemy recorded the day and hour of the observation; and we thus obtain the true longitude of the moon for the moment of the eclipse.

If we correct this for the periodical inequalities which affect it, and which may be exactly estimated by the tables, we shall have the mean longitude.

Now if we compare this result with another mean longitude deduced from modern observations, the difference of the longitudes, increased by the proper number of circumferences, will make known the arc described by the moon parallel to the ecliptic, in virtue of its mean motion, during the interval of the observations; and, dividing this quantity by the number of centuries elapsed, we shall have the mean secular motion.

Now the mean motion thus determined by MM. Delambre, Bouvard, and Burg, is found to be less by 3 or 4 minutes than that deduced from modern observations. The same result is obtained from two other eclipses observed by the Chaldeans in 719 and 720, B. C. This agreement leaves no doubt that the result is correct. On the other hand, the accuracy of modern observations is so great as to leave no room for such considerable errors. We must, therefore, conclude that the mean motion of the moon is more rapid now than formerly; or, in other words, that the estimates of this motion involve a considerable inequality, for which allowance must be made before it can be regarded as uniform.

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* It was observed at Babylon the 19th of March, 721 years before Christ. The moon began to be eclipsed about an hour after rising. Lalande has calculated this eclipse in his memoir upon the secular equation of the moon.—*Académie des Sciences*, 1757, pp. 429.

New evidence is obtained on this point when we compare the results of modern observations with those deduced from eclipses later than the preceding; for, if the motion of the moon has really been accelerated during this interval, this new comparison ought to give a motion inferior to the present; but the epoch of the ancient observation, here used, being nearer our own time than the former, the difference must be less than in the preceding case. This is actually found to be the result, when we make use of the eclipses observed at Cairo, by Ebn-junis, an Arabian astronomer of the tenth century.

Such a coincidence must put beyond question the reality of the phenomenon, and the inevitable conclusion is that the motion of the moon has been accelerated from the time of the observations of the Chaldeans to that of the Arabs, and from the time of the Arabs to the present.

332. In order to represent this acceleration, we must add to the mean longitude of the moon a term proportional to the square of the time, and another smaller proportional to the cube. This is called the *secular equation* of the moon. With this correction the present tables satisfy the most ancient observations, and might be extended to a thousand or twelve hundred years beyond the epoch under consideration.

But it must not be supposed that this acceleration will continually go on, and that the preceding formula will always be applicable. The theory of attraction, which makes known the cause of this acceleration, has shown that it is periodical, and connected with variations in the eccentricity of the earth's orbit; so that after increasing for a certain time it will be changed into a retardation. But the extent of this period is very great, and only a small portion is comprised in the interval which separates us from the ancient observations. The inequalities which result from this cause, in the motions of the moon, will amount at least to a fortieth part of a circumference. Future generations, upon witnessing these great phenomena, will undoubtedly acknowledge their obligations, and do justice to the geometers of the present age whose sagacity enabled them to foresee and calculate them, and furnished their successors with the means of judging of the past and future condition of the system, as we judge of its present state.

The celebrated astronomer, Dr Halley, was the first who recognised the existence of this acceleration; and it was Laplace who, by a most profound analysis, made known the cause of so remarkable a phenomenon. The merit of the discovery must, therefore, be divided between the observer and the geometer. The last achievement is undoubtedly the greatest, because it is of the nature of a demonstration; at any rate it is of more importance to the science which it perfects. But those who consider how difficult it is for the most enlarged minds to renounce ideas long and generally cherished, will perceive that it required the firm and resolute spirit of a philosopher to question the uniformity of motions which had been regarded as uniform for 2000 years.

333. The variation in the motion of the moon has an influence upon the duration of its tropical, synodic, and sidereal revolutions, all of which vary in different centuries. It will therefore be always impossible to find periods during which the sun and moon shall long agree in performing each an exact number of revolutions. Such periods can be extended only to a small number of centuries, and any attempt to carry them farther will be of no value.

Secular Equations which affect the Elements of the Moon's Orbit.

334. The motion of the lunar perigee is uniform; it is subject to a secular equation connected with that of the mean motion, and depending on the same cause. The comparison of ancient and modern observations, places this fact beyond a doubt; but we owe its discovery to the theory of attraction. This inequality is equal to that of the mean motion multiplied by the coefficient — 3,00052.

If from the mean longitude of the moon we subtract the mean longitude of the lunar perigee, we shall have the mean anomaly of the moon. This anomaly is therefore subject to a secular equation, equal to the difference of the two preceding, that is, to that of the mean motion multiplied by 1 + 3,00052 or 4,00052.

335. The motion of the nodes is also subject to a similar inequality, and of the same sign with that of the perigee. It is equal to that of the mean motion multiplied by — 0,735452. As the motion of the nodes is retrograde, it is obvious that this correction tends to augment their longitudes for the centuries posterior to that taken for the origin. All these results are confirmed by observation.

336. It is evident from what is here said that, on account of the secular equations, the motions of the perigee and the motion of the nodes, are retarded, while that of the moon itself is accelerated. Moreover, these inequalities are connected together by very simple ratios, since they are represented by the numbers 1; — 3,00052; — 0,735452.

337. The anomalistic revolution, depending at once upon the motion of the moon and that of the perigee, is likewise modified by the secular equation. The same is true of all the quantities which depend upon the mean longitude, the perigee, or the nodes.

338. The same analysis which disclosed these great phenomena, has shown that the distance of the moon from the earth, the eccentricity and inclination of its orbit, are, in like manner, subject to secular equations connected with those of the mean motion. But their effect has hitherto been very inconsiderable. Still, in the lapse of ages, it will be necessary to take account of it. The expression for the transverse axis of the orbit contains no inequality of this kind; this is moreover a result of theory.

The discovery of all these relations is due to M. Laplace. He was led to it by a knowledge of the cause which produced the secular equation of the mean motion, a cause which will be explained hereafter. It is seldom that a great discovery does not admit of important and multiplied applications.

Periodical Inequalities of the Lunar Motions.—Methods of ascertaining them by Observation.

339. All the inequalities hitherto discovered in the motions of the heavenly bodies, are confined within certain limits, and are subject to periods more or less extended; but some of them complete their periods only after a great number of centuries; and

their increments may, for a long time, be regarded as uniform; to such we have given the name of *secular inequalities*. The term *periodical* has been confined to those which return successively to the same values, after intervals of time so small, that their recurrence admits of being frequently observed, and their laws of being determined. This distinction has already been adverted to several times.

340. We at once perceive the effect of these inequalities in the motions of the moon, when we compare its actual positions, as given by observation, with those it ought to have in virtue of its elliptical motion. For if we calculate beforehand its longitude, latitude, and apparent diameter, according to this hypothesis, taking account also of the motion of the perigee and nodes, we still find considerable deviations, which are appreciable even for a small number of days, and which did not escape the ancient astronomers notwithstanding the imperfection of their instruments.

These deviations are not always the same; they vary periodically, and are reproduced successively in the same order, after regular intervals. Careful observers have followed out and determined their phases, and recourse has been had to theory to explain and confirm them. They are introduced into tables as so many corrections to be made in the elliptical motion.

341. These various inequalities would have been estimated with difficulty if they had all been confined to periods that differed but little from each other; for they would then have been confounded together in such a manner as to render it almost impossible to separate them by observation. Fortunately it happens that some have a short duration, and recur several times a year, while others increase or decrease for whole years. The mean values of some are considerable while those of others are very small. Lastly, some are often at their minimum value, while others are at their maximum, so that by a judicious choice of circumstances we may seize upon the instant when any inequality is most sensible, while others are least; and thus we may succeed in separating them from one another, beginning with the greatest, which from the circumstance of being the most perceptible, are the first to be observed.

342. It is not difficult to perceive that all these inequalities are closely connected with the relative positions of the sun and moon

with respect to the earth, or with respect to the perigee and the nodes of their orbits. They return to the same values when these return to the same positions, and then pass again through the same periods of increase and diminution. Now these positions are themselves determined by known angles, whose variations may be observed and calculated at all times. By comparing the course of these variations with those of the different inequalities for long intervals, we at length ascertain those which correspond to each other. We thus know the angles upon which each inequality depends, and from the variations which take place in these angles, we are enabled to predict the changes to which each inequality is subject.

343. In order to represent, in a convenient manner, the laws of these changes, we have had recourse to quantities, which had also the property of increasing and decreasing periodically, and which were connected with the observed angles by very simple relations. Indeed the idea of comparing with these quantities the course of the periodical inequalities, was a very natural one. The sines of angles are well adapted to this purpose, as we have seen in the theory of the sun; we have accordingly made use of them.*

344. When we have thus obtained the expression for an inequality, it is easy to calculate the duration of its period, that is, the time requisite for passing successively through all its values. This time should be such that the *argument*, that is, the angle on

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* In order to present thus *empirically* an inequality, when we have determined by observation, the angles on which it depends, we seek the values of these angles answering to its maximum and minimum state. We hence deduce the sine which is fitted to represent it, and to which it is proportional. This supposition gives terms of the form $k \sin E$, in which E is the angle, or the *argument*, of the inequality, and k a constant number, called the *coefficient*, by which $\sin E$ is to be multiplied, in order to obtain the absolute value of the inequality corresponding to the angle E. We suppose, as usual, that the sines made use of, are taken in a circle whose radius is equal to unity; and as the greatest value a sine can have, is only equal to the radius of the circle, the greatest value that can belong to $\sin E$, is unity. In this case $k \sin E$ reduces itself to k; this is the greatest value of the inequality. Under all other circumstances $\sin E$ is less than radius, that is, less than unity. It will accordingly be expressed by a fraction, and $k \sin E$ will be less than k.

which the inequality in question depends, should vary during the interval by an entire circumference, since it is then only that the sines revert to their first values. Accordingly, in order that the period may be terminated, it is necessary that this condition be fulfilled. We may then calculate the duration of this period by a simple proportion, when we know the variation of the argument for a given time; and this is always easy, because we know the angles of which this argument is composed.

The theory of attraction has confirmed these results, by showing that the inequalities of the celestial motions may in reality be developed in this manner. It has moreover determined the form of the terms by which they are expressed; so that observers have only to introduce into the general formulas the numerical data which are to particularize them. The course is the same as that which is adopted in the theory of the sun.

345. Uniting these two kinds of data, the form of the inequalities deduced from theory, and their numerical values deduced from observation, it is obviously easy to calculate beforehand the *true place* of the moon for any given instant. This instant being known, we first ascertain the place of the node and the inclination of the orbit. We next obtain the expression of the moon's longitude for the same epoch, employing all the inequalities made known by theory and observation. We shall thus have the true place of the moon projected on the ecliptic, and knowing the inclination of the orbit and the position of the node, we can calculate the true direction of the radius-vector in the orbit; after which, the parallax will give the length of the radius-vector or the distance from the earth; and from all these the real position of the moon in its orbit and in space will be known.

If the same calculations be made for a great number of instants very near to each other, the union of these results will enable us to predict the place of the moon for any instant. These results, reduced to a convenient form, constitute what are called *tables of the moon*.

These tables being founded on theory, as it respects the form and number of the inequalities contained in them, their accuracy depends simply on that of the numerical data introduced, and consequently not on the precision of the observations from which these data were derived. The tables of M. Burg, published by the Board of Longitude, having for their basis the profound researches of Laplace, and a vast number of observations made at Paris and Greenwich, and being constructed with the utmost care and address by the method of *equations of condition*, combine in the highest degree the qualities necessary to give them a durable success.[†]

347. It would undoubtedly be interesting to consider the wonderful means by which, from the single principle of universal attraction, have been deduced all the inequalities of the lunar motions, numerous and complicated as they are. But, though we cannot present to the learner the profound analysis by which these relations were discovered, we will at least indicate the method, in some degree experimental, which was first employed to ascertain the existence of the greater part of them, and make known nearly their value. We shall do this the more readily, since this method is well adapted to illustrate the law of the different inequalities, and also affords great assistance in discovering the angles on which they depend. This subject will occupy the three next sections, and as it is attended with some difficulty, they may be omitted by the learner in the first reading.

Periodical Inequalities which affect the Moon's Longitude.

348. THE periodical inequalities of the moon's motion are of three kinds, which refer themselves to the three coordinates by which we determine the position of the moon in its orbit. Some affect its longitude, some its latitude, and others its radius vector. We shall consider them in order.

The inequalities which affect the elliptic longitude of the moon, increase or diminish the equation of the centre. Their effect becomes sensible by the greater or less deviation of the moon from its mean place, than that which would result from the simple laws of the elliptical motion.

[†] Later and more correct tables of the moon have been calculated by M. Burckhardt and published by the French Board of Longitude. M. Damoiseau has also constructed a set of lunar tables in which all the inequalities are derived from theory.

349. The most considerable of these inequalities and that which was first detected, is called *evection*. It was discovered by Ptolemy. Its general and constant effect is to diminish the equation of the centre in the syzygies, and to increase it in the quadratures. If this diminution and increase were always the same, the evection might depend solely on the angular distance of the moon from the sun; but its absolute value varies also with the distance of the moon from its perigee. After a long series of trials and observations, we have succeeded in representing this inequality with great exactness, by supposing it proportional to the sine of double the angular distance of the moon from the sun, minus the mean anomaly of the moon. The coefficient of this proportionality is 1° 20′ 29″,54. The period of the evection is 31^d 19^h 29′ 11″,53.

350. We observe also a great inequality in the lunar motion, which disappears in the syzygies and quadratures, and attains its maximum in the octants. This inequality depends therefore on the angular distance of the moon from the sun; and, as it completes its period while this distance increases 90°, we conclude that it is proportional to the sine of double this angular distance. Its greatest value is 0° 35′ 41″,6. It is called the *variation*, and was discovered by Tycho Brahe, a celebrated astronomer, and the tutor of Kepler. Its period is half a synodic revolution.

351. Lastly, the angular motion of the moon is accelerated annually when the sun's distance from the earth is increasing, and retarded when it is diminishing; hence results a third inequality called the *annual equation*. Its period is an anomalistic year. Its existence becomes obvious by calculating several places of the moon for the different seasons, and comparing them with the results of observation; for, although we make allowance for evection and variation, the results are still found to deviate from the truth in a very sensible manner; and these deviations observe, with respect to their extent and sign, regular periods. The observed motion is slower than the mean, during the six months of the sun's passage from the perigee to the apogee; and faster during the other six. If, for example, the first observation be made in the month of January, when the sun has just passed the perigee, and we calculate from this its place for a day in March, by means of the time intervening between these two epochs, we find the increase of its longitude less, by obser-

vation, than the calculation, founded upon the mean motion, indicates. The difference increases until the sun reaches its mean distance; then it gradually diminishes, and becomes nothing when the sun is at its apogee. Here the results of calculation and observation agree. As the sun moves from this point toward the perigee, the same deviations occur in the inverse order; that is, the arcs described by the moon are at first greater than the calculation indicates; the difference increases till the sun regains its mean distance; thence it decreases to the perigee, where it becomes nothing. These regular and periodical variations make known the existence of an inequality depending on the distance of the sun from the perigee, or on its mean anomaly. Such is in fact the argument for the *annual equation.* It is of exactly the same form as the equation of the sun's centre; but it always has a contrary sign. It is evident that its period is an anomalistic year.

352. We might make allowance for this inequality; we might even suppress it entirely, in the theory of the moon, by modifying the equation of time, which would amount to the same thing as calculating the place of this body for an epoch a little different from the true one, and more or less advanced, according to the position of the sun in its orbit. By means of this alteration we might combine the error of the time in such a manner as to compensate for the variation of velocity. If, for example, we would calculate the moon's place for the 21st of March, when its true motion is slower than the mean, and the mean time corresponding to this epoch is accordingly slower than the true, by the whole quantity answering to this diminution of velocity, it is evident that, by calculating it according to this imaginary time, we should find the moon in its true position. Accordingly, Tycho Brahe, who discovered the annual variation, presented it under this form, employing in the calculation of the places of the moon, an equation of time different from the true; in this he was followed by several observers; but now that we know the true law of the annual equation, we abandon this imperfect and complicated method of using it, and introduce it directly into the theory of the moon, under the form already indicated. The theory of attraction explains very simply the phenomenon of the annual equation. The sun in approaching the earth, approaches the moon also, and the attraction which it

thus exerts upon the moon, becomes greater. This attraction being opposed to the gravity of the earth, draws the moon further from the earth, and makes it describe a larger orbit; hence its velocity and angular motion diminish. The contrary takes place when the sun recedes toward the apogee. The earth's gravity, becoming greater, contracts the orbit of the moon and accelerates its motion. The annual equation of the moon is, therefore, a consequence of the eccentricity of the earth's orbit, which causes a variation in the sun's distance. Whatever then affects this eccentricity, must affect also the motion of the moon. Indeed, it is precisely in this manner that the secular variation of the eccentricity of the earth's orbit, produces the secular equation or acceleration of the moon's mean motion.

353. The three inequalities of which we have spoken, called *evection*, *variation*, and *annual equation*, are the most obvious of those which affect the elliptic longitude of the moon, during short intervals; but they are far from being the only ones. The profound researches of the analyst, compared with long series of observations, have made known many others, which, by means of their complexity and minuteness, would never have been detected by observation alone. These small inequalities must therefore be considered as so many corrections to be made in the estimates of the preceding inequalities, in the same manner as these last serve to correct the elliptic motion; or rather the whole are to be applied together to the correction of the mean longitude. These different inequalities are all calculated in the tables.

But among the different inequalities of the moon's longitude, made known by theory, there is one which must not be omitted. It is not only very important in the formation of tables of the moon, but the discovery itself, which is recent, affords a striking example of the manner in which the inequalities are developed, and shows, at the same time, the use which is to be made of observations for rectifying the mean motion by successive corrections. We shall briefly describe this discovery which has brought our knowledge of the moon's motions to the highest degree of precision.

354. The lunar tables published in the third edition of Lalande's Astronomy, suppose a secular tropical motion equal to 481267° 53′ 11″,9796. The epoch for 1750, that is, the mean longitude of the moon on the 31st of December 1749, at mean noon,

was, according to these tables, 188° 17′ 14″,568. With these data we can calculate any other epoch anterior or posterior to 1750. It is sufficient to add or subtract the mean motion for the interval.

Suppose we would have the epoch for 1756.

	°	′	″
We take the epoch for 1750, which is . .	188	17	14,568
To this we add the mean motion for six common years, rejecting the entire circumferences	56	18	30,859
Plus the motion of two days on account of the two bissextiles	26	21	10,051
Also the secular equation, which gives for six years	0	00	00,033
Adding these results together, we have . .	270	56	55,511

which is the mean longitude of the moon, or the *epoch* for 1756.

It is evident that this epoch might also be obtained by observation, if we had any which were made about 1756. Now such observations are furnished us by Dr Bradley, and by means of them MM. Mason and Bouvard obtained a result exactly equal to the preceding.

But in determining in the same manner the epoch for 1691, by means of more than two hundred observations by Lahire, it was found to be less than that of the tables. The error of the latter is 4″,4, that is, at this epoch, the mean longitude of the moon was less by 4″,4 than the tables of Lalande indicated. Accordingly, from 1691 to 1756 the moon described a greater arc than these tables suppose. Its true motion in this interval was therefore greater than that of the tables; for, otherwise, being behind them in 1691, it could not have agreed with them exactly in 1756.

On the contrary, since 1756, the motion indicated by the tables has become greater than the real motion; for, by calculating the epochs for the subsequent years, we find them always less than those derived from observation. The series of errors is even increasing, as may be seen by the following statement;

Years.	Errors of the Tables.
	″
1766	— 3,0
1779	— 9,1
1789	— 17,6
1801	— 28,5

355. These results evidently indicated that the mean motion of the tables, determined from observation, involved in its value the results of some inequality of a long period, the existence of which was not known, but the effects of which, in alternately augmenting and diminishing the moon's longitude were rendered sensible by time. Hence the necessity of correcting continually the mean longitude indicated for the commencement of each year, as the tables became defective, and also the impossibility of extending them to distant times.

The only means of avoiding these difficulties was to discover the inequality in question. But there was little hope of doing this by simple observation; for, as it had hitherto escaped all observers, the probability was that it was subject to a very long period, perhaps dependent on a great number of elements, and consequently, in all its relations, difficult to be determined.

It only remained then to have recourse to theory, and the aids astronomy had already received from this source, encouraged the hope of new success.

356. Laplace has realized these anticipations. By a new and profound examination of all the parts of the lunar theory, he discovered a new inequality which had not before been suspected, and the argument of which is equal to double the longitude of the node of the moon's orbit, plus the longitude of its perigee, minus three times the longitude of the sun's perigee. It is proportional to the sine of this angle. Its period is 184 years.†

In 1691, this inequality was subtractive and diminished the mean longitude of the moon. It must have become nothing some time after, for it was additive in 1756, and had then nearly attained its maximum. The difference between the mean longitudes during this interval is then found to be increased by the progressive influence of this inequality. Accordingly the mean motion, given by the tables, appeared to be too small, when we wished to go back from 1756 to the epoch of 1691. On the contrary, after 1756, the inequality which had attained its maximum, began to decrease; and its effect in augmenting the longitude of the moon, became less. Then the increase of the mean longitude from 1756, became less than it ought to have been, in virtue of the mean motion; and consequently the motion assigned by the tables appeared too great. Lastly, from this epoch the error must necessarily increase, since the foreign increment, involved in the mean longitudes compared, is found to have diminished more and more in the second epoch, according as this was farther removed from 1756, and must even soon change to a diminution.

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† In M. Burckhardt's tables the period of this inequality is reduced to 179 years, and the argument to twice the mean longitude of the moon's node, plus the longitude of the moon's perigee. These points, however, are not considered as definitively settled.

357. This discovery required several corrections. It became necessary to determine the absolute value of the new inequality, and to subtract it from the mean longitudes adopted in 1750, for calculating the mean motion at the time of forming the tables; and lastly, to calculate this mean motion anew, with the corrected longitudes. But these corrections being all connected together, could not be obtained separately. When observations make known the errors of the tables, they only make known the general result of all the causes of error with which these tables are affected; they do not discriminate what belongs to each one separately.

We must then regard all these corrections as so many unknown quantities, connected together by certain relations, and requiring to be represented by so many indeterminate quantities. We calculate from these relations the analytic expression of the influence which each error must have upon the definitive result; and the sum of these hypothetical errors must be equal to the total error actually indicated by observation.

358. Thus, Laplace found 15″,3932 for the coefficient of this inequality, — 13″,4599 for the correction of the epoch of the tables in 1750, and 31″,9639 for the secular diminution of the mean motion, which must be multiplied by the number of centuries after 1750, and which becomes consequently additive before this epoch.

Periodical Inequalities which affect the Moon's Latitude.

359. Hitherto we have considered only the inequalities which affect the elliptical longitude of the moon; there are others which affect its distance from the plane of the ecliptic, or its latitude. The most remarkable is that which causes a variation in the inclination of the moon's orbit; it has for its maximum 0° 8′ 47″,15, and its argument is double the distance of the sun from the ascending node of the moon's orbit. It is proportional to the cosine of this angle. Its period is consequently equal to half a revolution of the sun with respect to the nodes of the moon, since, in this interval, its argument varies 360°.

Tycho Brahe first discovered this inequality by comparing together the greatest latitudes of the moon, observed at different epochs, and in different positions of this body with respect to the nodes of its orbit. He perceived that these latitudes are not always the same. They deviated sometimes positively and sometimes negatively from the mean value, which we have said was equal to 5° 8′ 59″,93; and as the greatest latitudes of the moon give directly the inclination of its orbit to the plane of the ecliptic, it necessarily followed that this inclination varied. The same astronomer went farther, and by a long series of observations, determined the law of this inequality, and presented it under a different form from that which is here given, but amounting to the same thing in the result. The motion of the moon's nodes involves also an inequality depending on the same angle as the preceding, with this difference, that it is proportional to the sine of this angle and not to the cosine. We might represent these two inequalities together by impressing a small motion of nutation upon the axis of the moon's orbit; in the same manner as we have represented the periodical oscillations of the earth's equator and the equinoxes, by means of the nutation of the earth's axis. For this purpose, it would be sufficient to place the mean pole of the moon's orbit at a distance from the pole of the ecliptic, equal to 5° 8′ 59″,93, and then to make the true pole oscillate about the mean pole in a small ellipse to be described in a period equal to a semi-revolution of the sun with respect to the moon's nodes, that is, in 178^d 7^h 26′ 20″,54.

There are several other inequalities which affect the latitude of the moon and the inclination of its orbit, and which are combined in the tables with the preceding; but they have been determined only by theory. For this reason we shall not speak of them in this place, our aim being only to give some idea of the aid which observation and theory mutually lend each other.

Periodical Inequalities which affect the Radius Vector of the Moon.

360. It remains to consider the periodical inequalities which affect the moon's distance from the earth, or its radius vector, the last element necessary for determining its position in space.

The variations of the moon's radius vector do not exactly conform to the laws of the elliptic motion; they deviate in a very sensible manner, sometimes positively and sometimes negatively. These deviations are rendered obvious by comparing together the distances of the moon from the earth, observed in the same point of its orbit at different epochs, or what amounts to the same thing, by comparing the parallaxes from which these distances are calculated; for we observe that these parallaxes are not always the same, and their changes have a marked reference to the positions of the moon with respect to the sun. We thus find that the smallest parallax 0° 53′ 51″, takes place when the moon is at the apogee and in conjunction; and the greatest 1° 1′ 29″, when it is at the perigee and in opposition. But the union of these circumstances is necessary in order to obtain the extreme values. Often the parallax at the perigee is less than 1° 1′ 29″, and the parallax at the apogee greater than 0° 53′ 51″.

361. These results show that the lunar ellipse does not always preserve the same dimensions. It dilates and contracts successively; we have already noticed this fact in connexion with the annual equation. These phenomena must, therefore, be considered as produced by periodical inequalities, which affect the radius vector of the moon. Their effect consists in causing the moon alternately to approach to and recede from the earth, and thus to oscillate within limits of small extent about a mean distance, which it is our object to determine, and to which all the

periodical inequalities must be applied alternately as positive and negative.

The theory of attraction having made known the forces which act upon the lunar spheroid, furnishes the means of determining this mean distance, and the nature of the inequalities which affect it; but as astronomical observations give immediately the value of the parallax, and as we can deduce from this the value of the radius vector by a simple proportion, it is found more convenient and exact to ascertain the inequalities of the parallax directly by theory, that we may be the better able to follow out and verify them by observation. But then it is necessary to determine a mean parallax from which all the others shall deviate equally on the one hand and on the other, in consequence of the inequalities; this is what is called the *constant of the parallax.*

In order to determine it, we in the first place have recourse to observation; by applying to the parallaxes observed, all the inequalities made known by theory, we obtain for the result the constant of the parallax; but by a process more profound, Laplace succeeded in obtaining this important element from theory alone; its value is 0° 57′ 4″,1648. It gives a mean distance equal to 60,23799 radii of the earth. The apparent diameter which corresponds to it, is equal to double 0° 57′ 4″,1648 multiplied by the constant ratio 0,27293, or 31′ 7″,73.

In order to determine this mean parallax by theory, we must consider the moon as a body subjected to the same action of gravity, as other terrestrial bodies, and revolving about the earth by the effect of a motion of projection. The velocity of this motion is known by observations of the moon. We know also by experiments upon the pendulum, the intensity of terrestrial gravity at the surface of the earth. Thus, supposing this gravity to be inversely proportional to the square of the distance from the centre of the earth, we can calculate to what degree it must be diminished, and consequently to what distance the moon must be removed, in order that the action of gravity upon it may be such as observations show it to be from its mean motion. We thus obtain the moon's distance from the earth, corresponding to this motion; and hence we deduce the mean value of the parallax, it being inversely as this distance. It may be proper to remark, that the value 0° 57′ 4″,1648 is not the arithmetical mean between the values 1° 1′ 29″ and 0° 53′ 50″ of the extreme parallaxes; but approaches nearer to the second than the first.

Libration of the Moon and Situation of its Equator.

362. It is found by carefully observing the spots on the moon that it always presents to us nearly the same face. Accordingly, during a revolution about the earth, its opposite hemisphere corresponds successively to all the points of the heavens which are in the plane of its orbit. It has, then, a real motion of rotation about its axis.

We shall be convinced of the existence of this motion in a manner still more striking, by referring the successive positions of the moon to a very distant point, as the sun; for the face which it presents to us is sometimes turned towards the sun, and sometimes in a contrary direction. An observer placed at the sun would, therefore, see the moon turn on its axis in the course of a synodic revolution. This rotation seen from the stars, would be of the same duration as the sidereal revolution of the moon; with respect to the equinoctial points, it would be the tropical revolution.

In order to determine the axis of rotation and the plane of the lunar equator, observers have been led to a very minute examination of the spots. Two circumstances served to facilitate this inquiry; these spots are permanent, and, in general, may be observed during a whole revolution.

They present, nevertheless, some varieties in their apparent position on the moon's disc; we see them alternately approach to and recede from the limb. Those which are near the limb disappear and reappear successively by periodical oscillations. But as the spots themselves do not seem to undergo any sensible change in their relative positions, and as they always present themselves under the same form and magnitude, when they return to the same situation, we infer that they are fixed to the surface of the moon. Their oscillations appear, therefore, to indicate in the lunar globe a kind of motion, which has received the name of *libration*, from a Latin word which signifies *to balance*.

But in adopting this expression, which represents very well the appearances observed, we must not affix to it an absolute meaning; it is only the result of several optical illusions.

363. To analyze these illusions, we must make use of precise language. Let us then suppose a visual ray drawn from the centre of the earth to the centre of the moon. A plane drawn through this latter centre perpendicular to the above ray, cutting the globe of the moon, will by its intersection form the circumference of a circle; this is the boundary of the apparent disc. If the moon had no real motion of rotation, that is, if each point of its surface were invariably directed towards the same point of the heavens, its single motion of revolution about the earth would discover to us successively all the parts of its surface; and the visual ray would cut it successively in different points, which we should see pass one after the other over the centre of the disc. The actual motion of rotation counteracts the effects of this apparent rotation, and causes the lunar globe to present constantly the same face to the earth; this is the reason why we never see the opposite face.

Let us now suppose that the rotation of the moon is sensibly uniform; that is, that it does not partake of the periodical inequalities; this supposition is at least the most natural that can be made, and theory has proved that it is correct. Then one of the causes which produce the libration becomes evident; for the motion of revolution, being affected by the periodical inequalities, is sometimes more and sometimes less rapid. The apparent motion which it occasions cannot, therefore, be always counteracted by the real rotation which remains constantly the same; and these two effects exceed each other alternately. The points of the lunar globe must therefore appear to turn alternately in different directions about its centre, and the appearance thus produced, is the same as if the moon had a slight vibratory motion on each side of the radius vector drawn from its centre to the earth. This is called the *libration in longitude*.

Several causes unite with the preceding and sensibly modify this first result.

The spots on the moon do not always preserve the same distance from the plane of its orbit; sometimes, by the effect of rotation, they even pass from one side of this plane to the opposite. These phenomena indicate an axis of rotation, which is not exactly perpendicular to the plane of the orbit. Now, as this axis appears under us its greatest and smallest obliquity, it must discover to us successively the two poles of rotation of the lunar spheroid; in the same manner as the axis of the earth presents successively its two poles to the sun at the two solstices. It follows, then, that we must sometimes see points situated about the poles, and then lose sight of them as they pass over the apparent edge. This is called the *libration in latitude.* It is inconsiderable, from which we infer that the equator of the moon differs but little from the plane of its orbit.

A third illusion arises from the observer being placed at the surface of the earth, and not at the centre. It is towards the centre that the moon always turns the same face, and the visual ray drawn from thence to the centre of the moon, would always meet its surface in the same point, the preceding inequalities being left out of consideration. It is not the same with the visual ray drawn from the surface of the earth; this ray makes a sensible angle with the preceding, in consequence of the proximity of the moon; this angle at the horizon is equal to the horizontal parallax. On account of this difference, the apparent contour of the lunar spheroid is not the same with respect to a spectator at the centre of the earth and one at the surface. In the latter position, when the moon is rising, some points are discovered towards its upper limb, which are not visible from the centre. As the moon increases in altitude, these points approach the upper limb of the disc, and at length disappear, while others become perceptible near the lower. The same effect continues as long as the moon is visible, and as the part of its disc which is uppermost when rising, appears lowest when setting, the difference is most obvious at these two epochs. Thus, by its diurnal motion, the body of the moon appears to oscillate about the radius vector, drawn from its centre to that of the earth. This is called the *diurnal libration.*

364. The preceding explanations satisfy the phenomena observed so readily and exactly that we cannot entertain a doubt of their correctness. These three kinds of libration are, then, purely optical effects; accordingly they disappear entirely, when we refer the position of the spots, as seen from the earth, to the position they would have with respect to an observer placed at the centre of the moon. Then, there remains only a uniform

motion of rotation about an axis nearly perpendicular to the moon's orbit.

365. To determine the position of this axis and the duration of the rotation, we make use of the processes already applied to the determining of the sun's rotation. We observe the differences of right ascension and declination, between one of the moons' limbs and a given spot. As we know also by observation or calculation, the apparent semidiameter of the disc, we deduce from these data, the difference of declination and right ascension between the spot and the centre, and consequently the difference of longitude and latitude.

The geocentric positions of a spot being known, we convert them into *selenocentric* positions, that is, the positions they would have as seen from the centre of the moon. The formulas necessary for this reduction, are the same as those employed in the case of the sun, with the exception of a small modification arising from the circumstance that the latitude of the moon is not 0, like that of the sun.

366. If the orbit described by the spot about the centre of the moon, is in the same plane, three observations, thus prepared, would seem to be sufficient to determine this plane, and consequently the position of the moon's equator which is parallel to it. But here a new peculiarity presents itself.

If we employ three observations made within a short interval, for example, within the space of 15 days, we find that the longitude of the ascending node of the moon's equator is equal to the mean longitude of the ascending node of the orbit; and as we find this equality to continue, at whatever epochs we repeat the operation, provided the interval be not great, we must conclude that the intersection of the lunar equator with the ecliptic is constantly parallel to the line of the mean nodes of the orbit; consequently this line of intersection has a retrograde motion equal to that of the nodes; and thus the axis of rotation of the lunar spheroid describes a conical surface about the axis drawn through its centre perpendicular to the ecliptic. As to the inclination of the lunar equator to the ecliptic, it appears to be constant; for, calculating it thus by observations made near to each other, we always obtain for it the same value, namely, 1° 30′ 11″.

367. The mean revolution of the moon's nodes taking place, as we have seen above, in $6793^d\ 10^h\ 6'\ 30''$, it follows that these 308. nodes retrograde 1° in $18^d\ 20^h\ 53'\ 40''$. The line of intersection of the lunar equator, therefore, following the motion of the nodes, must also retrograde 1° in the same interval. The slowness of this motion is such that observations, made within an interval of a few days, give nearly the same longitude for the moon's node; but we cannot combine in this manner observations made at distant epochs; for then the displacement of the lunar equator becoming sensible, the selenocentric positions of the spot, cannot be comprehended in the same plane having a constant longitude, as is supposed in the formulas established for the sun; hence another modification is to be made in these formulas in order to render them applicable to the moon. This modification consists in rendering the line of intersection of the lunar equator moveable on the ecliptic, conformably to the law we have determined. This is easily done without rendering the calculations any more complicated.

That nothing may be omitted which can conduce to precision, we remark that it is necessary to employ a different parallax for the centre of the lunar spheroid, and for the spots on its surface; since, on account of the magnitude of the moon's parallax, this difference is sensible. We can easily estimate it when we know the difference of the distances; and to obtain this last, it is sufficient to calculate the parallel of the spot by a first approximation, in which the parallaxes for the spot and centre are supposed to be equal.

368. By employing all the precautions which the case admits of, and applying them to a great number of observations, we establish with the utmost certainty the constancy of the inclination of the lunar equator to the ecliptic, and the parallelism of its nodes with the mean nodes of the orbit. These results may be represented by the following construction. Through the centre of the moon suppose three planes to pass; one representing the moon's equator, another the mean plane of its orbit, and a third parallel to the ecliptic. This last, which will be always comprehended between the two others, will pass through their common intersection. It will make with the first, an angle of 1° 30′ 11″, which is the inclination of the lunar equator to the ecliptic; and with the second, an angle of 5° 8′ 50″,93, which is the mean inclination of the orbit to the same plane.

369. We have seen that there are secular variations in the motion of the moon, which must, in the course of time, produce very considerable changes in it. If the rotatory motion remained always the same, and did not participate in these inequalities, it would soon cease to counterbalance exactly the motion of revolution, and the moon would gradually turn with respect to us until, in the lapse of centuries, it would present successively all the points of its surface. But theory has shown that this can never be the case. The rotatory motion, though independent of the periodical inequalities which affect the revolving motion, is subject to the same secular inequalities. These motions will, therefore, pass through the same changes, always counterbalancing each other, and the face of the moon now opposite to the earth, will forever remain so. By the same laws the constant inclination of the lunar equator to the ecliptic, and the parallelism of its nodes with the mean nodes of the orbit, will always be the same, notwithstanding the developement of secular inequalities. These valuable theorems are derived from the analysis of Lagrange.

Of the Form and Physical Constitution of the Moon.

370. We have seen that the ratio of the radius of the earth to that of the moon, on the supposition that they are spherical, is 0,273, or nearly $\frac{3}{11}$. The ratio of their bulks, on the same hypothesis, is equal to the cube of the ratio of their radii, namely, $\frac{27}{1331}$ or $\frac{1}{49}$. We hence infer that *the bulk or volume of the moon is about one forty ninth part of that of the earth.*

It appears, by the theory of attraction, that the mass of the moon is $\frac{1}{68,4}$ of that of the earth. This ratio is much less than that of $\frac{1}{49}$. The mass of the moon, compared with that of the earth, is, therefore, not in proportion to their bulks, and consequently the moon's density is less than that of the earth.

From the phenomena presented by the moon's rotation, it is proved that the moon is not spherical. The form must be that of a spheroid,† having its greater axis turned constantly toward the earth and situated in the plane of the lunar equator, its smaller axis in the direction of the poles of rotation, and for its third axis one perpendicular to these two and intermediate between them as to length. These curious results were discovered by Lagrange.

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† The term spheroid is used here and in some other places, with the latitude of the original *spheroïde,* to stand for any figure that ap-

371. By examining carefully the disc of the moon when it is not entirely illuminated by the sun, we observe on the dark part certain bright spots that gradually increase and become more conspicuous, as the illuminated part extends itself. These spots never appear far from the confines of the illuminated part, and when they are lost in the general light, they are always accompanied with a shade more or less deep which turns with the sun like a projected shadow, that is, so as to be always opposite to the sun. We hence conclude that the spots under consideration are mountains that elevate themselves more or less above the general surface of the moon, the summits of which are reached by the sun's light before their bases. They are found to be of various heights like those of the earth, the highest attaining to about the same elevation in both.

We observe also portions of the lunar disc that are distinguished by their obscurity. These, it would seem, are valleys, or depressions below the general surface. The result of the latest and best observations upon the different inequalities in the moon's surface, together with those which relate to a lunar atmosphere, will be given in the Appendix.

General Phenomena of Eclipses.

372. We proceed now to speak of certain phenomena which for a long time filled the world with terror, but which, owing to the progress of knowledge, have at length come to be regarded only as objects of interest and curiosity.

We shall first consider the general phenomena of eclipses, next the circumstances under which they can take place, and lastly, the means of predicting them.

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proaches nearly to a sphere; whereas it ordinarily denotes, in English, a solid generated by the revolution of an ellipse about one of its axes, and answers to what the French call an ellipsoid of revolution.

We shall begin with eclipses of the moon. The earth being an opaque body illuminated by the sun, will project a shadow behind it in space. When the moon enters this shadow it is cut off from the direct light of the sun, and is consequently eclipsed.

373. If through the extremity of the sun's diameter we suppose straight lines to be drawn touching the surface of the earth, as AB, $A'B'$, these lines produced, will represent the limits of the shadow; and, as the sun is much larger than the earth, they will meet behind the latter, and will thus give to the shadow the form of a circular cone, if the earth be spherical, and of an elliptical cone, if the earth be a regular spheroid. Accordingly, when the moon enters this shadow, a part of its disc still remaining illuminated by the sun, this part will not be terminated by a straight line, but will have the form of a luminous crescent, its concavity being turned towards the shadow. The same phenomena will take place when the moon begins to emerge from the shadow. Fig. 85.

374. When the moon approaches the earth's shadow, it does not become eclipsed suddenly, but loses its light by imperceptible degrees. In order to comprehend this phenomenon, we must remember that a small opaque body, upon being placed between an object and the sun, does not entirely obstruct the sun's rays, but the object becomes less illuminated on this account, and if the opake body be sufficiently near, receives a shadow. There is then between the part which is perfectly illuminated, and that which is perfectly dark, an intermediate shade called the *penumbra.* Such is the effect produced upon the moon when it approaches the earth's shadow.

To determine the limits of the penumbra, we suppose two straight lines $A'B$, AB', touching the surface of the sun and that of the earth, but on opposite sides, so as to cross each other between the two bodies, as in figure 86. The angles EBC, $E'B'C$, represent the space comprehended by the penumbra; for, from any point situated without these limits, the entire sun is seen; but from any point L within them, only the part AL' of its disc is visible. This portion diminishes from the line EB to CB, where it becomes nothing; and consequently the deepness of the penumbra increases from the first limit where it begins, to the second where it is confounded with the shadow. This explains the progressive obscurity observed in the moon's disc when it is eclipsed.

375. When the moon is totally eclipsed by the earth's shadow it is not entirely concealed from view. Its surface is still feebly illuminated with a reddish light, resembling that of the clouds after sunset. This light proceeds from the solar rays refracted by the earth's atmosphere, and bent to such a degree as to pass into the shadow; for those rays which are not sufficiently refracted to reach the surface of the earth, where they would be absorbed, continue their course through the atmosphere. After they have traversed it they are bent behind the earth and tend as it were to the focus of a lens. If the light thus tending to a point behind the earth, were not in a great degree absorbed by the atmosphere, its effect would be very considerable; for, if we consider one of the luminous points of the sun's disc, this point could only send directly a single ray to each point of space; but, by the interposition of the earth's atmosphere, it causes a cone of luminous rays to fall behind the earth. The focus where the rays unite, receives therefore a much greater number than it would do if it were illuminated directly.

376. Hitherto we have spoken only of eclipses of the moon. Eclipses of the sun are to be understood in a similar manner; for they are produced by the conical shadow which the moon projects behind it in space. When this shadow meets the surface of the earth, the points upon which it falls are deprived of the direct light of the sun, and are in complete obscurity. The points which are not strictly in the shadow but in the penumbra, lose only a part of the sun's light, and its disc would appear to a spectator thus situated to be hollowed out by the partial interposition of the moon; in a word, all we have said respecting eclipses of the moon is equally applicable to eclipses of the sun, if we suppose the spectator in the moon; for, what we call eclipses of the sun, are nothing but eclipses of the earth with respect to an observer thus situated.

377. Let us now subject the phenomena to calculation, and attend to the circumstances under which they may happen.

The first thing to be determined is the length of the conical shadow projected behind the earth, since it is necessary to know whether it always extends to the orbit of the moon. For this purpose we first leave out of consideration the earth's atmosphere and its oblateness.

Fig. 87. Let S be the centre of the sun, T that of the earth, AB a straight line touching the earth and sun, and forming the limit of perfect shadow; ST will be the axis of the conical shadow, and TC the length of the cone to be determined. Now it is sufficient to know the angle TCB at the vertex of the cone. This is equal to the semidiameter of the sun, minus its horizontal parallax, or 8″,78, as will be readily seen by the figure.

By means of this result we can easily find the length of CT; for, in the triangle CBT, right-angled at B, we know the angle C and the side TB, which is the radius of the earth.

It is obvious that the length of the conical shadow must vary with the apparent diameter of the sun, and consequently with its distance from the earth; the following are the values corresponding to the mean distance and those of the perigee and apogee.

Distance from the centre of the earth to the vertex of the conical shadow expressed in radii of the earth.

When the sun is at the perigee . . .	212,896
" " at its mean distance . .	216,531
" " at the apogee . . .	220,238

378. From these values it appears that the shadow lengthens as the sun's distance increases. We have before seen that the moon's greatest distance from the earth is only 63,94145 terrestrial radii, and therefore much less than the smallest length of the shadow. Consequently if the moon moved in the plane of the ecliptic, it would traverse this shadow each revolution, and there would be an eclipse of the moon every month.

379. In the same manner we may determine the length of the conical shadow, projected by the moon, on the supposition that it is spherical; it is sufficient to employ for the apparent diameter and parallax of the sun, their values referred to the surface of the moon. It is easy to deduce these values from those we have already employed; for the apparent diameter of the sun, as seen from the moon, will be equal to its apparent diameter, as seen from the earth, augmented in the ratio of the distances of the earth and moon from the sun. Also the sun's parallax at the moon is equal to its parallax at the earth, augmented in the ratio of the distances, and diminished in the ratio of the radii of the moon and earth. With these reductions, the formulas investigated in the case of the earth, will serve equally for the moon, and will enable us to determine the length of the conical shadow

which the moon projects behind it in space. This is the method by which the following results were obtained; they relate to the two extreme cases in which the length of the cone is greatest and least, and are compared with the distance of the earth.

	Length of the cone.	Distance of the Moon from the Earth.
Sun in the apogee, *maximum* parallax	59,730	55,902
Sun in the perigee, *minimum* parallax	57,760	63,862

In the first case the shadow of the moon will reach the centre of the earth and pass it; in the second, it will not even reach its surface. Thus, if we suppose the moon to move in the plane of the ecliptic, it would not always cause a total obscurity in any part of the earth, when passing before the sun, on account of the variations in its distance. Its only effect would then be to conceal a part of the sun's disc.

380. To calculate the extent and duration of lunar eclipses, it is necessary to know the diameter of the earth's shadow at the place where it is traversed by the moon's orbit. This is very easily found; for if *LL′* represent this orbit, considered as circular, the apparent semidiameter of the shadow as seen from the earth, at this distance, would have for its measure the angle *LTC*, which is equal to the difference of the angles *TCL* and *TLA*. The angle *TLA* is the apparent semidiameter of the earth, as observed from the moon; or, in other words, the horizontal parallax of the moon, at the moment of the eclipse. We have seen that the angle *TCL* is equal to the apparent semidiameter of the sun, minus its parallax. These two angles being known, the angle *LTC*, or the semidiameter of the shadow, becomes known also. We have only to add the parallax of the sun to the parallax of the moon, and subtract from their sum the apparent semidiameter of the sun. Fig. 87.

By calculating the above value according to what we have before found for the greatest, mean, and least, parallax of the moon, we shall have for the results the following table.

Diameter of the Earth's Shadow at the Distance of the Moon.

		″
Sun at the perigee	Moon at the apogee . .	4524,3
	Moon at its mean distance	4982,3
	Moon at the perigee .	5440,9
un at its mean distance	Moon at the apogee . .	4556,9
	Moon at its mean distance	5014,9
	Moon at the perigee .	5472,9
Sun at the apogee	Moon at the apogee . .	4588,3
	Moon at its mean distance	5046,3
	Moon at the perigee .	5504,3

The greatest apparent diameter of the moon being only 2011″, it is evident that it can always be comprehended and covered by the earth's shadow which is much greater. Accordingly whenever the moon traverses the centre of the shadow, it must be totally eclipsed.

The diameter which we have calculated is that of the shadow projected by the great body of the earth. It appears that the lower strata of the atmosphere obstruct so much light as to occasion a sensible shadow; for, the apparent diameter of the shadow which is deduced from observation, is always a little greater than the one above calculated. This difference is commonly estimated at 100″, as indicated by the instant of the entrance of the moon into the shadow, and that of its emergence from it. But this estimate is wholly empirical. Besides, it is evident that the greater or less purity and transparency of the inferior strata of the atmosphere must produce considerable accidental variations. Lastly, it is manifest that this shadow, produced by the atmosphere, must grow less and less distinct from the centre to the circumference, like the density of the atmosphere itself, and this diminution, added to that of the penumbra, must prevent the limits of the shadow, projected on the moon, from being accurately defined. The time during which the moon continues in the shadow, depends on the difference between its diameter and that of the shadow, and upon the rate at which it traverses the shadow by its horary motion. We shall resume this subject in another place.

Strictly speaking, the shadow is not circular, because the earth is not spherical. Indeed its form is elliptical; its dimensions, therefore, when projected on the moon, cannot be the same in all directions. But the limit of perfect shadow is not sufficiently marked in lunar eclipses to allow the difference of its axes to be appreciated by observation.

381. If we would determine, in like manner, the semidiameter of the moon's shadow at the distance of the earth, we have only to substitute in place of the data now used, those which belong to an observer stationed at the moon. Thus the semidiameter of the moon's shadow to an observer thus situated, would be equal to the sun's parallax, referred to the moon, plus the earth's parallax, minus the apparent semidiameter of the sun as seen from the moon. The earth's parallax is simply the apparent semidiameter of the moon as seen from the earth. We have then, from what precedes, all the required data. The formula is rendered more simple by neglecting the sun's parallax, the influence of which in the calculation is scarcely sensible, not amounting to a sixth of a second. *Then the semidiameter of the moon's shadow is equal to the excess of the apparent semidiameter of the moon over the apparent semidiameter of the sun.*

If we calculate the length of the shadow under the most favourable circumstances, which are those in which the sun is in its apogee and the moon in its perigee, we find that the semidiameter of the moon's shadow, as seen from the moon, is equal to about 60″. The apparent semidiameter of the earth, at this distance, is equal to the horizontal parallax of the moon at the same instant, that is, 3689″. Thus under these circumstances, which are the most favourable, the breadth of the moon's shadow is to the diameter of the earth's disc, nearly, as 60 to 3689, or as 1 to 61,5, that is, it does not cover a sixtieth part of the breadth of the earth's hemisphere. It is obvious that in less favourable circumstances the dimensions of the shadow will be still smaller. It would become nothing if the apparent diameter of the moon, as seen from the surface of the earth, were exactly equal to the apparent diameter of the sun, and negative, if it were greater. In the first case, the apex of the cone would barely reach the observer at the earth; in the second, it would not extend so far.

But it might still reach other observers differently situated on the terrestrial globe; for, in consequence of their situation, they might be more or less distant from the moon. Those, for example, who see it in the horizon, are farther by about $\frac{1}{60}$ than those who see it at the zenith; the apparent diameter of the moon will therefore appear smaller by about $\frac{1}{60}$ to the first than to the second; and as the effect of these different positions is insensible with respect to the apparent diameter of the sun, on account of its very great distance, it follows that to one the apparent diameter of the moon will seem less than that of the sun, while to the other it will appear greater. The first will not be reached by the shadow, while the second may be immersed in it. We perceive then, that within these limits the position of the observer becomes very important, since it may cause him to enter the shadow or escape it, by a mere difference of parallax in altitude. We shall pursue this subject farther in another place. For the present it is sufficient to remark, that the smallest apparent diameter of the sun being 1891″, and the diameter of the moon at it smean distance being 1878″, and consequently smaller than the first, there can never be a total obscurity at this limit, and still less when the moon is beyond its mean distance. We see moreover, that in consequence of the small dimensions of the moon's shadow, there can only be a small portion of the earth's hemisphere overshadowed at once; that very often the shadow does not reach the earth; and that, for this double reason, eclipses of the sun are very much rarer, in any given place, than those of the moon; which is confirmed by experience.

382. Thus far we have considered only the perfect shadow;
Fig. 87. we proceed now to fix the limits of the penumbra. Let AB' be a straight line touching to the opposite limbs of the sun and moon, and LL' the moon's orbit; the angle $L'TC$ will give the distance of the penumbra from the axis CT. Now this angle being the exterior angle of the triangle $C'TL'$, is equal to the sum of the two opposite interior angles, $TL'C'$, $TC'L'$. The first of these is the horizontal parallax of the moon; the second is equal to

$$C'AT + C'TA,$$

that is, to the sun's parallax plus its semidiameter, as seen from the earth. Consequently the angle $L'TC$, or the extreme radius of the terrestrial penumbra, is equal to the sum of the parallaxes of the sun and moon, plus the apparent semidiameter of the sun.

Subtracting from $L'TC$ the semidiameter of the perfect shadow, we have the breadth of the penumbra. This semidiameter is
380. equal to the sum of the parallaxes of the sun and moon, minus

the apparent semidiameter of the sun; consequently the breadth of the penumbra is exactly equal to the apparent diameter of the sun.

383. If we would calculate, on the same principle, the extent of the penumbra, projected by the moon upon the earth, in eclipses of the sun, we have only to substitute, in the preceding formulas, the data belonging to the moon instead of those relating to the earth. Thus the extreme radius of the penumbra will be equal to the sum of the parallaxes of the sun and earth, plus the apparent semidiameter of the sun; these diameters and parallaxes being supposed to be calculated for the moon. If we would simplify the process we may neglect the parallax of the sun, and then the extreme radius of the moon's penumbra, as seen from the moon, becomes equal to the sum of the apparent semidiameters of the sun and moon, as seen from the earth. To his sum must be added $5''$ on account of the sun's reduced distance. The total breadth of the lunar penumbra, as seen from the moon, is equal to the apparent diameter of the sun seen also from the moon, that is, to this diameter seen from the earth, plus very nearly $5''$ on account of the reduced distance.

384. In what precedes we have paid no regard to the refraction which the rays of the sun undergo in traversing the earth's atmosphere. We must now consider what allowance is to be made on this account.

When we leave the atmosphere out of consideration, the limit of the perfect shadow is determined by the ray which touches the exterior limbs of the earth and sun. The intersection of this ray with the line drawn through the centres is, therefore, the vertex of the shadow. But if we take into consideration the atmosphere the case is different. The luminous ray, coming from the exterior limb of the sun, a tangent to the earth, is no longer a straight line; it is curved where it enters into the atmosphere, and where it emerges from it, bending behind the earth so as to meet the axis of the shadow much nearer the earth than it would otherwise do; by this means the vertex of the perfect shadow has its distance reduced. Suppose such a ray coming from the limb A of the sun, and touching the earth somewhere in B. If S be the centre of the sun, and T that of the earth, the refracted ray will meet the line ST somewhere in O, but always nearer T than it would have done if it had been rectilinear. Let us Fig. 88.

now consider another point of the disc, as *A′* for instance Amongst all the luminous rays emanating from this point, there is likewise one which will pass into the earth's atmosphere and be refracted so as to touch its surface in a point *B′* different from *B*. After bending behind the earth, it will meet the axis of the shadow in some point *O′* farther from the earth than *O*. As the same may be said of all the points of the disc, it follows that all these points, even the other extremity *A‴* of the diameter, will, on account of refraction, send luminous rays to unequal distances on the other side of the earth, so that the whole image of the disc will be distributed from *O* to *O‴*, in the same manner as a lens, by refraction, disperses the images of objects transmitted through it.

This being the case, if an observer happen to be placed at *O*, or a little beyond this point in the direction *TO‴*, it is evident that he would not be completely in the shadow; for, he would begin to see, by refraction, the exterior border *A* of the sun, which would appear to him to surround the circumference of the earth's disc, like a luminous ring. If he removed farther, as to *O′* for example, he would see, in this manner, the rays coming from *A′*; but he would see the whole interval *AA′*, no longer, indeed, by rays touching the earth's surface, but by rays passing higher into the atmosphere. If he removed to a still greater distance, the luminous ring would become larger. Finally, if he placed himself at *O‴*, in the direction of the ray coming from the inferior border of the disc, he would see the whole disc of the sun around the surface of the earth. If he placed himself beyond this limit, he would no longer receive the solar rays touching the earth, since the diameter of the sun is supposed to end in *A‴*; but he would receive rays from certain points in the heavens, situated below *A‴* in the figure; consequently the luminous ring would be removed from the circumference of the earth, and the azure of the sky would appear in its place. Hitherto we have supposed the observer to be situated in the cone of the perfect shadow, and consequently all the portions of the sun's disc which are visible to him, becoming so by refraction only, they will be wholly comprehended within the space occupied by the earth's atmosphere. But at a still greater distance, the observer being entirely without the cone of the perfect shadow, formed by the extreme rays, he will begin to see the border of the sun directly and without refraction, while the rays which come from the interior of the disc, would still traverse the atmosphere, and be still refracted.

385. It is very easy to determine these different limits. For this purpose let us suppose a ray *SB*, proceeding from the sun and touching the terrestrial surface. The curve described by this ray about the surface of the earth, supposed to be spherical, will be symmetrical for the different parts of this surface, and if its direction be *ST* when it enters the atmosphere, and *T'O* when it emerges, the angles *BTI*, *BT'I*, formed by these directions with the tangent *TT'*, will be equal to each other. Now the angle *BTI*, *DTS*, is very nearly equal to *DBS* or the horizontal refraction, because the point *T* being little elevated above the point *B*, the lines *SI*, and *SB*, drawn from these two points to the sun are nearly parallel. Thus the angle *BT'I* is also nearly equal to the horizontal refraction; consequently, the angle *SIt'*, which expresses the inflection of the ray, is equal to *BTI* plus *BT'I*, or double this same refraction. Fig. 89.

386. We may therefore regard the effect of the atmosphere as augmenting by this quantity the apparent semidiameter of the sun, for, having once emerged from the atmosphere, the luminous rays continue their course in a straight line, as if they had been emitted originally in this direction; and the ray *SBO*, for example, emanating from the upper limb *S* of the true sun, arrives at *O* in the axis, as if it had come from the superior limb of an imaginary sun whose semidiameter exceeded that of the true one by a quantity equal to the angle *S'It*. We perceive then, that in order to determine the distance of the vertex of the cone of perfect shadow from the centre of the earth, regard being had to the refracting power of the atmosphere, it is sufficient to augment the semidiameter of the sun by double the horizontal refraction, and with this single correction the preceding formulas may be employed. If instead of considering the ray *SB* as coming from the exterior border of the sun, we suppose it to come from any point in the disc, situated at a known distance from the centre, we can substitute this distance for the semidiameter of the sun, and we shall then have the distance from the earth at which this part of the sun begins to be visible. We can thus determine successively the zones which become visible at each removal.

In this manner the following table was calculated, the distance being expressed in terrestrial radii.

	⊙ at the Perigee.	⊙ at its Mean Dist	⊙ at the Apogee.
Limb of the ⊙ begins to appear . . .	42,228	42,013	41,842
A quarter of the ⊙ at the circumference of the earth	46,674	46,502	46,408
Half the disc of the ⊙ at the circumference of the earth	52,236	52,236	52,236
Three quarters of the ⊙'s disc at the circumference of the earth	59,316	59,820	60,317
Entire disc of the ⊙ at the surface of the earth	68,283	69,125	69,645
Azure of the sky equal to a quarter of the ⊙'s disc between the sun and the circumference of the earth	81,499	82,144	82,898
Azure of the sky equal to half the ⊙'s disc between the sun and the circumference of the earth	100,190	101,560	102,980
Azure of the sky equal to three quarters of the ⊙'s disc between the sun and the circumference of the earth	130,140	131,570	132,840
Exterior border of ⊙ begins to be visible directly	185,490	192,590	198,200

The smallest distance of the moon from the earth is, as we have seen, 55,916. Thus, under the most unfavourable circumstances, even at the centre of the shadow, an observer placed at the moon, would still see, by refraction, three quarters of the sun's disc through the atmosphere of the earth. It is not then surprising that the moon, when eclipsed, appears coloured with a reddish light; it would appear much more luminous if the absorbing action of the atmosphere did not weaken this light to such a degree as to leave only a very small proportion to be transmitted.

387. Hitherto we have supposed the moon placed exactly in the axis of the earth's shadow. It would be actually in this axis at every opposition if it moved in the ecliptic; but as it sometimes rises above and sometimes descends below this plane, in consequence of the inclination of its orbit, it may happen that only a part shall enter the shadow of the earth, or that it shall merely touch the border of the shadow, or lastly, that it shall pass entirely without it. In the first case, the eclipse is *partial*; in the second, it is called an *appulse*. Eclipses are said to be *total* when the moon passes wholly into the shadow, and *central* when the moon's centre coincides with the axis of the shadow. We shall soon determine by calculation the circumstances under which these phenomena take place.

388. In like manner with respect to eclipses of the sun, we call those *partial* in which the moon passes over only a portion of the sun's disc; *total*, those in which the whole disc is concealed. *Annular* eclipses are those in which the moon is entirely projected on the sun's disc, without competely covering it; so that a luminous border, like a ring, surrounds it on every side. *Central* eclipses are those in which the observer is situated in the centre of the shadow, or in the line which joins the centres of the sun and moon.

From what we have said respecting the extent of the moon's shadow, compared with that of the earth's disc, it is evident that total eclipses of the sun can happen only through a small part of the earth's surface, and can continue only for a short time; while eclipses of the moon are observed throughout the whole hemisphere of the earth from which the moon is visible at the time; and, on account of the magnitude of the earth's shadow, the duration of the eclipse may be very considerable.

389. A total eclipse of the sun is very remarkable for the degree of darkness which accompanies it, and which extends to all the points of the earth on which the moon's shadow falls, precisely in the same manner as the shadow of a cloud, borne along by the wind, passes over hills and plains, and for a time deprives them of the sun's light.

The darkness which attends a total eclipse of the sun, is the more striking, as it follows so suddenly the full splendor of day. The inferior animals are filled with consternation. The heavens are at once enveloped in darkness; the larger stars become visible, and a pale whitish halo appears about the black disc of the moon, which is supposed to be the zodiacal light, or the atmosphere of the sun. A faint light also often skirts the horizon which is a real twilight, proceeding from portions of the earth's atmosphere that are without the cone of the moon's shadow and within the view of the spectator. Under the most favourable circumstances total darkness may continue eight minutes. The first ray of the sun which escapes, darts forth like an arrow, and this deep gloom instantly vanishes.

Method of Determining the Circumstances of an Eclipse of the Moon.

390. We proceed now to show how all the circumstances of an eclipse may be determined. We begin with eclipses of the moon which, being unaffected by parallax, are more easily understood than eclipses of the sun.

Let us suppose that, at the moment of opposition, the point *T* Fig. 90. represents the centre of the earth's shadow. Let *TE* be the ecliptic, *TY* the circle of latitude in which the conjunction takes place; *L* the centre of the moon then in this circle, and *LN* its orbit inclined to the ecliptic. In consequence of the sun's motion in the ecliptic the centre of the earth's shadow, always diametrically opposite to the sun, moves together with the sun, and with the same velocity from west to east, that is, from *N* toward *E*. During this time the centre of the moon also moves from west to east in its orbit, that is, from *N* toward *E'*. The velocities of these two motions are given by the tables. The question is to determine from these data the instant when the two circles which represent the moon and shadow, will meet whether before or after conjunction.

The calculation is very much simplified by considering that the apparent distance between the centre of the moon and that of the shadow, during the continuance of an eclipse, being necessarily very small, may be considered as rectilinear, and that we may also regard the difference of longitude and latitude of the centres, as small straight lines respectively parallel and perpendicular to the ecliptic; so that the motion of the two centres may be considered as referred to rectangular coordinates x and z, the first being taken on the ecliptic *NE*, the second on the circle of latitude which passes at each instant through the centre of the shadow.

The duration of a lunar eclipse is always so short that the motion of the sun, and consequently that of the shadow, in the ecliptic, during the interval, may be considered as uniform. We may also regard as uniform the motion of the moon in longitude and latitude, at least in the first approximation. With these modifications the problem presents no difficulty.

391. Let T', L', be two simultaneous positions of the shadow in the ecliptic and of the moon in its orbit, at a given instant, before or after opposition. Since we know the velocity of the moon and of the shadow, both in longitude and latitude, we shall have also, for the same instant, the values of $T'P'$ and $P'L'$, which express the relative motion of the centre of the moon with respect to the centre of the shadow. The sum of the squares of these lines will give the square of $L'T'$, that is, of the distance between the centres; since the triangle $L'P'T'$ is right-angled at P'. From the value of this distance we shall know whether the eclipse has begun or not; and by taking the algebraic expression of this value for any given time, it is easy to determine the precise instant of each of the phases of the eclipse. For this it is sufficient to resolve an equation of the second degree.

The above results are obtained upon the supposition that the motion of the sun and that of the moon are uniform. If we wish to attain to the utmost precision, they can only be regarded as a first approximation. Since we know, in this manner, very nearly the instant of each phase, we take this instant for the origin of the times, and having calculated by the tables the motion of the sun and that of the moon which correspond to it, we recommence the calculation of the phase with these new data; and we thus find the correction to be made in the epoch determined by the first approximation. The result will now have all the exactness which can be desired, since the supposition of uniformity in the hourly motion applies only to a very small interval of time. By proceeding in the same manner with each successive phase, we shall obtain with great precision all the circumstances of the eclipse, regard being had to the variation in the motions of the moon and sun.

It should also be added that in order to diminish the great uncertainty to which the different phases of a lunar eclipse are liable, astronomers are in the habit of observing the entrance of different spots into the shadow and their emergence thence. These spots being well known to all observers, and their position on the moon's disc being fixed, the mean epochs between their immersion and emersion answer to the middle of the eclipse; and by observing these phenomena we are able to determine this instant with more precision.

In what precedes, we have left out of consideration the diurnal motion of the heavens; for, as it carries along simultaneously and with an equal motion the sun, moon, plane of the ecliptic, and all the celestial circles, it does not change the relative positions of these bodies with respect to each other. This motion has no other effect than that of presenting the eclipse successively to different portions of the earth. It does not affect its absolute existence, but only the chance of its being visible or invisible.

Method of Determining the General Circumstances of a Solar Eclipse.

392. The questions which present themselves with regard to a solar eclipse are; 1. Whether there will be an eclipse in any part of the earth; 2. What will be the duration of the eclipse; 3. What will be the time of its commencement and termination.

These questions are attended with no difficulty, if we consider a solar eclipse as an eclipse of the earth with respect to an observer at the moon. Then the object is to determine for any given instant the apparent distance of the centre of the earth's disc from the centre of the moon's shadow; and from the known diameters of the earth and the shadow, we calculate the instant when their discs will meet. This problem therefore is absolutely of the same nature with that respecting a lunar eclipse, as seen from the earth. Accordingly it may be solved by the same formulas. The only difference is in the graphic construction made use of to represent the centre of the moon as the centre of the visual rays.

Fig. 92. Let T represent the centre of the earth, for any given instant, L that of the moon, S that of the sun; and let us consider the rectilinear triangle TSL, formed by lines connecting these three points in space. If we produce the side SL, which passes through the sun and moon, it will be the axis of the moon's shadow, and the angle OLT, formed by this prolongation with the visual ray, drawn from the moon to the earth, will be the apparent distance of the centre of the earth from the centre of the shadow, or the quantity to be determined. Now this is very easily found; for the triangle SLT furnishes all the necessary data, since we

know the two sides *ST* and *LT*, the distance of the sun and that of the moon from the earth, and the angle at *T*, which is the apparent distance between these two bodies as seen from the earth, and which is easily deduced from their difference of longitude and latitude, as was done in the case of lunar eclipses. We thus obtain, for any given instant, the expression for the apparent distance of the centre of the shadow from the centre of the earth's disc, as seen from the moon; and, by making this expression equal to the differences of this distance which correspond to the different phases of the eclipse, and considering the time as the unknown quantity, we are enabled to determine the epochs at which these phases will happen.

393. We have here paid no regard to the diurnal motion of the heavens; for, this motion being common to the sun and moon and all the celestial circles, it cannot change their relative positions, and it is with these only that we are concerned. In this case, as in that of lunar eclipses, the diurnal motion has no other effect, except that of presenting the eclipse successively to different parts of the earth; or rather, if we would attribute the diurnal motion to the earth, a supposition which equally satisfies the phenomena, and in a much more simple manner, this motion causes the observer to turn on his parallel, and presents the eclipse to him successively at different elevations above the horizon where he is situated. This rotation may even be the occasion of an eclipse being seen, or render it invisible by bringing the observer to the hemisphere where it is to take place or to the opposite. It may also affect the eclipse in another way, by causing the observer to approach to or recede from the moon's centre by a change of parallax in altitude. But all these variations which render the problem complicated, when we consider a physical and determinate point of the earth, have no such effect when we have reference to the earth in general.

394. There is a very simple means of removing these difficulties; and that is by reasoning with respect to any point of the earth's surface, as we have just done with respect to its centre. Only we must employ in the calculation the data which are suited to these new circumstances, that is, the apparent elements of the two bodies, as seen from the point in question, instead of the true elements which belong to the centre; and

as these two kinds of data differ only in the values of the parallax in longitude, latitude, declination, and right ascension, which we have learned how to calculate, it is obvious that when we know the instants of the different phases of the eclipse for the centre of the earth, we can calculate the corrections which must be applied to them, on account of parallax, in order to refer them to the surface.

395. If we examine also, in a general manner, the course of the lunar shadow or penumbra on the earth's disc, we shall perceive that it must move, with respect to us, from west to east, in the direction of the proper motion of the moon; for, as this body moves more rapidly than the sun, in its revolution about the earth, its shadow must follow it and move in the same direction. Thus an observer placed at the moon and observing the earth eclipsed, would first see the western part of the disc obscured and then the eastern. To us who are situated on the earth, the disc of the moon appears to pass over that of the sun from west to east by the excess of its relative motion. This is all that is constant in the phenomenon; for the magnitude of the part eclipsed and its position on the sun's disc, are extremely variable for several reasons, which will be soon explained.

Method of Calculating the General Circumstances of Occultations of the Planets and Stars by the Moon.

396. Occultations of the planets and stars by the moon are calculated upon precisely the same principles as eclipses of the sun. Fig. 92. If T represent the centre of the earth, L the centre of the moon, and S the centre of the planet or star, the line SL will represent the axis of the lunar shadow, and the angle TLO will be the apparent distance of the centre of the shadow from the centre of the earth. The general expression of this angle in a function of the time, may be obtained here as before, by calculating the angles at S and T. Making this expression equal to the different values of the angle TLO, which belong to the different phases of the eclipse, and considering the time as unknown, we obtain as before the epochs of these phases.

We here understand by *lunar shadow* the portion of space which the rays from the planet or star cannot reach, in consequence of the interposition of the body of the moon. In this sense the expression *lunar shadow* is correct, though we are not to attach to it any idea of sensible obscurity.

In solar eclipses, when the object is to calculate the angle *STL*, or the apparent distance between the two bodies as seen from the centre of the earth, we have considered this distance, which is always very small in eclipses, as the hypothenuse of a right-angled triangle, the sides of which are the latitude of the moon, and the difference between the sun and moon's longitude. In the occultation of planets and stars by the moon, we can likewise make use of the same supposition; but as the latitude of the eclipsed planet or star is not zero, as is the case with the sun, and as that of the moon may not be very small, one of the sides of the triangle will be the difference of latitude, and the other the difference of longitude, reduced on account of its distance from the ecliptic, by multiplying by the cosine of the latitude. This is all the difference that exists between the phenomena of occultations and those of solar eclipses. With these modifications, the formulas of the preceding sections will be completely applicable; and it is sufficient to substitute for the parallax of the moon and that of the planet, their numerical values suited to the different cases which may happen. If the body eclipsed be a star its parallax is nothing, and its motion in latitude and longitude is also nothing. Top. 35.

Method of Determining the Circumstances of a Solar Eclipse or an Occultation for a Definite Point of the Earth's Surface.

397. Hitherto we have considered the phenomena of solar eclipses only in their general circumstances, and as they would appear to an observer at the moon. If we would determine their particular appearance at a given point of the terrestrial spheroid, it is necessary to introduce into our calculations some particular conditions relating to the position of the observer.

In this case all the questions that can arise, resolve themselves into the two following. 1. To find the apparent distance of the two bodies, as seen from a determinate point of the earth's surface, at a given instant; 2. To find the instant when a determinate distance will take place for a given point.

398. The first problem is the easiest. We calculate the *apparent* latitudes and longitudes of the two bodies for the given instant; that is, the latitudes and longitudes as seen at the place where the observation is made. We then calculate the apparent distance of the centres, considering this distance as the hypothenuse of a right-angled triangle, one of whose sides is the difference of the apparent latitudes, and the other, the difference of the apparent longitudes, reduced on account of the moon's distance, from the ecliptic by multiplying by the cosine of the latitude. Top. 35.

Knowing the distance of the centres and the apparent diameters of the two bodies, at the given instant, we can easily determine whether there is an eclipse. We have only to inquire whether the apparent distance of the centres is less or greater than the sum of the semidiameters. In the first case, there is an eclipse, in the second, it has not commenced or has already passed.

In this calculation we must employ the *apparent* diameters as seen from the place of observation, that is, augmented according to the altitude of the bodies above the horizon. The moon only is sufficiently near the earth to make this apparent augmentation sensible.

399. Of the two questions proposed at the beginning of this section, that which we have just solved is much the easier. We now pass to the second, which is the converse of the first, and in which it is proposed to find the time, the distance of the centres being considered as known.

This second question is employed to determine, for example, if an eclipse is possible in a given place, and at what instant it will happen.

In order to arrive at a solution, we begin by calculating the circumstances of the eclipse for the earth in general, by the methods laid down in the preceding section. We shall thus know the instant of the beginning and end of the eclipse, and consequently its total duration. Then we divide this duration into a certain number of intervals, for example, into half hours, and calculate for each of these intervals the elements of the apparent place of the two heavenly bodies, as seen from the given point. We then determine the apparent distance of their centres by the method which we have just explained, and by comparing this distance with the sum of the semidiameters, we shall discover if an eclipse takes place or not. We shall thus learn, at least within half an hour, the time of the beginning and end of the eclipse for the place of observation.

In the present state of astronomy all eclipses of the sun, and for the most part, occultations of the planets and fixed stars, are known beforehand very nearly. There is no occasion then for this preparatory calculation which we have explained, or at least rarely. Still it was proper to consider it in order to complete the plan we had proposed to ourselves in this work, that is, to show how an insulated observer could build up the whole science of astronomy simply by his own observations and calculations.

Taking it for granted, then, that we know very nearly the times of the beginning and end of the eclipse, at the place for which the calculation is made, it is required to determine them more exactly. We calculate for these times the apparent longitudes and latitudes of the two bodies, their apparent horary motions and their apparent semidiameters, regard being had to their altitude above the horizon. It is very easy to make the small correction which this altitude requires, by means of a formula to be found in the Appendix. It is necessary only in the case of the moon.

We then suppose that the times sought differ from the preceding only by quantities so small that the apparent horary motions may be considered as uniform during this interval. Thus, regarding the correction of the epochs calculated as an unknown quantity t', we shall have an analytic expression for the distance of the centres in a function of t', and this expression, being equal to the true distance which must take place for the phase under consideration, will give an equation by which t' is determined.

If we wish to obtain this with still greater precision, we calculate again, for this new epoch, the elements of the apparent place of the two heavenly bodies, their horary motions, and their semidiameters, regard being had to their altitude above the horizon. Then we take this new epoch for the origin, and calculate in the same manner the necessary correction t''. This correction will be incomparably smaller than t', and it will give the epoch sought with the greatest exactness. This mode of proceeding is precisely the same as that laid down for solar eclipses, when we wished to correct the results of the first approximation.

400. It might be proper here to notice a curious phenomenon in eclipses of the sun; we allude to the progress of the shadow of the moon over the earth's disc. We have already remarked, that it must proceed from west to east in the direction of the proper motion of the moon; for, this body moving more rapidly than the sun, in the apparent orbit which it describes about the earth, its shadow must follow it in the same direction and at the same rate. Thus an observer, situated in the moon, and viewing the earth thus eclipsed, would see the eclipse commence upon the westerly part of the disc, and terminate at the easterly part. To observers upon the earth it is always the westerly part of the sun which is first eclipsed, and the easterly last. Hence the disc of the moon appears to us to pass over that of the sun, from west to east, by the excess of its motion over that of the sun. As to the portion of the disc which is thus eclipsed, it depends upon the respective positions of the moon, the sun, and the observer, which together occasion very great variations. In small eclipses, the inclination of the lunar orbit may be so great that the part eclipsed shall be entirely on one side of the disc. For example, if the moon approaches from its ascending node, rising towards the north pole of the ecliptic, it is possible that its disc may not meet that of the sun except in its most northern part, or rather a little on the side of this point towards the west. In this case, the eclipse would take place wholly on the western part of the sun's disc. The contrary might happen at the descending node, and the eclipse would take place only on the western part of the disc near the southern extremity. Similar effects may be produced by change of parallax in longitude and latitude, which make considerable variations in the apparent inclination of the lunar orbit.

Hence it will be perceived that the motion of the shadow and penumbra upon the earth must, in like manner, be subject to great variations. Still it is always very rapid, and the entire extent of the earth's disc is always passed over in a little less than 4^h. To illustrate this clearly, we leave out of consideration the inclination of the lunar orbit. Let the moon then be supposed to move in the plane of the ecliptic; and, for the sake of greater simplicity, we neglect also the parallax of the sun, which is extremely small. The case being thus simplified, let *T* be the centre of the earth, *MM'O* a section of its surface Fig. 96. made by the plane of the ecliptic, *TLS* the projection of the circle of latitude corresponding to the conjunction. Under these circumstances the course of the moon's shadow upon the earth's surface, will follow the circle *MO*, made by its intersection with the ecliptic. Suppose now that, at a certain instant after conjunction, the sun is in the prolongation of the visual ray *TS'*; and the moon at *L'* upon the visual ray *TL'*, both lines being drawn from the centre *T*. Then, by drawing from the point *L'* the straight line *L'O*, parallel to *TS'*, and meeting in *O* the terrestrial surface, the point *O* will be the place of the shadow upon this surface, and the angle *TL'O* will be equal to *L'TS'*, that is, to the relative motion of the moon with respect to that of the sun from conjunction. Let us take these bodies at their mean distances, and consider only their mean motions. That of the sun is 59′ 08″ a day, or 2′ 28″ an hour. That of the moon for a day is 13° 10′ 35″, which gives for its hourly motion 32′ 57″. The difference 30′ 29″ between these, is the relative motion of the moon, or its velocity over that of the sun. This is then the hourly increase of the angle *TL'O*. And as double this value exceeds the mean parallax 57′ 4″, which represents the 361 apparent diameter of the earth as seen from the moon, it will be perceived that the axis of the shadow will become a tangent to the earth's surface, before two hours have elapsed after conjunction. Then the angle *L'TO* will be equal to 90°, minus the horizontal parallax of the moon; so that the centre *O* of the shadow will, during this time, have passed over, upon the earth, an angle equal to this quantity, plus the angle *LTL'*, described by the moon about the earth, reckoning from conjunction.

Hitherto we have made no allowance for the diurnal motion of the heavens; but it is necessary to attend to this circumstance if we would consider the variations of the centre of the shadow, with respect to a determinate point upon the earth. For the sake of greater simplicity, we shall neglect the obliquity of the equator to the ecliptic, and suppose their planes to coincide with each other; then the circle *MM'O* will represent the terrestrial equator, and in order to simplify the reasoning, we attribute to this the apparent motion of rotation of the heavens, in a contrary direction, which, as we have before remarked, makes no alteration in the phenomena as seen from the earth. This being done, suppose an observer at *M*, at the instant of conjunction, that is, so situated as to have the sun and moon at his zenith. This observer, by the effect of the diurnal motion, will be carried on his circle of latitude from *M* towards *M'*, from west to east. He will move after the lunar shadow, which proceeds in the same direction, but he cannot overtake it; for, the rotation of the heavens, being 360° in a day, is at the rate of 30° in two hours, while, during the same time, the progress of the moon's shadow upon the circle *MM'*, exceeds it by 90°. It will then separate from the observer with the excess of its velocity; but, on account of this disposition, the observer will see the eclipse for a longer time than he would have done if he had been stationary. Nevertheless his velocity can never be equal to that of the shadow, if it could be, it would render the shadow permanent upon this point of the terrestrial surface.

The very great rapidity with which the shadow moves may be seen immediately from the triangle *L'TO*. In this triangle the sides *L'T*, *OT*, are to each other very nearly as 60 to 1. Hence the sines of the opposite angles *L'OT*, *TL'O*, are in the same ratio. The first then varies sixty times as fast as the second. The angle *T* which completes the triangle, partakes of this rapid increase according to the same proportion.

Beginning with this ratio, which becomes exact when the angles *TL'O*, *L'TO*, are very small, let us seek to deduce from it the time which the observer, situated in *M*, will remain in total darkness. In order to do this it is necessary to know the relative motion of the shadow with respect to the observer. The motion of the moon over that of the sun is, as we have seen, 30′ 29″ in an hour, or 30″,5 in a minute, nearly. This will be the value of the angle *TL'O* a minute after conjunction.

In order to reduce this motion to the centre of the earth, it is necessary to multiply it by 60, which gives 30′ 29″. This is the value of the angle *L'TO* for the same instant. Moreover, by virtue of its proper motion in longitude, the moon has also described the angle *L'TL* which, in the same interval, is equal to 33″ nearly, since its hourly motion is 32′ 57″. The sum 31′ 2″ of these two partial angles, will be the total angle *MTO*, described by the shadow about the centre of the earth, in a minute of time. But, during this interval the motion of the observer about the same centre, arising from the rotation of the earth, will be $\frac{15^{\circ}}{60}$ or 15′. The relative motion then of the shadow with respect to the observer, seen from the centre of the earth, will be 31′ 2″ — 15′ or 16′ 2″. It is in virtue of this excess that the shadow passes the observer.

Now we have seen that the semidiameter of the perfect shadow, as seen from the moon, is, at a maximum, equal to 1′ 391. nearly; consequently, its whole diameter will be 2′. Multiplying this quantity by 60 we shall have 2° nearly for the breadth of the perfect shadows as seen from the centre of the earth. This value being more than seven times the quantity 16′ 2″, which expresses the relative motion of the shadow with respect to the observer, in one minute, it will be seen that the whole duration of total darkness may exceed seven minutes under the circumstances above supposed.

401. By means of the principles which we have now laid down, we can predict and determine the moment of an eclipse with the utmost precision. But to avoid all superfluous calculations, it will be advantageous to devise some simple method of ascertaining beforehand whether there will be an eclipse. This is easily done. If, for example, we first consider eclipses of the sun and moon, we shall perceive immediately that it is sufficient to find a period after which the sun and moon return exactly or very nearly to the same positions with respect to the node of the moon's orbit. For, after this interval, the motions of these bodies recommencing in the same manner, the eclipses which depend upon them must, in like manner, recur successively and nearly in the same order. There can only be the differences arising from the inequalities to which the motions of the sun and moon are subject. The whole is then reduced to finding such a period; and in order to this, we have only to combine the synodic revolution of the moon and the synodic revolution of its nodes, in a period that shall just admit of each being completed a certain number of times.

Now we have before seen that the synodic revolution of the 313. moon is $29^{d}\ 12^{h}\ 44'\ 03''$, and the synodic revolution of the nodes $346^{d}\ 14^{h}\ 52'\ 16''$. These two quantities are to each other very 314. nearly as 19 to 223; that is, in 223 synodic revolutions of the moon, the node returns 19 times to the same position with respect to the sun. The difference is 10^h $50'$ nearly, as may be easily shown, and it answers to $28'$ $09''$, by which the longitude of the node is greater than it was in its former synodic position. Now 223 lunar months are equal to $6585^d,321124$, or 18 mean tropical years and 11 days; so that after this interval all eclipses both of the sun and moon must also recur in the same order, which affords us a very simple means of predicting them when we know those which happened during the preceding period. The return of eclipses after 223 lunations has been known from the time of the ancient Chaldean astronomers. They called this period *Saros*; and we frequently find it thus designated in the works of modern chronologists.

In like manner, if we compare the sidereal revolution of the
308. moon, which is 27^d 7^h $43'$ $11'',51$, with the sidereal revolution of its
305. nodes which is 6793^d 10^h $06'$ $29'',95$, we perceive that they are
in the ratio of 17 to 4227, that is, in 4227 sidereal revolutions of the moon, there are very nearly 17 sidereal revolutions of the node. The difference is 12^h. Then the position of the moon with respect to its nodes and to the different points of the heavens, becomes again very nearly the same. Occultations of the stars must, therefore, recur again in the same order. From the duration of a sidereal revolution we find this period to be $115488^d,661$ or 316 tropical years, 72 days, 2 hours, 24 minutes.

But the above ratios not being rigorously exact, these laws are to be regarded only as approximations, and the order observed in the several periods, must at length be changed, as is well ascertained. The periodical inequalities which affect the motions of the moon, disturb also these returns, since they have not all for their periods a multiple of the synodic or sidereal revolution. Still, as these inequalities are inconsiderable, we make use of the ratio above established in finding approximately the epochs of eclipses, and then we calculate by astronomical tables, the precise instant at which they are to happen. If we wish to approximate nearer at first, it will be sufficient to allow for the principle inequalities of the moon's longitude.

Another cause of deviation arises from the great secular inequalities, the nature of which we have already explained, and which in the immense duration of their course, sensibly change the values of the small periods, as will be easily conceived; so that all attempts to ascertain exactly the periods under consideration will be fruitless. In order to find directly, and independently of such periods, the syzygies when an eclipse is possible, we begin by calculating from the tables of the moon, the epoch of each mean conjunction. We commonly employ for this purpose the method of *astronomical epacts.*

The *epact of a year* is the age of the moon at the commencement of this year, that is, the time which has elapsed, or which ought to have elapsed, from the last conjunction of the preceding year to the 31st of December, at noon, mean time, if the year in question be a common year, or to the first of January, at noon, mean time, if it be a bissextile. I speak here only of the ancient tables. We unite these epacts to the other elements of the moon for the commencement of the year. The epact of each month then is the quantity which it is necessary to add to the epact of the year, in order to have the age of the moon at the commencement of this month.

Knowing the epact of any month it is easy to find the mean conjunctions which take place in this month. For, in the first place, by subtracting the epact ~~of~~ a synodic revolution of the from moon, that is, ~~of~~ $29^d\ 12^h\ 44'\ 03''$, the remainder will indicate the from day of the month and hour of the day when the first conjunction will happen. I say the first, because there are sometimes two conjunctions in the same month, separated by a synodic revolution; but there can never be more than two, since the months are at most but 31 days in length.

When we know the epochs of mean conjunction or of new moon, we find those of opposition or of full moon, by subtracting from the first half a synodic revolution, or $14^d\ 18^h\ 22'\ 01''$.

The times of conjunction and opposition being thus determined, in order to know if there will be an eclipse of the sun at the one, or of the moon at the other, we must calculate the distance of the moon from its node, and see if it falls within the limits required for an eclipse to take place. These limits, as calculated by M. Delambre, are as follows.

If the distance of the ⊙ from the ☽'s node is	less than 9°	the eclipse of ☽ is	certain.
	greater than 12½°		impossible.

Between 9° and 12½° it is doubtful, and recourse must be had to more precise calculations.

If the distance of the ⊙ from the ☽'s node is	less than 13½°	the eclipse of the ⊙ is	certain.
	greater than 19°		impossible.

Between 13½° and 19° it is doubtful, and a more exact calculation is necessary.

We perceive that these limits are more extended in the case of solar eclipses than for those of the moon. The first, therefore, must happen more frequently. But they are not visible, except from certain parts of the earth, on account of the parallax which modifies them; while those of the moon are visible from all points of the terrestrial hemisphere which have the moon above the horizon during the eclipse.

In the new tables published by the French Board of Longitude, we have not the epacts laid down, but as we find the values of the mean longitudes of the sun or the moon, as well as the distances of these bodies from the node of the moon's orbit, these arguments answer for calculating the epochs of the syzygies, and all the other phases, with no less despatch and greater precision than the method of epacts. The mode of proceeding is explained in the tables.

It remains to speak of an indispensable precaution to be used by astronomers in observing an eclipse. In solar eclipses the face which the moon presents to us does not receive the sun's rays; it is therefore completely invisible; for the light reflected by the earth to such a distance is entirely lost in the superior brightness of the sun. Hence we do not perceive the passage of the moon over the sun's disc, except by means of the interruption in the sun's light when it begins to make an indentation. But it is impossible to seize the first commencement of this phenomenon, unless we know beforehand the instant when it will happen, which may be calculated within a few seconds by means of the tables, and unless we know also in what part of the sun's limb the indentation first commences. The same problem presents itself in the transits of the planets over the sun's disc, and in emersions of stars and planets, in the case of an occultation by the moon. It is necessary to know precisely at what point of the moon's disc the star or planet will emerge, in order to be ready to observe the first indication of its reappearing.

Let us consider this last case, which is the most general and from which all the others may be deduced. We first calculate, by the tables, the precise instant of the emersion of the star or planet, and the difference of longitude and latitude between its centre and that of the moon at this instant. From this it is easy to determine the angle formed by the line of the centres with the circle of latitude which passes through the moon; for the tangent of this angle is equal to the difference of longitude, reduced on account of the distance of the star or planet from the ecliptic, and divided by the difference of the latitudes. We shall thus have the arc of the lunar disc comprehended between the point of emersion and the point of the disc nearest the plane of the ecliptic. This arc is denoted by *SE* in figure 95, ♈*NL* representing the ecliptic, ♈ the vernal equinox, C the centre of the moon, *C l* its circle of latitude, and *S* the point of the disc where the emersion takes place.

Now let *Z* be the zenith of the place of observation; then ♈*L* will be its longitude, and *LZ* its latitude, both being referred to the ecliptic. The great circle *ZC* will represent the plane of a vertical in which the centre of the moon is found, since this vertical must contain the centre of the moon and the zenith. These two conditions are sufficient for determining the angle *NC l*, formed by the circle of latitude of the moon with the vertical *ZC*, and by adding to it the angle *l CS*, just calculated, the sum will give the arc *SH* of the lunar disc comprehended between the point of emersion *S* and the point *SH* of the disc which is most distant from the zenith *Z*, and consequently the nearest to the horizon. We reckon this arc from the lowest point of the disc proceeding towards the east, and from 0° to an entire circumference. The formulas, being once established by means of this construction, are applicable to all cases, the rules respecting algebraic signs being carefully observed. If we wish to refer what is here laid down to eclipses of the sun, it is only necessary to substitute, in the formulas, the centre of the sun for the centre of the moon, and as the latitude of the sun is always nothing, the formulas become more simple (*App.*)

By means of this preparatory calculation we always know, with an exactness abundantly sufficient, to what point of the moon's disc or that of the sun, it is necessary for the observer to direct his attention; thus informed, he can seize the instant of the phenomenon with all the precision of which his organs are susceptible.

Of the Connexion between the Motions of the Moon and the Periodical Ebb and Flow of the Sea.

402. Before passing to another division of my subject, I ought to give some account of a phenomenon extremely remarkable, upon which the moon appears to have an important influence. I refer to the ebb and flow of the sea called the *tides.*

Twice every day the sea rises and falls with a regular oscillatory motion. The water elevates itself at first during about one fourth part of a day; it overflows the shores and runs into the channels of rivers, even to great distances from their mouths. When the water has arrived at its greatest height, it is said to be *high water*, *full sea*, or *flood tide;* it remains in this state only for a few moments. It soon begins to decline, and it falls at the same rate at which it rose. It withdraws from the places it had overflown; and continues to descend nearly a fourth part of a day. The water having reached its greatest depression, it remains stationary for a short time. It is then said to be *low water* or *ebb tide.* Very soon the same motion recommences, the sea advances and retreats again by the same steps, and according to the same laws.

403. These oscillations of the sea may be increased by the action of the wind; but they do not arise wholly from this cause, for they are observed, in like manner, when the weather is calm and serene. Aside from this their periods are so regular and constant, that we are able to predict the return of low and high water. This constancy clearly indicates a regular and durable cause whose effects are produced periodically. In order to discover this cause, we must observe the phenomena for a long time, follow out all the results, and determine the periods to which their smallest variations are subject. We may afterward undertake to discover, in the course they take, the nature of the causes which produce them. By thus examining the phenomena of the ebb and flow of the sea, we perceive, even in their minutest details, a marked connection with the conjunctions of the moon and sun; the influence of the moon is particularly apparent.

This is evident, in the first place, from the intervals at which they return. These intervals are not always exactly the same, but they still have a mean duration from which they differ but little, and this, expressed in mean time, is $1^d\ 0^h\ 50'\ 28'',32$. During this time it is twice low water and twice high water. Now this is precisely the time which the moon employs in returning to the meridian by the effect of its mean motion, and this period of ebb and flow may be termed a lunar day.

We perceive then, that if high water happens to day in a certain harbor at 0^h, tomorrow it will happen at $0^h\ 50'\ 28'',32$, the day after at $1^h\ 40'\ 56'',64$, and so on continually, the daily retardation being $50'\ 28'',32$; but in this interval there will be another high tide, which may be called the morning tide; this will happen the first day at $12^h\ 25'\ 14,''2$, the second at $12^h\ 25'\ 14'',2 + 0^h\ 50'\ 28'',32$, and so on.

404. But this is not exactly the course observed. The daily retardation from one tide to another is sometimes a little more than $50'\ 28'',32$, and sometimes less. This is only an additional proof of the influence of the moon upon these phenomena, for the course of this body is not more regular, it is subject to many inequalities which have a striking agreement with the retardations and variations observed in the tides. We may, therefore, consider this action of the moon, whatever may be its nature, as established beyond a doubt.

405. But the moon is not the only body whose action is perceived in the tides; the sun exerts also a sensible influence. We always remark, that the highest tides happen at the syzygies, and the lowest at the quadratures; so that by observing simply the phases of the moon, we are able to predict their return.

These elevations and depressions of the sea are more remarkable and conspicuous when the moon and sun are nearest the earth; and they diminish as these bodies recede; but even in this secondary effect the action of the moon maintains its ascendency, and the variations in its distance are particularly sensible. Finally, the position of these two bodies on the one side and the other of the equator, produces likewise modifications.

All this is to be understood of a sea very extended and free on all sides like the ocean. In small seas, and near the shores, the motions of the water must be obstructed and opposed by the obstacles it meets with. In like manner, the instant of the tide differs according to the time necessary for the undulations to be propagated. This happens in our harbors although they all belong to the same ocean. The hour of high water is very different in one port from what it is in another, although constant in each harbor. At Dunkirk, for example, high water happens half a day after the passage of the moon over the meridian; at St Malo it is one fourth of a day; at the Cape of Good Hope it is $0^d\ 1^h\ 30'$. The hour when this phenomenon happens on the day of full moon is determined in each place by observation, after which, by adding to it successively $50'\ 28'',32$, we obtain the successive retardations of the tides from one day to another, and the times at which they ought to happen. As there are two high tides every lunar day, two observations of this kind are necessary to enable us to foretell the times of every high tide.

The preceding method gives only the *mean* epochs of the phenomena under consideration, that is, those which would take place, if the course of the moon which governs them was subject to no irregularity. In order to predict the exact instant we must have recourse to the theory of attraction, which, having made known the cause of the oscillations of the sea, has shown the connexion which exists between the variations which they experience and the inequalities of the bodies which produce them. In the second volume of the *Mécanique Céleste* will be found all the formulas necessary for this purpose.

406. Among the inequalities in the motion of the sea we remark this general law; the higher the water rises at flood tide, the more it descends at ebb tide. We call the total tide the half sum of two consecutive high tides above the level of the intermediate low tide. The greatest value of this total tide at Brest is 19,32 feet. It takes place at the syzygies. The smallest is 9,15 feet. This takes place at the quadratures.

407. It is an object of great importance to obtain observations equally exact for all other harbors; but unfortunately they are yet wanting, and the members of the Institute of France thought it their duty to invite the attention of intelligent men inhabiting seaport towns to this subject. It is sufficient to set up a vertical rod properly graduated, and to observe as often at least as once every day, the exact time of high and low water, and the point of greatest elevation and depression of the water. But as the height of the water varies very slowly in approaching these extreme limits, it is necessary in determining it with precision, to

employ corresponding observations made before and after the extreme limit when the water has attained the same height. If these observations are near the *maximum* and *minimum* state, the limit sought will answer to the middle of the interval which separates them; but it is necessary in order to this, that they should not be too distant, for the tide employs a little longer time in falling than in rising. At Brest this difference amounts to nine or ten minutes.

408. The theory of attraction, having made known the cause of the ebb and flow of the sea, has furnished us, as we have said, with the means of calculating all the variations, which are to be looked for in these phenomena; it has taught us how they may be foreseen. The oscillations of the sea are thus found to be connected with the celestial motions, and they have served more clearly to make known several important particulars.

THEORY OF THE PLANETS AND COMETS.

Motion of the Planets about the Sun.

409. We have recognised and determined the motions of the sun and moon. These are to us the most interesting bodies in the system; the first, as being the source of life and fertility to the earth; the second, on account of its proximity, its light, and its phases; and both on account of the indications they furnish for the measurement of time.

But the other bodies which compose the solar system invite our attention and hold out to us the prospect of results no less curious and useful. The study of them will lead to a more perfect knowledge of the general scheme of the material world; and in this way it will connect itself with our first results, and enable us to correct them by more accurate calculations, or it will serve to bring them before us under a more enlarged view; or lastly, it will help to make known relations and analogies hitherto unobserved. Encouraged by these considerations, we proceed to examine the planetary motions. We shall endeavour to discover the laws which govern them, and in deducing these from the phenomena that present themselves, we shall continue to pursue the plan which has hitherto guided us.

410. We begin with *Venus*, which is the most conspicuous. There are few who have not observed a beautiful star in the west, a little after sunset, and called for this reason the *evening star*. This star is Venus. If we observe it for several days we find that it does not remain constantly at the same distance from the sun. It departs to a certain distance which is about 45°, or $\frac{1}{4}$ of the celestial hemisphere after which it begins to return; and as we can ordinarily discern it with the naked eye only when the sun is below the horizon, it is visible only for a certain time

immediately after sunset. By and by it sets with the sun, and then we are entirely prevented from seeing it by the sun's light. But after a few days, we perceive, in the morning, near the eastern horizon a bright star which was not visible before. It is seen at first only a few minutes before sunrise, and is hence called the *morning star.* It departs from the sun from day to day, and precedes its rising more and more; but after departing to about 45°, it begins to return, and rises later each day; at length it rises with the sun, and we cease to distinguish it. In a few days the evening star again appears in the west very near the sun; from which it departs in the same manner as before; again returns; disappears for a short time; and then the morning star presents itself.

These alternations, observed without interruption for more than 2000 years, evidently indicate that the evening and morning star are one and the same body. They indicate also that this star has a proper motion, in virtue of which it oscillates about the sun, sometimes preceding and sometimes following it.

These are the phenomena exhibited to the naked eye; but the admirable invention of the telescope enables us to carry our observations much farther.

411. Observing Venus with this instrument, we perceive that it has phases like the moon. At evening when it is approaching the sun, it presents a luminous crescent, the points of which are turned towards the east, that is, from the sun. The apparent magnitude of this crescent diminishes from day to day as Venus draws nearer to the sun. But after it has passed the sun, and reappears on the other side of it in the east, the points of the crescent are turned in the opposite direction, that is, to the west. The crescent gradually increases in magnitude till it reaches the form of a semicircle. Venus is then in its *first quarter.* As it approaches the sun, the illuminated part of its disc becomes enlarged; and when it comes up with the sun it appears full. After passing the sun and reappearing in the west, the edge of its disc begins to be hollowed out, and its illuminated part gradually diminishes, returning through the same phases that it exhibited when on its increase, but in the inverse order.

412. These phenomena represent Venus as a kind of moon, revolving about the sun and illuminated by its light. All observations confirm this fact.

When Venus appears as an entire circle its apparent diameter is very small, not amounting to more than 10″. It is then beyond the sun with respect to the earth. On the contrary, when its phases diminish, and its illuminated face is turned more and more from us, its apparent diameter is on the increase, which indicates that it is now nearer the earth. Lastly, in the interval between its disappearing in the evening and reappearing in the morning, we sometimes see it moving over the sun's disc, with the appearance of a round, black spot, and its apparent diameter is then the greatest it ever attains during a revolution, amounting sometimes to 59″,8, or very nearly 1′.

These diminutions and augmentations are not sensible to the naked eye, on account of the irradiation which dilates a little the apparent diameters of objects, and the more so, the more they are illuminated; now as the phases of Venus increase in extent, when it is beyond the sun, its increase of light compensates, to our eyes, for its increased distance; these illusions are removed by the telescope, and the real variations of its distance are indicated by those of its apparent diameter.

413. The orbit of Venus does not embrace the earth; since, if it did, this planet would sometimes be in the part of the heavens opposite to the sun, the earth being between them, which never happens. Neither is its orbit wholly beyond the sun with respect to the earth; for if it were, Venus would never appear between the sun and the earth; whereas it is sometimes seen passing over the sun's disc. Lastly, while the sun moves or appears to move in the ecliptic, Venus never deviates beyond its customary limits; and its oscillations about the sun, or its *elongations*, are always nearly of the same extent. These facts, taken together, manifestly prove that Venus moves about the sun in an orbit which returns into itself, and which accompanies the sun in its apparent elliptical motion. The progressive order of its phases, moreover, indicates that this planet is opaque, that it shines with reflected light, and is nearly spherical.

414. The orbit of Venus being presented to us under different points of view in its successive positions, there must result from this circumstauce a variety of apparent irregularities and anomalies, when we refer its motions to the centre of the earth; but htese disappear when we consider it with reference to the sun which is its true centre. All this is confirmed by observation, as we shall soon see.

415. Venus is not the only planet which exhibits the appearances we have described. Those of *Mercury* are perfectly similar, though confined to narrower limits. This planet, therefore, moves about the sun like Venus, but in a smaller orbit. We infer from its phases that it is opake, that its form is nearly spherical, and that it receives its light from the sun. Its greatest apparent diameter is 11″,3, its least 5″. Mercury and Venus are called *inferior* planets, because their orbits are below or within that of the earth.

416. These two planets, as seen from the earth, always appear to accompany the sun. Their angular distances never exceed certain limits which are nearly constant. The same cannot be said of the other planets; these diverge from the sun to all possible angular distances.

The apparent motion of Mercury and Venus is very unequal; it is sometimes direct, and sometimes retrograde. By comparing them with the stars which they meet with in their course, or what is still more exact, by observing from day to day their declination and right ascension, we perceive that they do not continue in the same parallel to the equator; nor do they move exactly in the plane of the ecliptic. But all the planets, except those lately discovered, deviate very little from this last plane; and it is this circumstance which has given, from the remotest antiquity, a particular denomination to the zone which comprehends them. It is termed the *zodiac*, and extends to 8° or 9° on each of the ecliptic. But since the discovery of the four new planets, this denomination has become useless; for Ceres, Juno, and especially Pallas, go far beyond these limits.

417. All the planets except Mercury and Venus, in their revolutions, come into *conjunction* and *opposition* with respect to the sun; that is, they are sometimes on the same side as the sun with respect to the earth, and sometimes the earth is between them and the sun. This last circumstance is peculiar to them, and never happens with regard to the inferior planets. We are led by analogy to inquire whether these planets do not move about the sun, like Mercury and Venus, but in more extended orbits, and we find that all their phenomena indicate such a motion.

418. Let us take Mars as an example. When we observe this planet with a telescope, its disc appears constantly illuminated and round; it never exhibits a crescent like Venus; but

yet it experiences very sensible variations in its phases. In conjunction and opposition its form is completely circular. In passing from one of these positions to the other, it gradually contracts and assumes the shape of an oval more or less oblate; and the change always takes place slowly and progressively. These phenomena indicate that Mars is an opaque body, illuminated by the sun, and nearly spherical. They are accurately represented by supposing the planet to move in an orbit returning into itself, and comprehending the sun and earth. Indeed this consequence follows so necessarily from the phenomena that they could not exist without involving it. If the orbit of Mars did not embrace the earth, it could never appear in opposition; and if it did not include the sun, it would pass, when in conjunction, between the earth and the sun; its disc also would be hollowed out, like those of Venus and the moon, instead of appearing always round.

Moreover the apparent diameter of Mars increases in passing from conjunction to opposition; and diminishes in passing from opposition to conjunction. Accordingly, in the first case, Mars approaches the earth; and in the second recedes from it. The variations of its apparent diameter are considerable. Its greatest value is 17″,1; its least 3″,6; the corresponding distances then are to each other as 3,6 to 17,1, or nearly as 1 to 5; that is, Mars is 5 times further from the earth in the second case than in the first.

These great differences indicate that the earth is not at the centre of the orbit of Mars. From analogy, it is much more natural to suppose that this planet moves about the sun, like Mercury and Venus; in which case the sun must carry the orbit of Mars also along with it, in its apparent motion through the ecliptic; the greatest and least distances of Mars from the earth must take place when the sun is in apogee; and its greatest and least apparent diameters must then be perceptible. All this is confirmed by observation, and hence we conclude that Mars revolves about the sun.

419. The phenomena presented by the other planets whose orbits embrace the earth, and which are hence denominated *superior* planets, take place in precisely the same order, and accordingly lead to the same conclusions; only the variations, observed in the apparent form of their discs, are much less

sensible, and this form deviates less from a circle. Hence we infer that their distance from the sun is much greater than that of Mars; for, if there were a planet at so great a distance from the sun, that the solar orbit might be regarded as a point with respect to it, and this planet could yet be seen from the earth, the appearances exhibited would be sensibly the same as if we were placed at the centre of the sun, about which it revolved. Its disc would, therefore, appear always under the form of a circle, the variations of its phases being too small to be appreciated.

420. To render more sensible the variations which take place in the distances of the planets from the earth, we have placed together in the following table their greatest and least apparent diameters, as given by observation. It must be remarked that Ceres, Pallas, Vesta, and Juno, are so small that their apparent diameters have not yet been satisfactorily determined.

				″		″
Mercury,	greatest	apparent	diam.	11,3	least	5,0
Venus	"	"	"	59,6	"	9,6
Mars	"	"	"	17,1	"	3,6
Jupiter	"	"	"	44,5	"	30,1
Saturn	"	"	"	20,1	"	16,3
Uranus	"	"	"	4,1	"	3,7

421. These considerations lead to the conclusion, that the sun is nearly at the centre of all the planetary orbits, and carries them with it in its apparent motion through the ecliptic; or rather, to the more natural supposition, that this apparent motion of the planetary orbits with the sun, is only an appearance occasioned by the real motion of the earth, which, by transferring us to different points of the ecliptic, causes the sun and the planetary orbits of which it is the centre, to appear as if they revolved about us. For if we are guided by analogy, which is always so evident in the works of nature, we cannot help considering the sun as the common centre of the whole planetary system, as well as of the earth's orbit; thus establishing the most perfect symmetry throughout. But as all the arguments for this opinion have not yet been stated, and as they crowd upon us more and more as we advance in the study of the heavens, we content ourselves for the present, with barely intimating

that such an arrangement is possible. We shall now for a moment transfer ourselves in imagination from the earth to the centre of the sun, and we shall endeavour to consider our observations as taken from that position.

Method of Determining the Position of the Planetary Orbits.

422. The most simple way of representing the planetary orbits, is to regard them as plane curves, the planes of which pass through the sun. This supposition is favoured by the analogy which exists between the motions of the planets about the sun, and that of the moon about the earth. Let us see how it agrees with observation.

423. If the planets move in plane orbits and the planes pass through the centre of the sun, the points where each planet meets the plane of the ecliptic, must be opposite to each other in the same straight line, drawn through the centre of the sun, and moveable with it in the ecliptic. These points, therefore, determine the intersection of the plane of the orbit with the ecliptic, which is called the *line of the nodes.*

An observer placed at the sun might easily determine whether this condition is fulfilled. He might determine by a series of observations, the instants when the latitude of the planet is nothing, and then see if the heliocentric longitudes at the same instants were the same, or whether they differed by a semicircumference. Those who are situated on the earth can also determine, by geocentric observations, the instant when the planet passes its node; but, not being at the centre of the planetary motions, these nodes will not appear opposite to each other on the celestial sphere; and the straight line which joins them being apparently carried along the ecliptic by the sun, will be presented under different degrees of obliquity, which prevents this opposition being perceived.

But among all the situations which the plane of the orbit is capable of taking with respect to us, there are two, but very rare indeed, in which the difficulty may be avoided; these are when the planet is in the ecliptic, and at the same time in conjunction or opposition with respect to the sun; for then we see it from the earth in the same straight line in which it would appear as viewed from the sun. A number of observations of this kind would determine whether the node of the planet has always the same heliocentric longitude.

This method is easily applied to Mercury and Venus. As the periods of these two planets are very short, the first being only 88 days, and the second $224\frac{7}{18}$, it follows that they pass through their nodes very often; for they must evidently pass each of them every revolution. The short intervals of these phenomena must, therefore, render the condition in question comparatively frequent. We have a particularly favourable opportunity for the observation in question at the time of the transits of these planets over the sun's disc. For then we see them in the same straight line with the sun; and at the same time their latitude, necessarily very small, shows them to be near one of their nodes. The observation of these transits will therefore settle the question with respect to the constancy of the nodes, as seen from the sun. The following table is decisive.

Observed Transits of Mercury at its Descending Node.

Years.	Mean Time at Paris reckoned from Noon.				Geocen. Long. of Mercury in Conjunction.			Geocen. Lat. of Mercury.		
		h	′	″	°	′	″	′	″	
1674	6th May	15	0	37	46	43	19	13	51	South
1770	5th	12	45	0	44	43	5	0	37	North
1740	2d	10	47	36	42	43	23	14	52	North
1753	5th	18	25	37	45	47	28	2	22	South
1786	3d	16	27	0	43	47	44	12	3	North
1799	7th	0	5	48	46	54	27	5	44	South
Mean in 1763					45	3	14	5	35	North

All these observations concur to show that Mercury has a very small latitude, when its heliocentric longitude becomes equal to 45° 3′ 14″. Accordingly, it is then near one of its nodes, which we call the *descending node*, because when Mercury passes it, it is descending toward the south. This conclusion is confirmed by rigorous calculation; for, by determining the heliocentric longitude of the descending node of Mercury from its transit in 1799, by methods to be explained hereafter, M. Delambre found it to be 45° 57′ 5″ for this epoch. It is necessary to subtract 27′ 33″ from this longitude, in order to bring the origin of the angles to the same point of the heavens as that made use of in 1763; for the equinox must have retrograded this quantity, during the interval, the annual rate being 50″,1. The result of M. Delambre, thus reduced, gives for the longitude of the descending node in 1763, 45° 29′ 32″, differing but little from the geocentric longitude given by the mean of the transits. This comparison, it is obvious, can be considered only as an approximation, since regard is also to be paid to the latitude of Mercury. Moreover its nodes are not strictly fixed, as we shall see hereafter. All these causes must affect the results compared. If, therefore, we make any allowance for them, as we ought to do in this first attempt, it is evident that the longitude of the descending node of Mercury is nearly constant.

The same constancy is observable in the passage of Mercury through its *ascending node*, so called, because when the planet passes it, it is ascending toward the north side of the ecliptic. The following are some of these observations.

Years.	Mean Time at Paris reckoned from Noon.				Geocen. Longitude of Mercury in Conjunction.			Geocen. Latitude of Mercury.		
		h	′	″	°	′	″	′	″	
1697	2d Nov.	17	37	50	221	35	2	10	40	S.
1710	6th	11	28	30	224	11	12	2	19	S.
1723	9th	5	15	52	226	47	26	6	0	N.
1736	10th	22	59	40	229	23	38	14	21	N.
1743	4th	22	27	0	222	37	50	9	3	S.
1756	5th	16	18	0	225	14	3	0	42	S.

The heliocentric longitude of the ascending node, deduced from these observations, exceeds that of the descending node very nearly 180° or a semicircumference. These two nodes, seen from the sun, are then opposite to each other in the ecliptic, as they must necessarily be if the orbit be in the same plane. They are moreover nearly constant.

The same agreement is found in the passages of Venus through its descending node, as appears from the following observations.

Years.	Longitude of Venus in Conjunction.	Geocentric Latitude of Venus.
	° ′ ″	′ ″
1761	75 36 14	9 34 South
1769	73 27 28	10 13 North

The agreement of these phenomena, though very remarkable, is not sufficient to justify our extending the inference to all the other planets, the long periods of whose revolutions must render much more rare the opportunities for observing them, when the latitude is very small or nothing at the instant of conjunction. We must, therefore, have recourse to some method of determining whether the node is constant, without the necessity of observing it under circumstances so seldom to be met with. Now geometry furnishes a very general and simple process adapted to this purpose.

Fig. 97.

At the moment the planet enters the plane of the ecliptic at P, let T represent the earth, S the sun, and PSN the line of the nodes of the planet, which must pass through the sun, according to the hypothesis under consideration. If ♈ represent the equinoctial point from which longitude is reckoned, the angle ♈TS will be the geocentric longitude of the sun, which we shall call S; the angle ♈TP will be the geocentric longitude of the planet P. These two quantities are known from observation. Lastly, the angle ♈NS or ♈SP, will be the heliocentric longitude of the node N, which is the unknown quantity to be determined. Now in the triangle STP, formed by the sun, the planet, and the earth, the angle at the earth is known, and is equal to $P - S$; the side TS is also known, and is equal to the distance of the sun from the earth at the moment of observation; we shall call it R. The unknown parts are the distance SP of the planet from the sun, which we shall call r, and the angle at the planet, which may be expressed by $N - P$. But as in every plane triangle the sides are proportional to the sines of their opposite angles, it is obvious that the ratio $\frac{SP}{ST}$ or $\frac{r}{R}$, will be equal to the sine of the angle $P - S$ divided by the sine of the angle $N - P$. Hence we know a relation which exists between the two unknown quantities N and r of the problem.

Suppose now that after one or several revolutions of the planet, we again observe the instant of its passing the same node at P', and repeat the same construction. The longitude of the sun and that of planet will be different from what they were in the first observation. We shall designate them by S' and P'. But if the line of the nodes $P'S'N'$ remain nearly parallel to itself, as the transits of Venus and Mercury seem to indicate, the angle $\Upsilon N'P'$ will still remain the same as before, and consequently be equal to N. The side $S'P'$ will also remain the same, and be equal to r, whatever be the nature of the curve described by the planet in its plane, provided only that this curve is immoveable, or has only an insensible motion, during the interval between the observations; which the slowness of the motion of the solar ellipse, renders very probable. This second passage, therefore, will give us a new relation between the unknown quantities N and r. We shall thus have two conditions which these unknown quantities must satisfy; and this circumstance will be sufficient to determine them. By repeating the same process for several passages, we shall ascertain whether the values of N are always the same, and if they are, we conclude that the longitude of the node is constant, and experiences only the variation resulting from the precession of the equinoxes. If we find that there is a very slow motion, we must deduce the value of N from observed passages made as near each other as possible; and by repeating the same operation for distant epochs, the difference of the results will determine the motion of the node. In this manner we determined the motion of the solar perigee.

These calculations will also make known another very important element in the orbit of the planet, namely, its distance from the sun, which we have represented by r; moreover, the ratio which this distance bears to the radius R of the earth's orbit, is determined by the same process.

We might proceed in the same manner with the passages observed through the other node. The longitude thus obtained would differ from N by two right angles; but the distance SP of the planet from the sun might not be the same. This comparison would, therefore, afford some indication respecting the nature of the orbit.

Lastly, we may combine observations made upon the descending node with passages observed at the ascending node; but if, in order to determine N, we have only two passages at the opposite nodes, we cannot arrive at the required result without having recourse to an hypothesis; because the ratio of the distances r and r' would remain indeterminate. In the solar system where the orbits of the planets have little eccentricity, we might, in the first approximation, suppose the two distances of the planet from the sun at the two nodes to be equal to each other; but this would be true, in general, only for circular orbits.

We have been thus minute in describing this process, because it does not suppose any previous knowledge of the nature of the orbit; and because it leads directly to the determination of the first elements sought. In the Appendix will be found an example of the numerical calculations that are required. They are applied to Mars. The observations upon which they are founded, were made by Lacaille, and the result obtained differs but a few minutes from that generally adopted by astronomers.

424. In the present state of astronomy, the motion of the planets may be considered as very nearly ascertained, and new observations are made simply with a view to still greater accuracy. For this purpose, we adopt the motion as laid down in the tables, which may be regarded as sufficiently exact for an interval of a few days; and by means of it, we reduce to the ecliptic all the observations made upon the planet in the vicinity of the node; in other words, we determine from the position and epoch observed, the place of the node and the instant when the planet is in it. Each observation will thus give a new value of the longitude of the node. The principle of these reductions is the same as that employed in reducing to the solstice and perigee, observations of the sun made in the vicinity of these points.

If the observations and tables were perfectly accurate, all these longitudes would be equal; but in consequence of the slight inaccuracies which remain, there is generally some difference in the longitude of the node, and the epoch of passing it. We therefore take a mean between all the longitudes and epochs; and in this way arrive at great precision in our results.

This method is founded on the presumption, that the errors in the tables are errors with respect to the absolute positions of the planets observed, and not with respect to their motions. For these motions, having been determined with care, and corrected by a comparison of many distant observations, cannot for the interval of a few days, deviate sensibly from the truth.

425. The place of the node being known, we make use of it to determine the inclination. For this purpose, we observe when the sun *is in the node of the planet*, that is, when its longitude is the same as that of the node; we then calculate, from observation, the geocentric latitude of the planet, and by a simple trigonometrical process we obtain the inclination of the plane of the orbit.

It is difficult and perhaps impossible to fix exactly upon the instant when the sun is in the node of the planet; by observing the two bodies for several days in succession, before and after the epoch of the sun's passing the node, we can, by the method of *interpolation* applied to the results, determine exactly the instant when this phenomenon must have taken place. We then find, for this instant, the respective positions of the two bodies, which is also done by interpolation, according to the law of the observed motions. From these positions we obtain the inclination of the orbit. This method supposes the node previously known. But if this element is still liable to some small error, that of the inclination thus deduced would be much less, especially if the epochs selected are when the planet is near its quadrature. It is proper, moreover, to remark that all these separate determinations of the elements of the orbit, are only approximations, to be afterwards rectified simultaneously by the method of equations of condition.

The different methods now explained suppose that the astronomer is able to observe, at his leisure, the whole course of the planet in question. This is true with respect to the seven principal planets, but not with respect to comets and the newly discovered planets. We must therefore seek some method by which we can determine the position and nature of the orbit, by observing an arc of small extent. This is the object of a very difficult problem, which has been solved by applying to the unknown body, the same laws which govern the other planetary bodies whose orbits are known, and which depends upon the principle of universal attraction. These methods are explained in the *Mécanique Céleste* in all their generality.

426. When we have determined the longitude of the nodes of a planet at epochs widely distant from each other, and refer the origin of these longitudes to the same point of the ecliptic, allowance being made for the precession of the equinoxes, we find

that the nodes are not strictly fixed. They have all a retrograde motion in the ecliptic; but these motions are very slow, and belong to the class called secular. The inclinations of the orbits to each other and to the ecliptic, undergo, in like manner, slight variations.

The retrogradation of the nodes of the planets is analogous to that of the nodes of the moon. It is, as we shall see hereafter, a necessary consequence of universal attraction. The same may be said of the changes in the inclination. That analysis by which we ascertain the cause, enables us to calculate the effects.

427. The inclination of the several planetary orbits to the ecliptic, together with the position of their nodes, and the secular variations of these elements, will be given in the next section.

Nature of the Planetary Orbits.—Kepler's Laws.

428. THE position of the plane of the orbit being determined, it remains to ascertain the law of the planet's motion, and the figure described by it.

Both these particulars would be known if we could assign for each instant the length of the radius vector drawn from the planet to the sun, and the angle formed by this radius with a fixed straight line drawn in the plane of the orbit, and passing through the centre of the sun. It was by this method that we traced from observation the figure of the solar orbit.

The first element to be determined is the duration of a complete sidereal revolution of the planet. The most simple and direct method of finding this, is to observe the interval that elapses between two consecutive passages through the same node. As the plane of the ecliptic gradually changes its position by laws already investigated, allowance must be made for this change during the interval in question, that is, the observations must be reduced to a fixed ecliptic. This is done by a simple interpolation, the motion of the planet near its nodes being known.

But it is natural to expect in the planetary motions perturbations analogous to those already recognised in the motions of the sun and moon. Hence, in order to diminish their effect as much as possible, we find it necessary to determine the mean motion from observations which embrace a great number of revolutions In this way the periodical inequalities are several times compensated during the interval, and the error which remains in the definitive result is rendered insensible, by being distributed through so long an interval of time. The method is the same which we used in fixing with so much precision the length of the mean year, independently of the periodical inequalities of the sun's motion.

The mean motion being known, we proceed to determine from observation the angular motion of the planet about the sun, and the variations in its distance. Conjunctions and oppositions are particularly adapted to this purpose; for, under these circumstances, the radius vector drawn from the planet to the earth and that which is drawn to the centre of the sun, are projected Fig. 99. upon the plane of the ecliptic in the same straight line *TSM*. Accordingly, the point of the ecliptic to which we refer the planet, is precisely that to which it would be referred from the centre of the sun, or it is directly opposite. In these two cases, the heliocentric longitude of the planet, is deduced from its observed geocentric longitude; for at conjunction these two longitudes are the same, and when in opposition they differ 180°; and as the position of the orbit on the plane of the ecliptic is known, as also that of the node, by the preceding processes, a simple trigonometrical calculation will enable us to determine the angle *NSP*, or the distance of the planet from the node, and the ratio of *SP* to *TS*, or the distance of the planet from the sun expressed in parts of the earth's distance or major axis of the solar orbit. This last is already known; the radius vector of the planet will therefore be given in parts of the same scale.

The oppositions and conjunctions of the planets take place successively in different points of the heavens, and do not always correspond to the same points of their orbits. Accordingly, making several observations by the method here described, we shall obtain successively different angles and different radius vectors. Then we have only to express these radius vectors in parts of the major axis of the solar orbit, and place them about the sun in their true directions, in order to trace the figure of the curve described by the planet.

But these results may even be obtained without the necessity of observing the planets at the epochs of conjunction or opposition. One geocentric longitude and latitude observed at any epoch, together with the aid to be derived from trigonometry, will be sufficient for determining the radius vector and the distance of the planet from the node.

429. The curve thus obtained for each of the planets approaches very nearly to an ellipse, having the sun in one of the foci. This analogy to the earth's orbit, leads us to inquire whether the laws of elliptical motion will not satisfy the results of observation.

The experiment is easily made. Three points, being given in position upon a plane, will suffice for determining an ellipse in which one of the foci are known; having found therefore an ellipse which satisfies three observations, we have only to inquire whether it answers to all the others that may be made. This we find to be strictly the case.

430. *In this manner it is ascertained that the orbits of the planets are ellipses, in which the sun occupies one of the foci.*

The radius vector in each case describes about the focus areas proportional to the times.

These results, therefore, agree with those obtained in the case of the sun.

431. The dimensions of the planetary orbits, that is, their axes and eccentricities are given as we shall see, in parts of a common scale, the mean distance of the sun from the earth being unity.

These eccentricities undergo slight variations, the extent and laws of which are determined by theory. At present those of Mercury, Mars, and Jupiter are increasing; while those of the other planets are diminishing.

The *perihelia* of the orbits are not fixed; they move slowly in the planes of the orbits, after the manner of the solar perigee in the plane of the ecliptic. All but that of Venus have direct motions. That of Venus is retrograde. As yet we have not accurate observations made at intervals sufficiently distant, to enable us to determine these small changes with precision. They are obtained much more exactly by having recourse to theory.

It is obvious that if we would ascertain the quantity and direction of these motions, they must not be referred to the equinoxes which are moveable, but to some fixed and determinate point in the ecliptic.

432. It will be perceived from what precedes, that the knowledge of the elliptical motions of the planets, depends, in each case, upon seven elements. Two serve to determine the position of the orbit, which are the longitude of the node and the inclination; the other five relate to the motion. They are, 1. The duration of the sidereal revolution. 2. Half the major axis of the orbit, or mean distance of the planet from the sun. 3. The eccentricity, whence results the greatest equation of the centre. 4. The mean longitude of the planet at a given epoch. 5. The longitude of the perihelion at the same epoch. There being now 11 planets, 77 elements are to be determined before we can have a complete knowledge of our planetary system, in the present state of astronomy.

Although these elements may be determined, and actually were determined at first, by the processes above described, yet it is obvious that if applied to each element separately, they could furnish only approximations. Our knowledge of the causes which affect the accuracy of astronomical calculations is so matured as to teach us the necessity of considering all the elements together in order to estimate the reciprocal influence which they exert upon each other, in the perturbations which they experience; and the importance of determining their values not by a single observation, but by a great number. In a word we have recourse to the method of equations of condition, in order to correct our first approximations.

433. It will be seen, by inspecting the table, at the end of this section, that the most distant planets move the slowest. By comparing, in a variety of instances, their velocities with their distances, Kepler discovered this singular relation; *that the squares of the times of their revolutions are proportional to the cubes of their mean distances.* This is Kepler's third law.

The duration of the sidereal revolution of Jupiter, for example, is 4332,5848 days; and that of Mercury 87,9693. If we square these numbers, we shall have for the ratio of the squares 2425,7.

The mean distances of these two planets from the sun are expressed by 5,20278 and 0,3871, half the major axis of the solar orbit being taken for unity. If we cube these numbers, we find that the ratio is 2427,8, that is, the same as the preceding within less than $\frac{1}{1100}$.

This law being demonstrated by observation in the case of all the planets, it may be regarded as more exact than the observations themselves; accordingly, instead of borrowing from observation the ratios of the distances of the planets from the sun, an operation which it is always difficult to perform with sufficient accuracy, it is thought better to deduce them from the period of their sidereal revolutions according to this law; for these revolutions are capable of being estimated with the utmost precision by noting the returns of each planet to the same node of its orbit. Reciprocally, if we know the distance of the planet from the sun, and not the period of its revolution, the latter may be deduced from the former by the same law. This is the case with the planets recently discovered; for their major axes and all the other elements were capable of being determined by observation, before they had completed their revolutions.

434. Finally, it is a striking fact, that the earth itself participates in this general law, to which all the planets are subjected. If we take for granted its annual motion, and regard that of the sun as apparent, its course becomes that of a planetary body revolving about the sun, conformably to the laws of Kepler. The duration of its revolution, calculated according to this hypothesis, is found to be exactly equal to a sidereal year.

This coincidence presents a wonderful analogy between our earth and the other heavenly bodies; as if nature had adopted this method to make known to us the motion of our globe, because it could never be made perceptible to the senses. In conformity to this analogy, we have placed the earth among the planets in the subjoined table, and attributed to its orbit the values obtained for the elements of what we have hitherto called the sun's orbit. We do not, however, conclude even now that its motion is real, but defer the decision on this point, until we have collected all the indications furnished by the celestial phenomena.

435. The laws of Kepler, of which we have been speaking, are the basis of all theoretical astronomy; they lead directly to the law of universal attraction, which may, in some measure, be considered as a consequence of them, as we shall see hereafter.

436. The planetary motions do not take place in exact ellipses. They are subject to a variety of small inequalities, which are made known by observation and theory with great precision.

They are given in the tables of the planets, as so many corrections to be made in the elliptical motion.

The most considerable are those which affect the motions of Jupiter and Saturn. By comparing modern with ancient observations, the period of Jupiter's revolution is found to be diminished, and that of Saturn increased. Modern observations compared together give a contrary result. These variations seem to indicate in the motions of these planets great inequalities, the periods of which are very long, and which have an opposite effect upon the two planets, so that the motion of the one is accelerated while that of the other is retarded. These phenomena have been completely developed by the author of the *Mécanique Céleste*, who has made known their laws, and, by means of his calculations, given a degree of accuracy to the tables of Jupiter and Saturn, which we had not before dared to hope for. This subject will be considered farther, when we come to speak of the general cause of the celestial motions.

Table of the Elliptical Motion of the Planets.

Sidereal Revolution.

	d		y	
Mercury . .	87,9692580	or	¼	nearly
Venus . .	224,7007869		⅗	"
The Earth . .	365,2563835		1	
Mars . . .	686,9796458		2	nearly
Jupiter . .	4332,5848212		12	"
Saturn . . .	10759,2198174		29½	"
Uranus . . .	30686,8208296		84	"

Mean Distance or half the Major Axis of the Orbit.

Mercury . .	0,3870981	or	37000000 miles
Venus . . .	0,7233316		68000000
The Earth . .	1,0000000		95000000
Mars . . .	1,5236923		114000000
Jupiter . . .	5,202776		490000000
Saturn . . .	9,5387861		900000000
Uranus . .	19,182390		190000000.

Ratio of the Eccentricity to the Mean Distance at the Beginning of 1801, with the Secular Variation. The Sign — indicates a Diminution.

Mercury	0,20551494	0,000003867
Venus	0,00686094	— 0,000062711
The Earth	0,01685318	— 0,000041661
Mars	0,0933070	0,000090176
Jupiter	0,0481621	0,000159350
Saturn	0,0561505	— 0,000312402
Uranus	0,0466108	— 0,000025072.

Mean Longitude for the Minute, which separates the 31st December, 1800, from the first of January, 1801, Mean Time at Paris.

	°	'	"
Mercury	163	56	27,0
Venus	10	44	21,6
The Earth	100	09	13
Mars	64	06	59,9
Jupiter	112	12	51,3
Saturn	135	19	05,5
Uranus	177	48	01,1.

Mean Longitude of the Perihelion for the same Epoch, with the Sidereal and Secular Variation. The Sign — indicates a retrograde Motion.

	°	'	"	'	"
Mercury	74	21	46,9	9	43,5
Venus	128	43	53,1	— 4	27,8
The Earth	99	30	05,0	19	39,8
Mars	332	23	56,7	26	22,4
Jupiter	11	08	34,4	11	3,9
Saturn	89	09	29,6	32	17,1
Uranus	167	32	06,0	3	59,3.

Inclination of the Orbit to the Ecliptic at the Beginning of 1801, with the Secular Variation of the Inclination to the true Ecliptic.

	°	'	"	"
Mercury	7	00	09,1	18,2
Venus	3	23	28,5	— 4,6
The Earth	0	00	00,0	0,0
Mars	1	51	06,2	0,2
Jupiter	1	18	51,3	— 22,6
Saturn	2	29	35,7	— 15,5
Uranus	0	46	28,4	3,1.

Longitude of the Ascending Node at the Beginning of 1801, with the Sidereal and Secular Motions.

	°	′	″	′	″
Mercury	45	57	30,9	13	2
Venus	74	54	12,9	—31	10
The Earth	0	00	00,0	0	00
Mars	48	00	03,5	—38	48
Jupiter	98	26	18,9	—26	18
Saturn	111	56	37,4	—37	46
Uranus	72	59	35,5	—59	58.

Sidereal Revolution.

	Days.		Years.
Ceres	1681,3931	or	$4\frac{3}{5}$ nearly
Pallas	1686,5388	.	$4\frac{3}{5}$
Juno	1592,6608	.	$4\frac{1}{3}$
Vesta	1325,7431	.	$3\frac{2}{3}$.

Mean Distance.

Ceres	2,767245	263000000 miles
Pallas	2,772886	263000000
Juno	2,669009	252000000
Vesta	2,36787	225000000.

Ratio of the Eccentricity to the Mean Distance.

Ceres	0,078439
Pallas	0,241648
Juno	0,257848
Vesta	0,089130.

Mean Longitude at the Beginning of 1820.

	°	′	″
Ceres	123	09	41,4
Pallas	108	18	28,7
Juno	200	09	32,4
Vesta	278	21	45,1.

Longitude of the Perihelion at the same Epoch.

	°	′	″
Ceres	147	07	31,5
Pallas	121	07	04,3
Juno	53	38	46,0
Vesta	249	33	24,4.

Inclination of the Orbit to the Ecliptic.

	°	′	″
Ceres	10	37	26,3
Pallas	34	34	55,1
Juno	13	04	09,7
Vesta	7	08	09,0

Longitude of the Ascending Node at the Beginning of 1810.

	°	′	″
Ceres	78	53	24,5
Pallas	172	39	26,8
Juno	171	07	40,4
Vesta	103	13	18,2.

It is very remarkable that the distances of these four small planets from the sun are so nearly equal to each other. Hence results the slight difference in the periods of their revolution, as determined from their mean distances, according to Kepler's third law. This equality is particularly striking in the case of Ceres and Pallas. On the other hand Ceres and Juno, although they differ a little more in their distances, agree almost exactly in their eccentricities and in the position of their nodes. From these circumstances some astronomers have thought that these four small planets were originally united into one larger planet, which has since been separated by some internal explosion. But the theory of attraction shows that this hypothesis is inadmissible, because such explosion would necessarily have given to the fragments unequal velocities of projection from the same point; and hence would have resulted very unequal major axes, contrary to what we observe.

Method of Predicting the Return of a Planet to the same Situation with respect to the Sun.

437. In making observations upon the planetary motions, it is often necessary to know the time they will employ in returning to the same position with respect to the sun; this is termed their *synodic revolution.*

It may be easily deduced from the preceding results; we have only to proceed upon the general fact, that all the known planets perform their revolutions in the same direction, namely, from west to east; a coincidence which is one of the most striking phenomena of our system.

Indeed, whether the earth revolve about the sun, or the sun about the earth, if we were transported to the centre of the sun, we should believe ourselves to be immoveable; the earth and all the planets would seem to revolve about us in the same direction, and the quantity by which they deviated from one another in a given time, would depend upon the difference of their motions.

Consequently, if from the daily motion of the earth we subtract the daily motion of a planet, the difference will express the quantity by which the earth and the planet, seen from the sun, would depart from one another in the interval of a day; and if we suppose their motion to be uniform, we may determine by a simple proportion, the number of days necessary for them to diverge 360°; in other words, to return to the same relative position. This is the period of a synodic revolution of the planet.

The daily motion of Mercury, for example, is 14732″,555; that of the earth being 3548″,318; the difference is 11184,237; this is the quantity by which Mercury and the earth, as seen from the sun, depart from each other in the interval of a day; accordingly, in order to depart 360°, they will require a number of days equal to $\frac{360^\circ}{11184'',237}$, or $115^d\ 21^h\ 3'\ 25''$. This is the period of a synodic revolution of Mercury.

In taking for granted the earth's motion, our object at this time is merely to simplify the inquiry, by adopting a uniform method. This supposition can evidently have no effect upon the definitive result deduced from observation, since it can never be decided by observation whether it be the sun or the earth which moves. As we proceed, we shall have frequent occasion to remark how much our calculations are simplified by the hypothesis of the earth's motion.

438. By comparing the mean motion of the planets with the mean motion of the sun, we can determine the periods in which these bodies will return to the same points of their orbit, and the same positions with respect to the earth. These periods differ

from the synodic revolution by which the planet returns to the same angular distance from the sun. It is useful to know them, because at the epochs thus indicated, the hour when the planets rise, set, and pass the meridian, and all the inequalities which affect their motion, become nearly the same as at the preceding epoch; so that all the phenomena which depend upon the position of the two bodies with respect to the earth, recommence at this time in the same order as before, and hence they may be easily predicted.

We have already had an example of the advantage of these periods in the calculation of eclipses. To obtain similar ones with respect to the planets, we have only to find a number of sidereal years, in which the planet completes exactly or very nearly, a certain number of circumferences. The simplest method is to take the ratio of the annual motion of the planet to that of the sun, and reduce this ratio to its lowest terms.

The tropical motion of Mercury, for instance, in a Julian year, is equal to 5381066″,88; that of the sun for the same period being 129627″,45; and the ratio of these is $\frac{5381066,88}{129627,45}$. If the two terms of this fraction had a common divisor, we might avail ourselves of it in reducing the expression to a simpler form. If for instance, we could reduce it to $\frac{54}{13}$, we should conclude that the motion of Mercury is to that of the sun as 54 to 13; and that while Mercury performs 54 revolutions about the sun, the sun performs 13 about the earth; so that after 13 years, the sun and Mercury, having both performed an exact number of circumferences, would return simultaneously to the same position in which they were 13 years before. But as in general the ratio in question is not reducible, or admits only of slight reductions, we must seek for numbers which will divide them, if not exactly at least nearly; and the quotients will give so many periods, the greater or less accuracy of which will depend upon the remainders after division.

Accordingly we proceed as if we wished to find the greatest common divisor of the quantities under consideration. As this process would, in general, if carried to its limit, give unity for the last divisor, since the numbers are prime to each other, we stop at the first, second, third, &c., division; that is, we neglect the first, second, and third remainder. The smaller the remainder, according to these different suppositions, the more exact will be the periods.

439. Having in this manner determined the mean periods, we are enabled to predict all the epochs, when Mercury and the sun will appear to have returned to the same synodic and sidereal position, and consequently all the epochs of mean conjunction, which are of great importance.

If the orbit of Mercury coincided with the plane of the ecliptic, this planet would appear on the disc of the sun at each inferior conjunction. But owing to the inclination of the orbit, these phenomena occur less frequently than the inferior conjunctions; for it is obvious that Mercury in conjunction can only be projected on the disc of the sun, when its geocentric latitude is less than the sun's apparent semidiameter. In every other case, it passes above or below the disc.

Now there are several circumstances which are opposed to these conditions, and which cause the epochs of Mercury's transit over the sun's disc to vary. Among these are, 1. The great eccentricity of the orbit of this planet, which renders its motion very unequal. 2. The motion of its nodes, which prevents it from having the same latitude, when it returns to the same longitude; and lastly, the annual motion of the orbit, which, being carried along the ecliptic with the sun, is presented to our view under different elongations and aspects, which produce very considerable variations in the geocentric latitude of the planet.

Amidst all these inequalities, the only method of predicting exactly all the transits, consists in determining the epochs when they can take place according to the periods above determined, and then calculating by the tables all the conjunctions corresponding to these epochs, in order to ascertain those in which the planet will actually cross the disc. In this manner tables have been constructed containing the transits observed or to be observed for several centuries.

These tables exhibit the effect of the different periods which have been ascertained; and show also that the transits of Mercury always take place in May or November, most frequently in the latter month, owing to the position of Mercury's orbit on the plane of the ecliptic. This ellipse is now so situated that its perihelion is turned toward us in winter, and its aphelion in summer, as in figure 100; and since it is very eccentric, Mercury is much nearer the sun in November than in May. Now, if we suppose a luminous cone formed by the visual rays drawn from any point of the earth to the sun, this cone diminishes towards the earth and enlarges towards the sun whose disc forms its base. Mercury will, therefore, be more likely to meet this cone when it is near the sun than when at a greater distance, and consequently the transits over the disc must take place most frequently in November.

440. What has been said respecting the orbit of Mercury, applies equally well to that of Venus. The motion of this planet in a Julian year being 2106591″,52, its ratio to the annual motion of the sun is $\frac{2106591,52}{129627,45}$, from which we obtain the periods $\frac{13}{8}$ and $\frac{475}{291}$, which are accomplished in 8 and 291 years. We say nothing of shorter periods, because they would be very inaccurate, nor of longer periods, because they would extend beyond the most ancient observations. If we combine these two periods and their multiples by addition and subtraction, we shall obtain all those which have been recognised by astronomers.*

These periods will give the epochs of the transits of Venus over the sun's disc, which are much less frequent than those of Mercury, because Venus is much farther from the sun. The following table was calculated by Lalande.

Years.	Epochs of the Transits.
1631	6th December
1639	4th December
1761	5th June
1769	3d June
1874	8th December
1882	6th December
2004	7th June.

441. Venus is sometimes so bright that it becomes visible to

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* Such, for example, are 235, 243, and 251, which are formed from the preceding; for $235 = 291 - 8 \times 7$; also

$$243 = 291 - 6 \times 8;\ 251 = 291 - 5 \times 8.$$

In general, if we represent by m and n any two whole numbers, the formula $291 \times m \pm 8 \times n$ will represent all the periods that can be employed for Venus.

the naked eye in open day. This remarkable phenomenon takes place in the positions in which this planet reflects the most light. Now we have remarked that the phases of Venus are on the increase as this planet recedes from the earth. This increase, proportional to the sine of the elongation, tends to increase its brightness; but the increase of distance tends to diminish it, for the intensity of light decreases in proportion to the square of the distance. There is, therefore, an intermediate position in which these causes, taken together, produce the most favourable effect, and it is then that Venus sends us the greatest quantity of light. The interval between the returns of Venus to this position is about eight years, this being the period of its return to the same position with respect to the sun and the earth; but there are several other positions in which Venus may be seen at mid day, so that this phenomenon is very frequent.

442. The process described in this section will serve also to determine the epochs at which the several planets will return, at the same time, to determinate positions, such as opposition or conjunction. We have only to find the period when these planets and the sun, by their mean motions, shall have each completed an exact number of revolutions.

Physical Constitution of the Planets.

443. We have delayed the consideration of the physical peculiarities of the planets till now, that we might not interrupt the course of reasoning respecting their motions.

We have already proved that these bodies are nearly spherical. Very careful observations have been made to discover spots on their discs, in order to determine whether, like the sun, they have a motion of rotation; and such a motion has been found to take place with respect to all those upon which spots have been discovered; these are Venus, Mars, Jupiter, Saturn. The method is the same as that for the sun and moon. It is also proved that Mercury turns on its axis, but by another process, as we shall see hereafter.

The period of rotation of Venus is estimated at $23^h\ 21'$. The axis of rotation remains constantly parallel to itself, and the equator, which is perpendicular to it, makes with the ecliptic a considerable angle. On the surface of Venus very high mountains have been discovered.

The period of rotation of Mars is $24^h\ 39'$; the axis is inclined to the ecliptic 59° 42′.

The period of rotation of Jupiter is $9^h\ 56'$, about an axis nearly perpendicular to the plane of the ecliptic; its motion is therefore much more rapid than that of the preceding. It will appear still more so when it is considered that Jupiter is much larger than the other planets. The circumference of its equator is about 11 times larger than that of the earth, as will be shown hereafter. Attributing to the rotation of the earth the appearances of the diurnal motion, the period of this rotation will be to that of Jupiter as 24^h to $9^h\ 56'$; and while a point of the terrestrial equator describes one degree of its circumference, that of Jupiter will describe $\frac{1^\circ \times 24^h}{9^h\ 56'}$ or 2° 25′. Now these degrees are longer than those on the earth, in the ratio of the circumferences, that is, of 11 to 1. Accordingly in the same interval of time each point of Jupiter's equator passes through 26 times as much space as a point on the earth's equator.

444. If it be true that the flattening of the earth at the poles, is a consequence and an indication of its motion of rotation, we should expect to find Jupiter flattened at its poles in a similar manner but to a much greater degree; and this conjecture is confirmed by observation. The smallest diameter of this planet is to the greatest as 167 to 177. The first is in the direction of the poles, the second in that of the equator. The corresponding diameters of the earth have been shown to be to each other in the ratio of 309 to 310, the difference being much less than in the former case. This is a new analogy, which conspires with every thing else to indicate the reality of the earth's rotation.

445. Lastly, the period of Saturn's rotation is $10^h\ 16'$. Before observing this phenomenon, it might have been inferred from the flattening of the planet at the poles; which is so considerable that the polar diameter is $\frac{1}{11}$ shorter than the equatorial. Indeed this consideration led the author of the *Mécanique Céleste* to predict beforehand the rotation of this planet.

446. The rotation of the other planets has not yet been directly observed in this manner. Mercury is too near the sun to

admit of any observation of its spots; it is so enveloped in the solar rays that nothing is to be seen but a sparkling disc of light. But its rotation is ascertained by continued observation of the variations in the *horns* of its phases. In this manner Schroeter found its period to be $24^h\ 5'$. In the same way he determined the period of the rotation of Venus, the fact only of its rotation being indicated by its spots.

The new planets Ceres, Pallas, Juno, Vesta, are too small for such delicate observations. The shortness of the time, moreover, since their discovery, has hardly allowed a sufficient opportunity. Uranus, situated at the confines of the solar system, is too far from us to admit of our observing these motions; but the rotations of Mercury, Venus, Mars, Jupiter, Saturn, the sun and even the earth, being established with so much certainty, it is in the highest degree probable that the other planets have analogous motions.

447. A general law of these motions of rotation is that they are all directed from west to east, like the progressive motions. And this agreement, owing without doubt to the first cause which determined the planetary motions, is one of the most remarkable phenomena of our system.

Another remarkable coincidence is, that the four first planets, Mercury, Venus, the Earth, and Mars, perform their rotations in nearly the same interval of time, namely, 24 hours; while in the case of Jupiter and Saturn, the period of rotation, nearly the same in both, does not differ much from $\frac{4}{10}$ of a day. In this respect the planetary system seems to be distributed into two classes; but we do not know to what this singular circumstance is owing.

448. There is reason to believe that most of the planets are surrounded by an atmosphere analogous to our own, at least as to its aeriform state. This atmosphere is indicated in Venus, by the progressive manner in which the light diffuses itself about that planet. In Jupiter it is indicated by the irregular variations of some of its spots, which seem to resemble so many clouds driven about by the winds.

449. Very great variations are also observed in some of the spots of Mars; two especially, which form a sort of zone about its poles, increase or diminish according as they are exposed more or less obliquely to the sun. Hence it is thought that they may be masses of ice, such as exist in our polar regions.

450. All the spots of Jupiter are not subject to the variations of which we have spoken. Many of them have in general the form of zones or dark belts, parallel to each other and to the ecliptic. They are situated at a little distance from the equator of the planet. Similar belts are observed also near the equator of Saturn.

Satellites.

451. When we look at Jupiter with a telescope, we find it accompanied with three or four luminous points greatly resembling small stars. We might at first take them for real stars that Jupiter had come up with in its course through the heavens; but if we repeat the observation for several days in succession, we find that they change place about the planet. They appear at different distances from its disc, sometimes on the right and sometimes on the left; and, as they always attend it, they are called *satellites*.

We sometimes see these small bodies pass over the disc of Jupiter and project on it a shadow which describes a chord on its disc; these are true eclipses of Jupiter, analogous to those which are caused by the moon upon the earth. We hence infer that Jupiter and its satellites are not self-luminous, but opaque bodies, shining by the light of the sun.

This explains another very singular phenomenon. Jupiter being an opaque body, it must project behind it in space, a conical shadow opposite to the sun; and when the satellites enter this shadow they must become eclipsed; accordingly they often disappear when at a considerable distance from the planet, and too far to be concealed by its disc. This phenomenon takes place on the west, if the sun is on the east of Jupiter, and *vice versâ*. It happens near the disc of the planet, when the conical shadow is presented to us obliquely and under a small angle as near the opposition; and farther from the disc, when we see the cone transversely and under a greater angle, as at the quadratures. The reappearance of the satellites presents analogous phenomena, that is, it sometimes takes place at a considerable distance from the disc; which seemed a matter of surprise when they were first observed. The two satellites which depart the farthest from the planet, may, under certain circumstances, emerge from the shadow and reappear on the side of the disc where they were eclipsed. These phenomena leave no doubt that the satellites are so many small moons, which revolve about this planet in orbits returning into themselves. The satellite whose departure from the planet is least, is called the first, and the others are named second, third, and fourth, in the order of their distances.

452. Jupiter is not the only planet which presents these phenomena. Seven satellites are observed about Saturn, and six about Uranus; but it requires very good telescopes to perceive them. The moon may also be considered as a satellite to the earth; and this furnishes another instance of the analogy existing between our earth and the other bodies which compose the system.

453. The first question which arises with respect to the satellites and planets relates to the direction of their motions.

If we observe them with care, we perceive, in the first place, that the satellites are never eclipsed except when they pass from the west to the east of the planet. When they follow this direction we never see them on the disc; on the contrary, when they appear on the disc, it is on their return from their eastern elongation to the west of the planet. Hence it appears that the satellites revolve about their *primaries* from west to east; that is, in the same direction in which the primaries move about the sun; a coincidence which forms one of the most remarkable features of the planetary system.

Moreover when a satellite moves from the west to the east of its primary, it is not always eclipsed; it sometimes passes above and sometimes below the shadow; also when it returns from the east to the west of the primary, it does not always traverse the disc; but sometimes passes above and sometimes below. It is evident from these phenomena that the satellites have their orbits inclined to the orbits of the primaries; since, if they moved in the same plane they would be eclipsed each revolution.

454. The eclipses of the satellites make known the instant of their oppositions to the sun; the interval between two eclipses gives the *synodic revolution* of the satellite; hence we ascertain its angular motion with respect to the axis of the shadow which

joins the centres of the planet and the sun. Then, knowing the motion of the planet about the sun, we deduce from this result the period of the sidereal revolution. For the sake of greater accuracy, we employ the eclipses which take place when the planet is near its opposition, at which time it is nearly in the same straight line with the earth and sun. Especially we take the precaution to compare together several eclipses distant from each other, in order to compensate as far as possible, for the periodic inequalities which may exist in the motions of the satellites and primary. A table of these revolutions will be found at the end of the section.

With respect to the form of the orbits, the simplest method is, to suppose them circular; this may always be regarded as a first approximation, since the orbits of the satellites return into themselves; it is moreover indicated by the analogy which exists between the planets and satellites; lastly, it is confirmed by the phenomena; since the supposition exactly satisfies the observed eclipses. The orbits of the satellites are then very nearly circular; and in order to ascertain their distances from the centre of the primary, we have only to measure them with the micrometer, at the time of their greatest *elongations*. The values are given at the end of the section.

455. By comparing these distances with the duration of a sidereal revolution, we find them to conform to the laws discovered by Kepler in the case of the planets. In each system of satellites, the squares of the times of revolution are as the cubes of the mean distances. This law has enabled us to calculate the sidereal revolutions of the satellites of Uranus, by means of their observed elongations; for the second and fourth satellites of this planet, are as yet the only ones, whose sidereal revolutions have been directly observed. But as the durations compared with the elongations of these same satellites, satisfy the law of the squares of the times, we can have no doubt that this law extends also to the other satellites.

456. The frequent eclipses of Jupiter's satellites have furnished astronomers with the means of following their motions with an exactness much greater than would have been possible from the mere observation of their distances from Jupiter; for these distances being always extremely small, their variations are very difficult to be perceived.

The greater or less duration of the successive eclipses of the same satellite, and the series of positions in which these happen, make known the inclination of its orbit, and the position of its nodes in the plane of the orbit of the planet. These first results are only approximations; but we gradually correct them by comparing them with a variety of observations. This comparison moreover shows whether it is necessary to modify the laws of the circular motion, in order to represent these phenomena; and it is in this way that we determine whether the orbit in question has a sensible eccentricity.

It has thus been ascertained that the orbit of the third satellite of Jupiter has a small eccentricity, and that the orbit of the fourth has an eccentricity much greater; but none has been discovered in the other two.

Another very simple and exact method might be pursued in determining all the elements of the orbits of the satellites; namely, by considering them as spots moving about the primary, and applying to them the formulas investigated for the spots on the sun. The radius of each orbit being already known from the greatest elongation, we observe the difference of declination and right ascension between these small bodies and the centre of the primary, as in the case of the spots on the sun and moon. Two observations of this kind are sufficient, on the supposition that the orbit is circular, to determine its inclination and its node. The reason why three observations are not necessary, as in the case of the sun, is that here the distance of the plane of the orbit from the centre of the planet is nothing, whereas in the other case this distance was one of the unknown quantities to be determined. The arcs described, in a given time, would then show whether the motion might be regarded as uniform, or whether it is affected by a sensible ellipticity in the orbit.

457. The inclination of these orbits to that of Jupiter are variable; their nodes and their *perijoves* are in motion; by *perijoves* we understand their points of least distance from Jupiter. These satellites form, therefore, about the primary, a sort of separate system or world, affording a representation in miniature of the changes which take place, or which are to take place in the course of ages, in the motion of the planets about the sun.

458. Independently of these variations, the satellites of Jupiter are subject to very sensible inequalities which disturb their

elliptical motions, and render their theory very complicated; but the theory of attraction with the aid of a profound analysis, has disclosed the secret of all these irregularities.

The three first satellites especially, being more in the neighbourhood of each other, are connected by relations which are very remarkable.

The mean sidereal motion of the first, plus twice that of third, *is always* equal to three times that of the second. This may be easily verified by inspecting the table at the end of the section.

The same relation exists between their mean synodic motions, which are, for each equal to their sidereal motion diminished by that of Jupiter. If we substitute these synodic motions in the preceding ratios, the motion of Jupiter disappears and the condition is fulfilled.

The preceding law relative to the mean motions, affects only the variations of mean longitude, reckoned from a given instant. But the absolute mean longitudes are themselves subjected to another law not less remarkable; namely, that the mean longitude of the first satellite, minus three times that of the second, plus twice that of the third, is always equal to 180°. This relation extends equally to the mean synodic and sidereal longitudes. It is demonstrated by theory that this relation will always exist, and that consequently the three satellites can never be eclipsed at the same time, for then their longitudes would be equal, and the above sum would be nothing. These discoveries, which are among the most striking ever made in theoretical astronomy, are due to Laplace; but it would be impossible to explain in this place the method by which he was guided.

459. The vast distance of the satellites of Saturn and the difficulty of observing their positions, have prevented the determination of the ellipticity of their orbits, and still more the inequalities to which they may be subject. The same may be said of the satellites of Uranus. The ellipticity of the orbit of Saturn's sixth satellite, however, is evident from observation.

The apparent diameter of these bodies is so small, that we have not hitherto been able to determine with precision their magnitude. Attempts have been made to ascertain them, from the time employed in passing into the shadow of the planet; but from what we have already said on the subject of lunar eclipses, it will be seen how little this method is to be relied on.

460. By observing with care the periodical variations which take place in the intensity of the light of Jupiter's satellites, Sir W. Herschel remarked that they were alternately brighter and fainter. It is natural to conclude that certain parts of their surface reflect more light and others less, and then the epochs of their greatest and least brightness must take place when these same parts of the surface of the satellites are turned towards us. By comparing these periods with the positions of the satellites with respect to Jupiter, Sir W. Herschel found that they always present the same face to the primary; whence it follows that they turn about their axes in the same time in which they complete a revolution. Maraldi had already adopted this conclusion with respect to the fourth satellite, from observing the returns of the same spot on its disc. This law exists also with respect to Saturn's seventh satellite. When it is on the east of Saturn, its light is weakened to such a degree, that it is hardly perceptible; this can arise only from the spots belonging to the hemisphere which it presents to us when in this position. But in order that this hemisphere may be always the same at this point of the orbit, the motion of rotation must be exactly equal to the mean motion of revolution. We have already observed that the moon conforms to the same law with respect to the earth. From this analogy it would seem that the equality of the motions of rotation and revolution of the satellites, is a general law of nature; and in consequence of it they always present the same face to their primary. A great number of careful observations recently made by Schroeter, place this fact beyond a doubt.

461. By observing the eclipses of Jupiter's satellites, we have been led to the discovery of a very remarkable phenomenon, namely, the successive transmission of light.

If we compare the returns of these eclipses, we find that when Jupiter is in opposition, they take place sooner than they ought, according to the duration of the sidereal revolutions of the satellites. On the contrary, when Jupiter is in conjunction, that is, beyond the sun, they take place too late. These variations are exactly the same for all the satellites. They cannot be ascribed to inequalities in their motion; for, on account of the motion of Jupiter, the oppositions and conjunctions correspond successively to different points of the heavens; the same is true of the eclipses of the satellites in their orbits. The most simple explanation which suggests itself, is to suppose that the light of the sun, reflected by these small bodies, is not transmitted instantaneously to the earth, but that it occupies a sensible interval of time in traversing the earth's orbit. Indeed, if the orbit of Jupiter is concentric with the sun, as the phenomena of its motion evidently indicate, it is obvious that this planet is much nearer to us when in opposition than it is at the time of conjunction. If, for the Fig. 101. sake of simplicity, we suppose the orbit of Jupiter circular, the difference is equal to the diameter of the solar orbit; which is sufficient to account for the retardations observed.

The eclipses which take place at the time of conjunction of the planet are about 16′ 26″ later than those which take place in opposition; whence it follows that light employs 16′ 26″ in traversing the diameter of the solar orbit, or 8′ 13″, in coming from the sun to us. This hypothesis is so exactly conformable to the results of observation, that no doubt can remain of its correctness.

462. Observations of Jupiter's satellites are of great importance in determining the longitude of places on the earth. The very rapid motion of these bodies gives rise to frequent eclipses, the epochs of which are calculated beforehand, and put down in *Almanacs*. The tables of Jupiter's satellites, constructed by M. Delambre, according to the theory of Laplace, and from a great number of observations, leave nothing to be desired in respect to the motions of these bodies. By comparing these epochs, calculated for the meridian of Paris, with the results of direct observation, made in another place, at a given instant, the difference of longitude between the two places may be deduced from the difference of the times. The method is the same as for eclipses of Top. 117. the moon. Unfortunately it cannot be practised at sea, on account of the motion of the vessel, which renders it impossible to use such telescopes as are necessary to enable us to distinguish these small bodies. But observations of this kind may be very useful to the navigator when not exposed to agitation by the motion of the vessel.

463. Conformably to the plan we pursued with respect to the planets, we have collected together in the following table, the mean distances of the satellites from their primaries, and the duration of their sidereal revolutions. The mean distance is given on a scale in which the radius of the primary, seen at its mean distance from the sun, is unity. This in the case of Jupiter is 18″,37, in that of Saturn 8″,1, in that of Uranus 1″,9.

Mean Distance.		Sidereal Revolution.
		Days.
1st satellite of Jupiter	6,04853	1,769137788148
2d	9,62347	3,551181017849
3d	15,35024	7,154552783970
4th	26,99835	16,688769707084
1st* satellite of Saturn	3,351	0,94271
2d	4,300	1,37024
3d	5,284	1,88780
4th	6,819	2,73948
5th	9,524	4,51749
6th	22,081	15,94530
7th	64,359	79,32960
1st satellite of Uranus	13,120	5,8926
2d	17,022	8,7068
3d	19,845	10,9611
4th	22,752	13,4559
5th	45,507	38,0750
6th	91,008	107,6944

Saturn's Ring.

Fig. 102.

464. When we look at Saturn with a telescope, we ordinarily see it accompanied with a luminous belt, which encircles it like a girdle, and extends out each way in the shape of ears or handles. These handles gradually contract into a luminous line, and finally disappear. Saturn then presents a completely round figure. But after some time the handles reappear, and open anew, affording indications that they are detached from the body of the planet, for there is between them a void space through which we can perceive the sky or any small stars that may happen to pass in this direction.

These appearances constantly succeed each other in a regular manner; we must conclude therefore that the cause which produces them is also constant. They are ascribed to a solid body surrounding Saturn, and appearing and disappearing alternately.

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* The 1st and 2d of Saturn's satellites, being discovered long after the others had been known under the names of 1st, 2d, 3d, 4th, and 5th, they are still often designated as the 6th and 7th.

But, as it would not be natural to suppose that this body successively acquires and loses the property of shining, we must adopt the conclusion that it is an opaque body which reflects to us the light of the sun, and that the variations which it undergoes, result from its position and form. Then we are to inquire what form will satisfy the phenomena observed.

465. They may all be represented with great exactness, by supposing the equator of Saturn to be surrounded by a circular ring, opaque, very thin, and detached entirely from the planet. The body of Saturn being in motion in the plane of its orbit, carries this ring with it, and by thus presenting it under different aspects and inclinations, produces all the appearances which are observed.

Thus, it will be bright when it turns towards us that face which is illuminated by the sun, and cease to be visible when it presents the opposite face. We must also lose sight of it when it is so situated that its plane, if produced, would pass through the centre of the earth, in which case it can reflect no light to us. Again it must be invisible when its plane passes through the sun; for then its edge only is illuminated, and as it is very thin, it cannot reflect light sufficient to render it perceptible. This is in fact the case when we observe Saturn with ordinary telescopes; but if we employ telescopes of a very high power, we can discern the edge of this ring, and it appears like a luminous line on the disc of the planet. This line is so fine that it hardly subtends an angle of a second, but a second at this distance answers to a thickness of 4000 miles.

These phenomena afford strong confirmation to the hypothesis of an annular surface surrounding Saturn without adhering to it. Indeed it can be demonstrated that this ring, as well as the planet to which it belongs, are opaque bodies, illuminated by the sun, and not shining with their own light, for when the illuminated surface of the ring is inclined to the earth, as in figure 103, it projects on the globe of Saturn a sensible shadow.

But in order to determine whether this hypothesis satisfies all the particular phenomena, we must consider geometrically the different forms under which the circumference of a circle would appear, according to the different points from which it is observed. For this purpose the eye of the observer must be considered as the apex of a cone of visual rays which has the circle for its base. The line drawn from the eye of the observer to the centre of the circle, forms the axis of the cone, which is generally oblique to its base. If we draw a plane perpendicular to this axis, it will cut the cone so that its intersection will form an ellipse, the position and eccentricity of which will depend upon the greater or less obliquity, and consequently on the position of the observer. It is this ellipse which forms at the eye the apparent outline of the ring. It is consequently necessary to determine its dimensions and the direction of its axis. Top. 7.

To this end we draw through the axis of the cone a plane perpendicular to the plane of its base. This plane will divide the oblique cone into two symmetrical portions, and will consequently cut the ellipse symmetrically. It will be readily seen, that it will contain the minor axis, and that the major axis will be perpendicular to it.

It appears at first view, that the centre of the ellipse must be in the axis of the cone, and that its major axis, parallel to the plane of its circular base, must be equal to the diameter of this base; but this supposition is not rigorously true. The centre of the ellipse is depressed a little below the plane of the base, and its major axis is consequently somewhat greater than the diameter of this base. These differences arise from the fact that the half of the base nearest the observer, appears greater than the other half. But the inequality diminishes as the observer recedes, and becomes entirely insensible when the distance can be considered as infinite compared with the radius of the base; which is the case with Saturn's ring, as observed from the earth. Then the centre of the ellipse coincides with the centre of the base, its major axis is equal to the diameter of this base, and perpendicular to the axis of the cone; and lastly, the minor axis is to the major, as the sine of the inclination of the axis is to unity. Top. 7.

It will be perceived from these results, that if we measure with a micrometer the ratio of the two axes of the ellipse at any instant, we shall have, for the same instant, the inclination of the axis to the plane of the base. By making this observation at the time when the earth is 90° from the nodes of the ring, we can deduce directly the inclination of the ring to the ecliptic.

When the earth is 90° from the nodes of the ring, the plane drawn through the earth and Saturn, perpendicular to the ecliptic, is at the same time perpendicular to the plane of the ring; and consequently the straight line according to which it cuts this plane, measures the inclination of the ring to the ecliptic. Let $AA'E$ represent this line, IT the intersection of the cutting plane Fig. 104. with the ecliptic, T the earth, C the centre of Saturn, and AA' the real extent of the ring. Observation makes known the visual angles ATC, $A'TC$, which are sensibly equal to each other, on account of the great distance of Saturn; and each of them measures half the minor axis of the ellipse. Now, according to observation, this minor axis is equal to half the major axis, parallel to the ecliptic; that is, $a'a$ is equal to half of $A'A$; for the major axis of the ellipse is the diameter of the ring itself. Accordingly, the sine of the angle CAa, which is represented by $\frac{Ca}{CA}$, is equal to $\frac{1}{2}$, and consequently the angle CAa or CAT is equal to 30°. Trig. 18. If to this angle we add $CTE + CTA$, or the geocentric latitude of Saturn, plus half the minor axis of the ellipse, the sum will give the angle IEA, since this is the exterior angle of the triangle EAT. This angle IEA is the inclination of the plane of the ring to the ecliptic, and is found, by the above process, to be equal to 31° 20′.

When we have obtained an approximate value of this angle, and know the longitude of the nodes of the ring, it is easy to calculate, from the motion of Saturn, the variations, as to eccentricity, which the ellipse must undergo in the interval of a few days, before and after the epoch of greatest inclination. We thus reduce to this epoch the measures of the ellipse, made several days before and after. The mean result of all these measures gives, with greater exactness, the ratios of the two axes of the ellipse for the instant of greatest inclination.

But, in making these observations, the earth must be 90° from the nodes of the ring, in order that the projection of the axis of the cone upon the ecliptic, may be perpendicular to the intersection of the ring with the same plane. How then is this position to be ascertained as the longitude of the nodes of the ring is not yet known? We obtain it by another circumstance which is peculiar to this position of the ring, namely, that the opening of the ellipse is then the greatest possible; for this opening is proportional to the sine of the inclination of the axis of the cone to the plane of the ring. Now there is no position where this inclination is greater than when the earth is the farthest possible from the intersection of the ring, that is, when it is 90° from the nodes of the ring.

In order to determine completely the position of the ring, it only remains to ascertain the direction of its intersection with the plane of the ecliptic, or the situation of its nodes. This is easily done if we know the instant when its plane disappears by passing through the centre of the earth, for then Saturn and the earth are at the same time in this plane; and as the inclination of the ring to the ecliptic is already known, these two points are sufficient for determining its nodes. But how is this instant to be distinguished when the passing of the plane of the ring through the sun, may equally occasion its disappearance.

466. The difficulty is removed by attending to the periods in which these phenomena take place; for they cannot be the same in the two cases. The passing of the plane through the sun depends simply on the motions of the ring and Saturn, the other depends, besides, on the motion of the sun about the earth.

Now it is observed that the phenomena of the appearing and disappearing of the ring succeed each other, with great precision, and in the same order, each sidereal revolution of Saturn. It is therefore natural to suppose that they result from the passing of the plane of the ring through the sun. To place this fact in the clearest light, we have collected a series of these phenomena, as they have been observed.*

1715 . .	5th Feb.	R
1730 . .	10th Nov.	D
1744 . .	23d July	D
1760 . .	25th April	R
1774 . .	9th January	R
1789 . .	1st October	D
1803 . .	18th June	D

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* These epochs are taken from a work of Dionis du Sejour *Sur l' Anneau de Saturne*. They are to be regarded only as approximations; for the ring does not disappear instantaneously, but very gradually, and is the longer visible the more perfect the instruments are with which it is observed. It is impossible, therefore, to fix with precision the instant of the passing of the ring through the earth or the sun.

The letter D denotes disappearance, and the letter R reappearance. If to the epoch of 1715, we add a sidereal revolution of Saturn, or 29 Julian years and 166,827213 days, we shall have precisely the epoch of 1744; and in the same manner we obtain those of 1774 and 1803.

The epoch of 1730 will give, in like manner, those of 1760 and 1789.

467. This agreement is too exact to be ascribed to chance. It evidently indicates the law according to which these phenomena take place; and their returns being independent of the revolution of the sun about the earth, must of necessity be referred to the passing of the ring through the centre of the sun.

We perceive then that these phenomena form two distinct series, which proceed according to the same periods, but have different epochs. These two series are evidently to be referred to the two opposite situations, in which the ring must have passed through the sun each revolution.

468. But as in each series, the phenomenon always returns after a complete revolution of Saturn, it follows that these two situations of the ring always correspond to the same points of the orbit of the planet; that is, *the plane of the ring remains constantly parallel to itself in the orbit of Saturn*, and consequently, *its intersection with the plane of the ecliptic must always make with the intersection of the orbit a constant angle.*

These results are perfectly conformable to observation. If we calculate the heliocentric longitude of the nodes of the ring, from the passing of the plane through the centre of the earth, which is distinguished from that through the sun, since it is not comprehended in the series of article 466, we find this longitude to be constantly the same. The process is very simple (*App.*) The intersection of the plane of the ring with the ecliptic makes with that of the orbit of Saturn an angle of 59°.

469. By inspecting the table above given, we shall perceive that the intervals of the passing of the ring through the sun from one node to the other, are not of equal duration. Between the epoch of 1715 and that of 1730, the interval is 15 years and 9 months, while it is only 13 years and 8 months, between the epochs of 1730 and 1744, and so of the rest. The difference is owing to the eccentricity of Saturn's orbit. If we suppose a plane drawn through the centre of the sun parallel to that of the ring, this

plane will determine the instant of passing the sun. If the orbit is circular, it will be divided into two equal portions, and the intervals of time employed by Saturn in passing from one node to the other will be of equal duration; but if the orbit is elliptical, the plane parallel to the ring will divide it into two unequal portions, and that which contains the aphelion, where the motion is the slowest, will require the longest time to be described. The intervals then will be unequal; and this is actually the case in the present position of Saturn's orbit. This planet passed its aphelion between 1715 and 1730, and its perihelion between 1730 and 1744.

470. All the different aspects of Saturn's ring might be easily predicted, if we knew the period of its passing through the centre of the earth, as we do the intervals of its passing through the centre of the sun; but the annual motion of the sun about the earth renders the determination of these intervals rather more difficult. In order to ascertain them, it is necessary to find a number of sidereal revolutions of Saturn, which corresponds exactly or very nearly, to an exact number of sidereal years. If we take the ratio of these periods, which is $\frac{10758,219817}{365,256384}$, and proceed as in article 437, we obtain for the first approximate values $\frac{59}{2}$ and $\frac{324}{11}$. We say nothing of the remaining values in the series, because they would give periods too long to admit of our employing them.

The fraction $\frac{59}{2}$ indicates that in 59 sidereal years there are nearly two revolutions of Saturn; if we calculate with precision the duration of these two periods, we find that the first exceeds the second by 32,185796 days; accordingly, after 59 sidereal years, Saturn is so much in advance of the sun. Its velocity being known, it is easy to deduce from this the epoch at which the two bodies will return to the same relative position, or the number of days to be added to 59 years. The period given by the fraction $\frac{324}{11}$ is much more exact. It shows that 324 sidereal years are very nearly equal to 11 revolutions of Saturn. The difference is only 5,606304 days; and Saturn is so much behind the sun. From this it is easy to determine the epoch at which it must have been in the same position with respect to the sun, prior to the completion of 324 years.

These periods agree sufficiently well with the observed returns. But they can only be applied to the passing of the ring through the centre of the earth; they would necessarily be erroneous if we were to apply them to its passing through the sun. The period in the latter case, as we have already seen, is equal to a sidereal revolution of Saturn. But even with this restriction the preceding results may sometimes be faulty, on account of the great inequalities of Saturn's motion, occasioned by the attraction of Jupiter.

471. To complete this subject, it only remains to speak of the physical constitution of the ring, so far as it has been made known by observation.

If we take its dimensions with a micrometer, we find that its apparent breadth is equal to the distance of its interior border from the surface of Saturn. This distance is a third of the diameter of the planet, and its mean value is 5″,4. The actual dimensions are probably somewhat smaller, since they would naturally appear magnified, on account of irradiation. If we employ the best telescopes, we discover on the surface of the ring concentric lines, extremely fine and dark, which appear to separate it into as many parallel circumferences. There may, then, be several distinct and separate rings. But the telescope must be of the very first order to enable us to perceive the separation. Otherwise the whole appears to form one continued ring, the space occupied by each being somewhat extended by irradiation.

From certain spots, observed on the surface of the ring, it appears that it turns upon its axis in 10^h 29′; its axis of rotation is perpendicular to its plane, and coincides with the axis of Saturn.

The duration of this rotation presents a very remarkable relation. If we suppose a satellite to move about Saturn, having the mean circumference of the ring for its orbit, and calculate, according to the third law of Kepler, its sidereal revolution, it is found to be precisely equal to that of the ring.

472. This relation solves the question as to the manner in which the ring is sustained about the planet without touching it, or, at least, it refers the phenomenon to the general cause by which all the satellites are preserved in their orbits. We may consider each particle of the ring as a small satellite, and the

ring itself as a mass of satellites, connected together in an invariable manner. If these small bodies were free and independent of each other, their velocities would vary with their distances from the centre of the planet; those nearest to the centre would move the fastest, and the most distant ones slowest, according to Kepler's third law; and if we take for the mean term, the velocity which belongs to the mean circumference of the ring, the velocities of the other particles would deviate from it each way by equal quantities, the greater counterbalancing the smaller. Now if these particles were united and attached to each other so as to form a solid body, a sort of compensation would take place in their motions; the most rapid would communicate to the slowest a portion of their velocity; and the latter would exert a reciprocal influence upon the former, rendering them slower; and as these opposite efforts would counterbalance each other, there would remain only the mean motion common to all the particles, which would be that of the mean circumference; and the rings would be sustained about Saturn, in the same manner as the moon is sustained about the earth, or like the arches of a bridge, when the centre of gravity is at the centre of the voussoirs.

Mech. 212. This theory would hold even in the case where the ring might be composed, as that of Saturn appears to be, of several detached and concentric rings, only it must be applied separately to each one. The duration of their rotations would then be sensibly different. It must be left to time and observation to complete these results.

Comets.

473. In the first part of this treatise we spoke of the transient appearance of certain bodies which come gradually into view, increasing in brightness and velocity, till they arrive at their maximum, and then, after diminishing by the same degrees, pass off to be lost again in the distance. These are comets. From their unusual appearance they were, for a long time, objects of terror to mankind, and were regarded as the harbingers of some great calamity. The luminous train which accompanied them was particularly alarming, and the more so in proportion to its

length. It is but a little more than half a century since these superstitious fears were dissipated by a sound philosophy; and comets, being now better understood, excite only the curiosity of astronomers and of mankind in general. Those discoveries which give fortitude to the human mind are not among the least useful.

474. It was formerly doubted whether comets belonged to the class of heavenly bodies, or were only meteors engendered fortuitously in the air by the inflamation of certain vapours. Before the invention of the telescope, there were no means of observing the progressive increase and diminution of their light. They were seen but for a short time, and their appearance and disappearance took place suddenly. Their light and vapoury tail, through which the stars were visible, and their whiteness often intense, seemed to give them a strong resemblance to those transient fires, which we call shooting stars. Apparently, they differed from these only in duration. They might be only composed of a more compact substance capable of retarding for a longer time their dissolution. But these opinions are no longer maintained; more accurate observations have led to a different theory.

All the comets hitherto observed have a small parallax, which places them far beyond the orbit of the moon; they are not, therefore, formed in our atmosphere. Moreover their apparent motion among the stars is subject to regular laws, which enable us to predict their whole course from a small number of observations. This regularity and constancy evidently indicate durable bodies; and it is natural to conclude that comets are as permanent as the planets, but subject to a different kind of movement.

475. When we observe these bodies with a telescope they resemble a mass of vapour at the centre of which is commonly seen a nucleus more or less distinctly terminated. Some however have appeared to consist of merely a light vapour without a sensible nucleus, since the stars are visible through it. During their revolution, they experience progressive variations in their brightness, which appear to depend upon their distance from the sun, either because the sun enflames them by its heat, or simply on account of a stronger illumination. When their brightness is greatest we may conclude from this very circumstance that they are near their perihelion. Their light is at first very feeble, but

becomes gradually more vivid, until it sometimes surpasses that of the brightest planets; after which it declines by the same degrees till it becomes imperceptible. We are hence led to the conclusion that comets, coming from the remote regions of the heavens, approach, in many instances, much nearer the sun than the planets, and then recede to much greater distances.

476. The alternations of heat and cold thus produced, serve to explain the phenomena of the tails of comets. As these bodies approach the sun, their heat being greatly increased, the most volatile parts must be converted into vapour, and this vapour being extremely attenuated and retained by a very feeble force, may be supposed to be driven to an indefinite distance by the impulse of the solar rays. This hypothesis suggests also a reason why some comets have appeared without a nucleus; we may suppose their mass reduced entirely to vapour by excessive heat. But in order to appreciate these considerations, we must investigate the laws by which the motion of comets is regulated.

477. In this inquiry we must be governed by analogy. Since comets are bodies which seem to belong to our planetary system, it is natural to suppose that they move about the sun like planets, but in orbits extremely elongated. These orbits must, therefore, still be ellipses, having their foci at the centre of the sun, but having their major axes almost infinite, especially with respect to us who observe only a small portion of the orbit, namely, that in which the comet becomes visible as it approaches the sun.
Trig. 176. Accordingly the orbits of comets must take the form of a *parabola*, for we thus designate the curve into which the ellipse passes,
Fig. 105. when indefinitely elongated.

If we introduce this modification into the laws of Kepler, which relate to the elliptical motion, we obtain those of the parabolic motion of comets.

Hence it follows that the areas described by the same comet, in its parabolic orbit, are proportional to the times. The areas described by different comets in the same time, are proportional to the square roots of their perihelion distances.

Lastly, if we suppose a planet moving in a circular orbit, whose radius is equal to the perihelion distance of a comet, the areas described by these two bodies in the same time, will be to each other, as 1 to $\sqrt{\;}$. Thus are the motions of comets and planets connected.

478. By means of these laws we can determine the area described by a comet in a given time after passing the perihelion, and fix its position in the parabola. It only remains then to bring this theory to the test of observation. Now we have a rigorous method of verifying it, by causing a parabola to pass through several observed places of a comet, and then ascertaining whether all the others are contained in it.

For this purpose three observations are requisite. If we observe the right ascension and declination of a comet at three different epochs, and thence deduce its geocentric longitude and latitude, we shall have the direction of three visual rays drawn at these epochs from the earth to the comet, and in the prolongation of which it must necessarily be found. The corresponding places of the sun are also known; it remains then to construct a parabola, having its focus at the centre of the sun, and cutting the visual rays in points, the intervals of which correspond to the number of days between the observations.

Or if we suppose the earth in motion and the sun at rest, let T, T', T'', represent three successive positions of the earth, and Fig. 106. TC, $T'C'$, $T''C''$, three visual rays drawn to the comet. The question is to find a parabola $CC'C''$, having its focus in S at the centre of the sun, and cutting the three visual rays conformably to the conditions required.

These conditions are more than sufficient to determine completely the elements of the parabolic motion, that is, the perihelion distance of the comet, the position of the perihelion, the instant of passing this point, the inclination of the orbit to the ecliptic, and the position of its nodes. These five elements being known, we can assign the position of the comet for any epoch whatever, and compare it with the results of observation. But the calculation of the elements is very difficult, and can be performed only by a very delicate analysis, which cannot here be made known. The reader will find it in the *Mécanique Céleste* or in Delambre's *Astronomie*.

About 120 comets have been calculated upon the theory of the parabolic motion, and the observed places are found to answer to such a supposition. We can have no doubt, therefore, that this is conformable to the law of nature. We have thus obtained precise knowledge of the motions of these bodies, and are enabled to follow them in space. This discovery has given additional confirmation to the laws of Kepler, and led to several other important results.

479. Comets do not all move from west to east like the planets. Some have a direct and some a retrograde motion.

Their orbits are not comprehended within a narrow zone of the heavens like those of the principal planets. They vary through all degrees of inclination. There are some whose plane is nearly coincident with that of the ecliptic, and others have their planes perpendicular to it. Indeed a slight inclination of the orbit is no longer deemed an essential characteristic even of the planets; for the small planets lately discovered have great inclinations. It may be remarked also, in this connexion, that the orbits of the satellites of Uranus are nearly perpendicular to the ecliptic.

480. It is farther to be observed that the tails of comets begin to appear, as the bodies approach near the sun; their length increases with this proximity, and they do not acquire their greatest extent, till after passing the perihelion. The direction is always opposite to the sun, forming a curve slightly concave, as in figure 107.

The comet of 1680 was particularly remarkable for the length of its train; it occupied in the heavens a space of nearly 60°. That of the comet of 1744 was still more striking; it resembled a kind of fan 15° in length, and 117° in breadth. It was so brilliant as not to be effaced by the light of the full moon, and was even visible after sunrise.

481. These results tend to establish the idea suggested by the first consideration of the phenomena, namely, that the tails are really the effect of the vaporization produced by excessive heat. The length does not reach its maximum till after passing the perihelion, and then the comet is most heated and penetrated by the rays of the sun; just as the surface of the earth does not attain its highest temperature till after the summer solstice. This analogy seems strongly to indicate that comets, at least before passing the perihelion, are solid bodies, progressively heated, and then totally or partially vaporized. Otherwise, how could they endure so powerful a heat; the comet of 1680, for instance, was, at its perihelion, 166 times nearer the sun than the earth is. Accordingly, it was exposed to a heat 27556 times greater, than that at the surface of the earth, if, as is

natural to suppose, the intensity of the sun's heat is, like its light, reciprocally proportional to the square of the distance. This intense heat exceeds several thousand times that of red hot iron, and indeed all the degrees of heat that we are able to produce. A simple mass of vapour, exposed to a thousandth part of such a heat, would be at once dissipated in space.

482. After having calculated the motions of comets, let us endeavour to predict their returns. If they describe very elongated ellipses, their parabolic motion can be regarded only as a first approximation. But the analogy which leads to the first result, furnishes also the means of correcting it, of calculating the true ellipse, and consequently of determining the length of its major axis in parts of the radius of the solar orbit. Then, by Kepler's third law, we obtain the sidereal revolution of the comet, and the epochs of its return.

Unfortunately these results are rarely susceptible of any thing like accuracy, because they are derived from observations that extend through only a very small arc of the comet's orbit, in which it is seen as it approaches to or recedes from the sun. Besides it may be that certain comets describe parabolas or hyperbolas, whose branches are indefinite, and then, when once observed as they pass through the part of their orbit nearest the sun, they disappear forever. But it is probable, that, if such bodies exist, they must, from the antiquity of the universe, long since have passed the sun; so that the periodic motions are the only ones which remain for us to observe.

When we have determined the elements of a comet's orbit, we compare them with those of comets before observed, and see whether there is an agreement with respect to any of them. If there is a perfect identity as to the elements, we should have no hesitation in concluding that they belonged to different appearances of the same comet. But this condition is not rigorously necessary; for the elements of the orbit may, like those of other heavenly bodies, have undergone changes from the perturbations of the planets or their mutual attractions. Consequently, we have only to see whether the actual elements are nearly the same with those of any comet before observed, and then, by the doctrine of chances, we can judge what reliance is to be placed upon this resemblance.

483. Dr Halley remarked that the comets observed in 1531, 1607, 1682, had nearly the same elements, and he hence concluded that they belonged to the same comet which, in 151 years, made two revolutions, its period being about 76 years. It actually appeared in 1759 agreeably to the prediction of this great astronomer. Its next return will take place in 1832. According to Kepler's third law, if we take for unity half the major axis of the earth's orbit, the mean distance of this comet must be equal to the cube root of the square of 76, that is, to 17,95. The major axis of its orbit must therefore be 35,9; and as its observed perihelion distance is found to be 0,58, it follows that its aphelion distance is equal to 35,32. It departs, therefore, from the sun to thirty-five times the distance of the earth, and afterwards approaches nearly twice as near the sun as the earth is, thus describing an ellipse extremely elongated.

The intervals of its return to its perihelion are not constantly the same. That between 1531 and 1607 was three months longer than that between 1607 and 1682, and this last was 18 months shorter than the one between 1682 and 1759. It appears, therefore, that the motions of comets are subject to perturbations, like those of the planets, and to a much more sensible degree.

484. A comet appeared in 1818 whose orbit presented elements differing so little from those of a comet observed in 1805, that their identity was thence inferred. This would give a short revolution of thirteen years, provided the comet had not returned to its perihelion during this interval. But M. Encke, by an examination of numerous observations made in 1818 and 1819, came to the conclusion that its period was still shorter, and only about 1203 days, or 3⅓ years nearly. He was accordingly led to believe that it would appear again in 1822; and to facilitate the means of finding it, he calculated the place in which it was to be seen each day. The southern declinations of the comet during its appearance would be such as to render it next to impossible to take observations in Europe. Fortunately Botany Bay afforded a favourable situation for this purpose, and Mr Rumker, a skilful astronomer, observed it from the 2d to the 23d of June, and the observed positions agree so well with the calculated places, that there no longer remains any doubt of the return of this comet predicted by M. Encke.

485. The comet of 1770 offers a striking example of the changes to which comets are exposed. The observations of this comet, first calculated by M. Lexell and afterwards by M. Burckhardt, can only be represented by an ellipse, in which the duration of the revolution is 5½ years. Still we have not seen it since. There can then, be no doubt that the orbit has been changed subsequently to 1770. We shall see hereafter that these effects are owing to universal attraction, which, while it preserves the motions of the larger planetary bodies, causes the lighter masses to undergo very great alterations.

Meteoric Stones.

486. Solid masses, composed of various mineral substances, are often seen to descend from the higher regions of the atmosphere. The fall of such bodies was for a long time doubted, and the prevailing opinion, as to their reality, regarded as a popular superstition. But all the particulars of the phenomenon have been established in such a manner, that the fact can no longer be questioned.

487. The most remarkable circumstances respecting these substances is, that they so exactly resemble each other; being all of them pyriteous masses glittering with metallic grains. The exterior surface is black as if it had been burned, while the interior is of a yellowish white. They all have nearly the same specific gravity, which may be fixed at 3,591, water being unity. On being analyzed, they always present the same ingredients, and in nearly the same proportions. They are composed of *silex*, *magnesia*, *sulphur*, *iron in a metallic state*, *nickel*, and some particles of *chrome*.

These common characteristics strongly indicate a common origin. It is, moreover, to be remarked that iron is hardly ever met with in a metallic state, in terrestrial bodies. Volcanic substances do not contain any which is not oxydated. Nickel is also very rare, and never found at the surface of the earth. Chrome is still more rare. These considerations make it probable that the substances in question have an origin foreign to our globe, or at least that they are not brought together by any phenomena hitherto observed.

488. These solid masses seem to proceed from a meteor-like appearance in the air. They are in fact ignited or inflamed bodies which suddenly present themselves in the atmosphere, and move with a velocity equal, in some instances, to that of the sun in its orbit, or twenty miles in a second. Their direction is generally inclined to the horizon. After emitting an intense light for a few moments, they explode with a loud noise, at a height from 18 to 100 miles above the surface of the earth. This is ascertained by their parallaxes, as deduced from several simultaneous observations, made at different places where they were visible at the moment of explosion. They do not appear to follow any determinate direction with respect to the earth's motions, but are seen moving towards all points of the compass.

489. As yet we are ignorant of the true origin of these masses, and various hypotheses have been invented to explain them. Laplace thinks they may be thrown upon the earth by lunar volcanoes. It is found by calculation that this would only require a projectile force about four times that of a cannon ball. This force meeting with little resistance from the very rare atmosphere of the moon, would be sufficient to send a body without the sphere of the moon's attraction, and then it would be brought to the earth's surface by gravity. Now it is by no means improbable that lunar volcanoes may give such an impulse to a body, since those on the earth are capable of a much greater force, although the greater intensity of gravity at the earth's surface, and especially the enormous resistance of our atmosphere, are such as to prevent a corresponding effect. By thus ascribing to meteoric stones a common origin, we account in some degree for the sameness of their composition.

Some philosophers maintain the opinion that the substances under consideration are small planets or fragments of planets, carried along in space like the other heavenly bodies, which, upon entering the earth's atmosphere, became enflamed by friction, gradually lose their velocity, and then fall to the earth by the influence of gravity. According to this hypothesis, the meteors called shooting stars, are revolving bodies or terrestrial comets, which enter our atmosphere at great heights, but with a velocity sufficient to traverse it, so that they are only inflamed on their passage. Time will probably throw some light upon this subject, especially since there is now no prejudice among scientific men to prevent the multiplication of observations. It seemed proper to introduce it here; for, according to some distinguished philosophers, these bodies would seem to have an origin belonging to astronomy.

Proofs of the Real Motion of the Earth.

490. We have at length nearly completed our course of observations; we have studied the properties of the atmosphere, calculated its height, and allowed for the effects it produces upon the phenomena of vision. Having thus disengaged ourselves from the illusions of the senses, we passed, in imagination, the bounds of our habitation into the immensity of the celestial regions. We found the fixed stars removed to an immeasureable distance, and our planetary world reduced to a point in the great scale of the universe. Shut up and confined as it were in a small corner of this vast theatre, we have succeeded in inventing instruments by which we have been able to follow and measure with exactness the motions of the celestial luminaries. We have traced out their paths, detected their deviations and inequalities, and predicted their returns. By a series of observations and reasonings, carefully conducted, we have attained to a precise knowledge of the forms of these bodies, and their physical constitution so analogous to that of our earth. Thus furnished with a multitude of results, it only remains for us to compare them together in order to develope their relations; we have studied the solar system in detached parts, we are now to contemplate it as a whole; guided by the analogies made known by observation, we pass from appearances to the real and absolute state of things.

491. The first question to be decided, relates to the motion of the earth, whether the sun and the celestial sphere actually turn about the earth, or whether the earth revolve on its own axis and about the sun.

That our ideas may be definite, let figure 119 represent the solar system, comprehending the sun, planets, and satellites, distributed according to the results of observation.

Since there can be no doubt that our earth makes a part of this system, let us designate it by the sign ⊕, placed at pleasure. And since it is equally evident that the earth is at a certain distance from the sun, let us represent this luminary by the character ☉.

As to Mercury, we have seen that its orbit embraces the sun, but not the earth; we must therefore represent it by a small ellipse passing between the earth and the sun, and having the sun in one of its foci. Let it be denoted by ☿. And since Venus presents similar appearances, let us place it in a similar manner, referring it however to a larger ellipse, and designating it by ♀.

The orbit of Mars embraces the earth and sun, and must consequently be represented by an ellipse comprehending the earth, and having its focus at the centre of the sun. In speaking of it we make use of the sign ♂.

We place, in a similar manner, the other superior planets, Ceres, Pallas, Juno, Vesta, Jupiter, Saturn, and Uranus, each at its proper distance from the sun. As to the satellites, a small circle described about the earth, will represent the orbit of the moon, and similar circles about Jupiter, Saturn, and Uranus, will serve to indicate those belonging to the other planets respectively. Lastly, parabolas very elongated and having their foci at the centre of the sun, will give a representation of the orbits of comets.

Beyond these bodies are the fixed stars, separated by an immense interval, and arranged in an order to us wholly unknown.

Such is the state of the universe as at present understood. It is proved incontestibly from observation, that the moon revolves about the earth, and that all the planets with their satellites, and the comets, revolve about the sun.

Three phenomena now remain to be disposed of between the sun, the earth, and the celestial sphere. 1. The state of rest which seems to belong to the earth. 2. The annual motion, which, according to the evidence of the senses, we refer to the sun. 3. The diurnal motion, which appears to be common to the celestial sphere and all those bodies which have a proper motion. Let us now bring together the considerations which may enable us to decide these three points.

492. In the first place, since the sun and all the planets which admit of nice observations, are found to have a motion of rotation about an axis passing through their centre, analogy leads us to suppose that the earth also turns upon its axis.

This argument is strengthened by the circumstance of the oblateness of the earth, so exactly similar to what we observe in Mars, Jupiter, and Saturn, and which appears to be a necessary effect of the centrifugal force.

The rotation of the earth is also indicated by the progressive augmentation of gravity from the equator to the pole, another effect of the centrifugal force.

Again, the motion in question is manifest from the deviation which bodies undergo in falling from a great height. For, they are found, as already remarked, to be carried a little towards the east, a necessary consequence, if the earth turn on its axis, since, being elevated above its surface, their distance from the axis of rotation, and hence their velocity, is increased. If we suppose the earth immoveable, these phenomena admit of no explanation.

To these very strong indications, may be added other consideration. If we suppose the earth to move, the phenomena of the diurnal motion, and those of the precession of the equinoxes and nutation, become extremely simple; on the contrary supposition they become in the highest degree complicated.

The earth is a globe of less than 4000 miles radius; the sun is incomparably larger. If the centre of this luminary coincided with that of the earth, its radius would reach to twice the distance of the moon, and its volume would fill a space of this vast extent.

Is it not then infinitely more simple to suppose, in our globe, a motion of rotation already indicated by so many striking analogies, than to consider the immense mass of the sun as describing each day, about us, a circuit of nearly 600 millions of miles? What force can we imagine sufficient to retain it in its orbit, and counterbalance its centrifugal force? But this is the least of the difficulties. We must suppose similar motions in the planets, comets, and satellites, each in exact proportion to its distance, and all perfectly agreeing with each other, as if previously concerted. Nay more, we must extend these motions to the immense multitude of fixed stars with which the heavens are crowded; and all these bodies, whose distances are too great even for the imagination to conceive, must revolve together each day about an imperceptible point, with the most perfect regularity, and with a velocity truly appalling.

If, in studying the phenomena of nature, the simplest laws are always to be preferred; if we are to follow analogy and to be guided by it in tracing a series of phenomena to their true causes; if, in fine, we are to be convinced of the truth of a proposition, when all the arguments point to it, centre in it, and conspire to give it an extreme simplicity; then we must regard the rotation of the earth, as one of the best established physical truths. We shall in future consider this doctrine as established, and shall refer to it all the appearances presented by the diurnal motion of the heavens.

493. The annual motion of the earth is indicated by proofs no less numerous and powerful.

In the first place, since all the planets with their satellites and the comets, revolve about the sun, analogy leads to the conclusion that this is the case with the earth also.

This reasoning derives additional weight from the circumstance of the motions of the satellites, by which it appears that the smaller bodies of the solar system revolve about the larger.

The conclusion is rendered still more probable by a reference to the third law of Kepler, that the squares of the times of the revolutions of the planets about the sun, are proportional to the cubes of their mean distances. The earth, on the supposition that it is in motion about the sun, is included in this law.

It is in vain to urge the difficulty of moving a mass like the earth, and impressing upon it a velocity of 20 miles in a second, without our being able to perceive it; since Jupiter, which is 1000 times larger than the earth, revolves also about the sun; and by considering the earth at rest we must necessarily suppose Jupiter to revolve about it, which would be a still greater difficulty.

But the forces of nature are not to be measured by our feeble standard; the greater or less velocity in the heavenly bodies, would not be a decisive argument, if these results were not accompanied by another more powerful consideration, that of extreme simplicity.

Indeed, if the earth revolves about the sun, the laws of its motion are not distinguished by any peculiarity; they are the same as in the case of the other planets; but if we suppose the earth at rest, we must admit that the sun in its annual revolution, carries with it through the ecliptic, the orbits of all the planets; accordingly all the analogies are destroyed, and the motions of these bodies become extremely complicated.

Lastly, the aberration of light affords a sensible proof of the annual motion of the earth, as the deviation of falling bodies does of its rotation. This we now proceed to explain.

Aberration of Light.

494. VISION appears to be produced by the impulse of luminous particles upon the nervous membrane, situated in the back part of the eye, called the *retina*. The direction in which these particles strike the globe of the eye, determines the straight line to which we refer the situation of the object from which they emanate; and if the *series of particles* which compose the *luminous ray*, has been bent in its course by any cause whatever, we suppose the objects placed in the prolongation of their last direction, as is the case in atmospherical refraction.

Let us now suppose an observer, at rest, to receive luminous rays, which proceed from the object to the eye in straight lines; this observer will see the object in the prolongation of these rays and in their true place. But if the observer himself be in motion, and his velocity sufficiently great to be compared with that of light, the eye, by the effect of this motion, would itself impinge upon the luminous particles; it would, therefore, receive in its turn, and in the opposite direction, an impulse or impression, compounded of the velocity of light and its own velocity. Accordingly, the luminous particles would appear to this observer to arrive at his eye in a direction different from their true course.

Thus, if a person on board a steady boat, under sail or propelled by steam, should endeavour to hold a tube so as to catch drops of rain or snow, that may be supposed to fall perpendicularly, he would be obliged to incline the tube in the direction of his motion in order that the drops might describe lines parallel to the axis, that is, the tube must form the diagonal of a parallelogram, the sides of which would be the velocity of the boat, and that of the falling drops.

If the earth have really a progressive motion round the sun, it must produce a similar effect upon the light coming from the heavenly bodies, and the impression of this light upon the eye cannot take place in the actual direction of the rays. This is what actually happens, and it constitutes what is called *the aberration of light.* Let us endeavour to determine the circumstances of the phenomenon.

The problem, considered under its most general aspect, resolves itself into this; the body from which the light comes and the observer being both in motion, according to any given laws, it is required to determine, at each instant, the angle comprehended between a visual ray drawn to the apparent place of the body, and one drawn to its true place.

To begin with the most simple case, let us first suppose the body from which the light comes, at rest, and the earth only in Fig. 188. motion. Then, at any given instant, let S represent the body, and T the earth, both being considered as points. The visual ray ST represents the direction in which the light actually comes from the body to the earth. But the observer will not see the body in this direction; for being himself in motion, according to the line TT^v, he impinges against the luminous particle with all his velocity; and, as he believes himself at rest, he attributes this effect to a proper motion in the light in a contrary direction. Hence a sensation is produced compounded of the real velocity of the particle, in the direction ST, and the supposed velocity in the direction Tt, opposite to the motion of the earth. This composition of motions produces in the eye of the observer an impression exactly similar to that which would have taken place if he had been actually at rest, and the luminous particle had impinged upon the eye in the direction of the resultant of the two velocities. Accordingly, to obtain the apparent direction of the visual ray TS', we proceed upon the principle of the composition of forces, and take, in the prolongation of the true ray ST, beginning at the point T, a line Ts which represents the proper velocity of light; then in Tt, which represents the earth's motion, and in a direction opposite to this motion, we take a line Tt, representing the earth's velocity; and lastly, with the lines Ts and Tt, we construct the parallelogram $RtTs$. The diagonal RTS' of this parallelogram, being indefinitely produced, will be the apparent direction of the visual ray, at the instant the observer receives it; and the angle STS', formed by this diagonal and the true visual ray, will be the aberration which light undergoes in consequence of the earth's motion.

We pass now to the general case in which the observer and the body are both in motion. At the instant the observer receives Fig. 109. the impression of the luminous ray, let T represent the earth, and S the true place of the body. Then TS will be the true visual ray. But the luminous particle which reaches the earth at this instant, will not have emanated from the body in this direction; for, on account of the successive transmission of light, when the body is in S, the light which it emits at this point, will not reach the earth till after a certain interval. To determine the point S', whence the luminous ray emanated which arrives at the earth in T, at the instant when the body is in S, it is necessary to go back a little in the orbit, and take the arc SS', equal to the path described by the body during the time light employs in passing from S to T, in virtue merely of its motion of emission. Indeed, when the body is in S', the light which it emits in the direction $S'V'$, parallel to ST, is acted upon by two velocities; one is the motion of the body in the direction $S'S$, and the other the motion of transmission, in the direction $S'V'$, equal and parallel to ST. These two velocities may be represented by the straight lines $S'S$ and $S'V'$, since these lines express the effects produced in the same interval of time. Consequently, if we construct, on these lines, the parallelogram $S'STV'$, the true absolute motion of the particle will take place in the direction $S'T$, and the particle will reach the earth in T, at the instant the body arrives at S in its orbit.

This construction supposes the arc $S'S$ to be so small that it may be considered as rectilinear; and in the cases which occur in the solar system, the motion of bodies is so slow compared with that of light, that the supposition involves no perceptible error.

If we would obtain this composition of velocities about the point T, as in the case first considered, it would be necessary to draw through this point a line Ts' parallel to SS', the path of the body, or to its tangent at S', and in the direction of its proper motion. In this line we take Ts' to represent the velocity of the body in S'. In the same manner we prolong the direction ST of the true visual ray, and take the line Ts to represent the velocity of transmission. We then construct the parallelogram $rsTs'$, and its diagonal rT, will be the true direction of the luminous ray, which reaches the earth in T, at the instant the body is in S.

Now the luminous particle, upon arriving at T, in the direction $S'T$, is met by the observer, with all the velocity of the earth's motion, and the compound sensation thus produced, is such that the observer does not see the body in S', in the true direction of the luminous ray, but in advance of this direction, for instance, in S''. This case is evidently comprehended in that first considered. To find the direction of the apparent ray TS'', a composition is necessary between the resultant Tr, which indicates the true direction of the luminous ray, and the velocity tT of the earth, taken in a contrary direction to its actual motion; in other words, we must obtain the resultant at the point T of the three velocities Ts, Ts', Tt, of light, the body, and the earth; the two first being taken in their actual directions, and the last in a direction opposite to its actual motion. This resultant will express the direction TR of the apparent ray; and the angle STS'', formed between this ray and the line TS, drawn at the same instant, from the eye of the observer to the true place of the body, will express the aberration sought.

Let us now endeavour to unite all the elements of this calculation, and then compare the results with the aberration.

It is proved by the eclipses of Jupiter's satellites that light employs 8′ 13″ in passing through the mean radius of the earth's orbit. For the sake of simplicity, we shall first consider the orbit as circular, and neglect the inequalities of the annual motion. This motion, upon the hypothesis under consideration, is to be attributed to the earth; which describes in its orbit, in the space of 8′ 13″, a small arc equal to $\frac{360^\circ \times 81' 3''}{365^d\ 6^h\ 9'\ 11'',23}$, or 20″,25. The velocity of the earth, therefore, is to the velocity of light as
Top.148. 20″,25 is to 206264″,80 the value of the radius expressed in seconds.

Accordingly, by supposing the primitive direction of light perpendicular to the earth's radius, as it would be, if, for example, the body from which it comes, be situated in the pole of the ecliptic, and at an infinite distance, the apparent direction of the ray will be the hypothenuse of a right-angled triangle, of which one side, perpendicular to the ecliptic, would be 206264″,80, and the other situated in the plane of the ecliptic 20″,25; so that the angle of aberration will have for its trigonometrical tangent $\frac{20'',25}{206264,''80}$; and as the denominator is the radius reduced to seconds, it is evident that the value of the angle itself will be 20″,25. This is the greatest possible deviation. A similar calculation will give the quantity of aberration in any other case. As the earth passes through its orbit about the sun, and continually changes its direction, it is evident that the luminous ray moving constantly before it, must also change its direction with it. Hence it follows that we never see the heavenly bodies in their true places; that, in consequence of this aberration, the apparent place must oscillate about the true one, and that the period of these oscillations must be exactly a year.

The earth's motion of rotation must produce analogous effects; but this motion being, even at the equator, only $\frac{1}{60}$ of the annual motion about the sun, the effect must be less in the same proportion, and may, therefore, be neglected in the first approximation.

495. These very remarkable phenomena are necessary consequences of the earth's annual motion; let us see whether they are confirmed by observation.

We find in fact that they are so, very exactly. All the stars appear to describe annually in the heavens small ellipses, the dimensions of which are precisely those which result from the preceding theory.

To verify this fact, we observe, each month, the declination and right ascension of a star, and make allowance for the changes which result from precession and nutation. If the star has no other motion, real or apparent, it must be found, after these corrections, always in the same place; but this is never the case.

Take, for example, the variations in declination and right ascension of the three very bright stars, Regulus, Capella, and Lyra, observed at Paris, each month during a year. The sign + indicates increase; and — diminution.

Longitude of the Sun		Change in Right Ascension			Change in Declination.		
		Regulus.	Capella.	Lyra.	Regulus.	Capella.	Lyra.
	°	"	"	"	"	"	"
*Vernal equi.	0	0,00	0.00	0,00	0,00	0,00	0,00
	30	— 16,94	— 12.99	+ 13,09	+ 2,69	— 2,79	+ 1,59
	60	— 17,17	— 20.70	+ 23,49	+ 6,19	— 6,71	+ 7,39
Summer sol.	90	— 26,63	— 20,99	+ 28,19	+ 9,59	— 10,79	+ 15,83
	120	— 33,37	— 14,00	+ 26,40	+ 11,83	— 13,80	+ 24,88
	150	— 35,51	— 1,30	+ 18,21	+ 12,60	— 15,10	+ 31,88
Aut. equi.	180	— 32,40	+ 13,51	+ 5,94	+ 11,40	— 14,19	+ 31,99
	210	— 24,88	+ 26,28	— 7,10	+ 8,68	— 11,40	+ 33,37
	240	— 15,20	+ 34,08	— 11,60	+ 5,18	— 7,48	+ 27,60
Winter sol.	270	— 5,70	+ 34,41	— 22,19	+ 1,81	— 3,40	+ 19.12
	300	+ 1,00	+ 27.28	— 20.38	— 0,47	— 0,39	+ 10.08
	330	+ 3,11	+ 14.71	— 12.18	— 1.20	+ 0,91	+ 8,01
	360	0,00	0.00	0.00	0.00	0.00	0.00

Each observation, represented by the numbers of this table, is a mean between those of several consecutive days. We have chosen those stars which, like Capella and Lyra, pass the meridian near the zenith of Paris, in order that the irregularity of refraction might not be resorted to as a possible cause of the variations in declination. As to the variations in right ascension, they are determined by the transits of the stars over the meridian; and that the changes might not be attributable to errors of observation, we have selected those stars whose variations are in opposite directions, so that it is absolutely impossible to entertain any doubt as to the result.

To render the progress of the variations more intelligible, let us construct the curve which they represent. The variations being themselves very small, the curve must also be very small; and we may therefore project it on the concavity of the heavens, as on a plane surface. Then each position of the star will be referred to two rectangular coordinates, having for their origin the apparent position of the star at the instant of the equinox, which we have taken as the point of departure. One will be the change of declination from this instant; the other will be

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* This equinox is taken as the point of departure, and the increase in right ascension and declination of the star is reckoned from the values, which they then had.

the difference of right ascension reduced to the origin of the coordinates, that is, multiplied by the cosine of the declination. The method is the same as that we pursued in calculating the small ellipse of nutation.

This construction gives figures 110, 111, 112. These are ellipses more or less flattened, and having different inclinations. That of Lyra approaches very nearly to a circle; that of Capella is sensibly flattened, and that of Regulus is almost a straight line. But the length of their major axes is always the same, and equal to 40″,5.

By comparing these results with the latitudes of the stars, we find that the flattening of the ellipse increases according as the star is nearer to the plane of the ecliptic.

			°	′	″
The latitude of	Lyra	in 1800 was	61	44	44
"	Capella	"	22	51	44
"	Regulus	"	0	27	30

The first therefore is very distant from the plane of the ecliptic, and the last nearly in this plane.

496. These results agree perfectly with the theory deduced from the motion of the earth. According to this hypothesis, let $T'T''T'''$, represent the ellipse annually described by the earth Fig. 113. about the sun, designated by S. Let E represent the true place of a star situated at any altitude whatever above the plane of the ecliptic, which is here that of the figure. The luminous rays ET', ET'', ET''', coming from this star to the different points T', T'', T''', of the earth's orbit, may be regarded as parallel to each other; for this orbit, seen from one of the stars, would subtend so small an angle that it may be considered as a point. This being premised, let us determine the position of the apparent ray when the earth is at T'. For this purpose we take, in the true direction of the ray, a straight line $T'e$, which represents the proper velocity of light; then from the point e, we draw, parallel to $T't'$, which represents the direction of the earth's motion, a line ee' to represent its velocity; and the diagonal Te' will be the apparent direction of the luminous ray. If we repeat the same construction when the earth is at T'' and T''', the length and direction of the lines $T''e$, $T'''e$, which represent the velocity of light, will be the same as before; but the direction and length

of the lines $e\,e'$, $e\,e''$, which represent the velocities of the earth, will change, because the direction and velocity of the earth's motion are different in different points of its orbit. These lines, however, will always remain parallel to the ecliptic, since the direction of the earth's motion is always in this plane. The apparent rays will thus form about the true ray, $T'E$, $T''E$, $T'''E$, a conical surface, having this ray for its axis, and having its base parallel to the ecliptic, and terminated by a plane, oval curve, the radius vectors of which will be the velocities of the earth in its orbit. If we confine ourselves to the first power of the eccentricity of the earth's orbit, which is always sufficient for calculating the aberration, the curve of the velocities, constructed in this manner, will be an ellipse similar to that described by the earth, but situated at right angles to it. It has for its focus the mean place of the star; and the point nearest the focus, or what may be called its *periaster*, being seen from the mean place of the star, has its longitude equal to that of the aphelion of the earth's orbit, plus a right angle. This same ellipse evidently serves as a base to all the cones of the apparent visual rays, whatever be the position of the star to which they belong; but the section of these cones perpendicular to their axes, and representing the apparent orbit of the star, seen from the earth, varies with their obliquity to the ecliptic. If we confine ourselves to the first power of the eccentricity of the earth's orbit, this intersection is still an ellipse, having its major axis parallel to the plane of the ecliptic, and constant for all the stars. The visual angle which it subtends on the celestial sphere, is equal to 40″,5, that is, double the greatest value of the aberration. The minor axis is to the major, as the sine of the latitude to radius. The star appears in the major axis when the axis of the cone becomes perpendicular to the direction of the earth's velocity. It appears in the minor axis, when the axis of the cone makes the smallest possible angle with this velocity, which takes place when the direction of the earth's motion becomes parallel to the circle of latitude drawn through the star. If we neglect the eccentricity of the earth's orbit, the direction of the earth's velocity is always perpendicular to the radius vector, drawn from the earth to the sun. Then the star appears in the major axis of its ellipse when it is in opposition or conjunction with respect to the sun; and it appears in the minor axis at the quadratures, when its longitude differs from the sun's by 90°. In general, its apparent position for any given instant may be easily assigned by the foregoing theory, and the result is always found to agree with observation.

We have several times observed, that when a simple law, and one which is capable of a mathematical enunciation, satisfies a great number of independent observations, with the exception of some slight irregularities and variations, this law may be regarded as more exact than the observations themselves, and may be employed to correct them. This is the case with the phenomenon of aberration. Since it agrees so well with all the observations of the stars, we cannot doubt its reality. If then we consider its existence as demonstrated, its exact measure is to be determined not from a single observation, but by a variety, in order that the errors, if there be any, may be compensated and destroyed in the mean result.

For this purpose we must begin with admitting the fact and the cause of the aberration as now explained; and then, with these data, proceed to construct formulas which shall express the apparent motions of any star, whose position is known. The whole process is founded on the single principle of the composition of forces.

But this only fixes the law of these variations; it still remains to determine their absolute numerical value. For, although we might deduce it from the velocity of the earth compared with that of light, as ascertained by the eclipses of Jupiter's satellites, yet, since this supposes the motions of the satellites perfectly known, some doubt might remain with respect to the accuracy of the results thus obtained. We proceed then to determine it simply from observations of the stars.

For this purpose we reverse the problem. We suppose the changes of the apparent places of several of the stars, in declination and right ascension, to have been accurately observed. We make these results equal to the algebraic expressions which represent them according to the theory; then the absolute quantity of the aberration is the only unknown quantity; and each equation of this kind will enable us to determine it. But to avoid the small errors to which observations are liable, we form a great number of equations of this kind, several thousand, for example, and take the mean of the whole. It is also evident, that to obtain the utmost accuracy, the changes observed must be the greatest of those to which the star is subject.

Such was the course pursued by the celebrated astronomer, Dr Bradley, to whom we are indebted for the discovery of the phenomena of nutation and aberration. In the memoir in which he gave an account of this last, he relates that he observed for a long time the zenith distances of a great number of stars, with a mural quadrant of 12 feet; and that he afterward corrected these distances for the precession of the equinoxes. Nutation was not then known, but the variations on this account are very small in the interval of a year. The corrections being made, he found that the greatest annual variations in declination were, as follows;

For	γ	of	Draco	39″
	η	"	Ursa Major	36
	α	"	Cassiopeia	34
	α	"	Perseus	23
	τ	"	Perseus	25
	β	"	Draco	39
	35	"	Camelopardalis	19
			Capella	16

By making these results equal to their algebraic expressions, he obtained for the major axis of the ellipse of aberration, the following values;

				″
For	γ	of	Draco	40,4
	η	"	Ursa Major	40,4
	α	"	Cassiopeia	40,8
	α	"	Perseus	40,2
	τ	"	Perseus	41,0
	β	"	Draco	40,2
	35	"	Camelopardalis	40,2
			Capella	40,0
			Mean	40,4

By combining, in this manner, several thousand observations of Dr Maskelyne, and correcting for the annual change of nutation, M. Delambre obtained for the result 40″,492; he also obtained one precisely similar by a profound examination of a great number of Jupiter's satellites.

Hitherto we have considered only the aberration of light arising from the progressive motion of the earth in its orbit. If it be true, as numerous considerations seem to indicate, that the diurnal motion of the heavens, is, in like manner, only an appearance caused by the actual rotation of the earth on its axis in a contrary direction, this motion must also produce an aberration in proportion to its velocity, much less consequently than the former but yet worthy of being subjected to calculation, if it were only for the purpose of being assured that it is insensible. We proceed in this case also upon the same principle of the composition of forces, and we thus obtain an expression for the effect in question upon the right ascension and declination of the heavenly bodies. But this effect is so minute that it has hitherto been confounded with the errors of observation.

If from the earth we pass to the other bodies which compose our system, it is obvious that the proper motions of these must also be impressed upon the light which they send to us, and hence must result another species of aberration. These effects, and all others of a like nature, are calculated upon the same principles. We consider all the motions of the observer and of the body from which the light comes, as so many forces acting upon the luminous particles; and, upon the principle of the composition of forces, we determine the direction of the resultant to which the observer always refers the luminous particles which enter the eye.

Finally, if the solar system be also in motion in space, by virtue of a primitive impulse common to all the bodies of the system, as the rotation of most of these bodies would lead us to believe, the velocity of this motion of translation, combined with the preceding, and with the proper velocity of light, will produce still another species of aberration. But the displacement of the planetary system, although very probable, is not yet rendered sensible by modern observations, the only ones which are sufficiently exact to enable us to calculate its direction and velocity. Accordingly, until we are able to determine these two elements, the unequal aberration which this motion must produce with respect to the different stars, in consequence of their different positions, will be confounded with their proper motions, without our being able to separate them.

497. As yet we have only considered the velocity which light

receives from the motion of the heavenly bodies, of which it partakes. But we know by experiment that all transparent bodies, through which light passes, modify its velocity by their action. Thus the atmosphere which surrounds us increases the velocity of the luminous particles which enter it from void space. This velocity is also very sensibly changed by passing through the glasses of our telescopes and the humours of the eye. Among all these velocities, which shall we select to be compounded with the motion of the earth? Is astronomical aberration liable to be affected by all such causes, or is it independent of them? These are questions, which, on account of the frequency of their occurrence and the consequences they involve, deserve to be seriously discussed.

That the problem may not be too complicated, let us neglect, for the present, the action of the atmosphere, as it is very inconsiderable. It will be shown hereafter that this action, whatever be its intensity, can have no influence upon the phenomena of aberration.

Almost all astronomical observations consist in determining the instant when the centre of a heavenly body is concealed by a wire which moves with the earth, but whose position on its surface is known. Such are the wires of our micrometers. Let us first consider how these occultations take place, regard being had to the simultaneous motions of light and the earth; and, to simplify the problem, let us leave the glasses of the telescope out of the question, and consider the observation as made with the naked eye.

Supposing the observer to look through a tube, the axis of which is determined by fine wires crossing each other at its extremities; our object is to find the real direction of this axis in space, when the centre of the heavenly body is concealed behind the intersection of the wires.

We have said that this direction is precisely that of the resultant of the two velocities of light and the earth in space. For instance, let S be the body from which the light comes; ST, ST', the real direction of the luminous rays, and TF the length of the tube employed. If the observer at T directs the tube according to the resultant of the two velocities, the lines TT', $T'F$, will be proportional to these velocities. Consequently, if there is no obstacle in the interior of the tube, the luminous par- Fig. 114.

ticle, entering it at *F*, in the direction *SF*, will pass freely through it, as it moves parallel to itself; and this particle will arrive at *T′* at the instant the earth arrives there. Reciprocally, if the luminous particle passes freely along the axis of the tube, we must conclude that this axis is directed parallel to the resultant of the two velocities, for this position is the only one in which the particle could pass through the tube without encountering its sides. Now this case is always that which takes place when we see the body at the intersection of the wires. The luminous particle intercepted is then the same which would have come to the observer at *T′*, in the direction *ST′*; and as all those which precede it come to the eye, it is at the point *T′* only, that the eye perceives the interruption of the luminous ray.

If we compare, under these circumstances, the apparent motion of the luminous particle coming from the body, with that of the luminous particles coming from the wires which determine the axis of the tube, we shall find that these two motions are absolutely the same. For, at the moment the light from the body arrives at *F*, the entrance of the tube, in the direction *SF*, and with the velocity *FT′*, if we suppose the wire, placed at *F*, at this same moment, to emit a luminous particle with the same velocity, in the same direction, these two particles will necessarily accompany each other from *F* to *T′*, and will arrive at the eye at the same time and in the same direction.

If we would consider simply the relative motion of these two particles with respect to the observer, we have only to decompose their proper velocity *SF* into two others *tF* and *S′F*, the first being equal and parallel to the motion of the earth; then the other *S′F*, will express their apparent direction and velocity; for the observer always leaves out of consideration the motion which is common to himself and the object observed. Thus the two particles would still pass the tube together, as before.

If now we suppose any cause to act upon the two rays in the direction of the component *S′F*, so as to increase their velocity, it is evident that this would produce no change in their direction with respect to the moveable axis of the tube; only they would arrive at the eye a little sooner than before. Thus, when a musket is properly aimed by a skilful marksman, the ball always reaches its destination, whatever the force of the charge; only this is effected sooner or later, according as the force is greater or less.

Such precisely is the effect produced by the object glasses of our telescopes upon the apparent rays, and such is, in general, the effect of all the refracting media employed in the construction of these instruments. The extreme surfaces of the media being perpendicular to the optic axis of the tube, and consequently to the direction of the apparent ray, they serve only to accelerate its velocity, but do not change its relative direction with respect to the observer. For example, in our common astronomical telescopes, where the wires of the micrometer are placed at the focus of the object glass, the course of the luminous particle coming from the object, is first accelerated in traversing the object glass; then, at its emergence, it receives the velocity which it had before. It experiences a similar increase in traversing the eye glass, and resumes its primitive velocity at its emergence; but, during this whole passage, and notwithstanding all these variations of velocity, its apparent direction does not change; it remains constantly the same as that of the luminous particles, which emanate from the wire of the micrometer, with the same velocity, whether absolute or relative, and the same direction. In a word, the moment we decompose the proper velocity of light into two directions, of which one is equal and parallel to the velocity of the earth, the luminous particle becomes a terrestrial object; only its apparent velocity is not that of ordinary light; it is represented by $S'F$. Fig. 114

We see then that the increase of velocity, produced in luminous particles by optical instruments, parallel to their apparent direction, has no effect except to hasten the instant of their arrival. But as the velocity of light is extremely great, the absolute time employed in traversing our longest instruments is insensible to our organs, and consequently the acceleration produced between the object glass and the eye, is likewise insensible. If we were to form object lenses of substances having the greatest refracting power; if we were to fill the tube with water, as Boscovich proposed; or even if we were to make this tube of diamond which accelerates the velocity of light more than any other known substance, still the acceleration thus produced, would not make the luminous particles visible a hundred thousandth part of a second sooner; and consequently would not vary by the hundred thousandth part of a second, the instant when the object would be actually found in the apparent direction above determined.

But if we observe the luminous particle coming from the object through a medium whose refracting action is not exerted parallel to the apparent direction of the ray, for example, through prisms which cause a deviation in the luminous ray, we must perceive this small difference of velocity which distinguishes the apparent direction, from that which the light would have taken, in virtue simply of its motion of emission; for the deviation which these bodies produce, differs from that which they would produce upon a ray of light emitted naturally through a terrestrial body in this same direction. By accurately observing this difference of deviation, we can deduce from it, by calculation, the difference of the velocities and their ratio. Then, in the parallelogram *S t FS'*, we know the ratio of the side *FS*, which expresses the proper velocity of light, to the side *S'F* which expresses its apparent velocity. We know also the angle *S'SF*, comprehended between the direction of the earth's motion, and the real luminous ray at the instant of observation. With these data we can calculate the angle of aberration *SFS'*, and the ratio of the sides *SF* and *SS'*, which represent the proper velocity of light, and that of the earth. By repeating the process for different stars, we can determine whether the proper velocity of light is the same for each. We may even hope to render sensible the small difference of velocity which takes place in the motion of the earth at different seasons of the year in consequence of the ellipticity of its orbit. This method, which determines the aberration by observations made with the prism, would seem to be the more exact, since by increasing the refracting angle of the prism, we increase the deviation which it produces, so as to render it much greater than the natural value of the angle of aberration observed directly. At the suggestion of Laplace, this method was employed by M. Arago; but he found, very differently from what we should expect, that all light, whether terrestrial or celestial, direct or reflected, undergoes exactly the same deviation, whatever be the direction in which it is emitted.* It might be thought that this anomaly was owing to the difficulty of observing exactly the centre of the refracted image, since in these experiments, the action of the prisms always decomposes the light and dilates the image of the luminous point, in the form of an oblong coloured spectrum. But no such cause of error existed in the experiments of M. Arago; he made use of an achromatic prism, composed of flint and crown glass, adapted to each other in such a manner as to recompose the light with great exactness; so that the image of the heavenly body, seen through this prism, was almost as much concentrated as if it had been observed through media with parallel faces.

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* The prism which M. Arago used in his experiments, was placed before the object glass of a repeating circle in such a manner as to cover only a part of it, so that he could observe successively the direct luminous ray through the telescope simply, and the same ray deflected by the prism. In taking an account of the time at which

This equality of deviation which appears at first view directly contrary to Newton's theory, admits nevertheless of being referred to it, as M. Arago has shown, by supposing that luminous bodies send out, in all directions particles of light endued with an infinity of different velocities, among which there is only one which is suited to our organs, and which can produce in us the sensation of light. This idea agrees with a discovery made some years since by Sir W. Herschel; namely, that when we decompose light by a prism, there are, without the coloured spectrum, invisible rays, which cause a higher temperature than the luminous rays themselves, and which are even capable of producing chemical effects, as Dr Wollaston and M. Ritter have since observed. But, on the other hand, if we admit that each point of a body sends an infinite number of particles of different velocities in the same direction, it would seem that many of these

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the two observations were made, the heavenly body was reduced, by calculation, to the same height above the horizon. The difference of the angles observed directly and through the prism, would give the deviation experienced by the luminous ray. By observing thus the stars of the ecliptic which pass the meridian at six o'clock in the evening, the earth which turns upon its own axis, as well as about the sun from west to east, would proceed in its orbit in the same direction with the light of these stars, and consequently this light would have, upon arriving at the prism, only the difference of the two velocities. The contrary would take place with respect to stars which passed the meridian at six in the morning, and the earth would be moving in a contrary direction to that of the light from the stars. But this opposition, which must have caused a difference of 50″ in the observed deviation, did not produce any appreciable change.

particles must impinge against each other in their passage from the body to us; and it is difficult to comprehend how there should always remain some which preserve the same velocity. These considerations and many others prove that our knowledge of the nature of light is yet very imperfect. We remark, nevertheless, that the experiments of M. Arago, only evince still more strongly the necessity of calculating the aberrations of all bodies, with the same value for the velocity of light, whatever be the direction in which their light arrives, and whatever be the nature of the instruments employed.

In what precedes we have neglected the action of the earth's atmosphere upon light; but by applying to it the general considerations above stated, it is equally easy to calculate its influence upon aberration. In the first place suppose the star situated at the zenith in order to avoid the effects of refraction; in this case, if we decompose the proper velocity of the luminous particle, at the instant it enters the atmosphere, and resolve it into two others, one of which is equal and parallel to the motion of the observer; the other, which is that of the apparent direction, will be parallel to the earth's radius, and consequently perpendicular to the atmospheric strata. The action of the air will have no other effect except slightly to accelerate the motion of the luminous particle without changing its apparent direction. But the atmosphere is so rare and light employs so little time in traversing it, that the effect of this acceleration is altogether insensible. For the distance of the sun from the earth is equal to 23578 times the earth's radius. If we suppose the height of the atmosphere to be $\frac{1}{100}$ of the earth's radius, it will be $\frac{1}{2357800}$ of the distance of the earth from the sun. Now, light passes through 23578 times the earth's radius in 8′ 13″. Accordingly, with this same 288.
velocity, it would traverse the atmosphere in $\frac{8' \, 13''}{2357800}$. If the atmosphere were diamond, its action would double this velocity, and the time would be diminished one half, that is, it would be expressed by $\frac{4' \, 06''}{2357800}$; this acceleration would, therefore, be altogether insensible, and yet it surpasses several thousand times that produced by the atmosphere.

But if the luminous particle enter the atmospheric strata obliquely, as is the case when we observe bodies in the horizon;

then the action of these strata, becoming oblique to the apparent direction of the particle, must, it would seem, not only accelerate it, but cause it to deviate unequally according to the direction and intensity of its velocity. The aberration must then be unequal for different stars according to their position and apparent altitude. But it is easy to perceive from the experiments of M. Arago, that this effect is absolutely nothing, for the action of the atmosphere in this case is analogous to that of the prism. Besides, independently of this result, the refracting action of the atmosphere is so feeble, and the total deviation produced so small, that the inequalities of velocity in the luminous particles cannot manifest themselves in a sensible manner; for, under the most favourable circumstances, supposing the proper motion of the particle to conspire with the motion of the earth, or to oppose it, we find, by calculation, that the change in the refraction of the ray, arising from this change of velocity, would not be more than $\frac{12}{100}$ of a second, even at the distance of 79° or 80° from the zenith; which is about the limit at which we can make accurate observations, the atmospherical refraction near the horizon being irregular. Hence the star would vary only about $\frac{6}{100}$ of a second on one side and the other of its mean place, in virtue of the changes produced in the apparent velocity of its light; and as the effect would be still less sensible when the zenith distance is less, we might neglect it altogether. But, as we have already observed, the observations of M. Arago prove that this difference is not only very small, but is to be considered as absolutely nothing.

It now remains to consider the variations of apparent direction and velocity which the luminous particles undergo in the humours of the eye. As to the change of apparent direction, we must conclude from what precedes, that it will be nothing when the strata of the eye are perpendicular to the direction of the apparent ray, that is, when vision takes place directly through the centre of the pupil, as is ordinarily the case when we look through optical instruments. The whole is then reduced to an increase of velocity in the luminous particle while traversing the eye; a distance so small that the effect must be perfectly insensible. The only error, then, must take place when the eye is in an oblique position; for, in this case, although the rays coming from the object and those from the micrometer with which we compare it, would fall upon the globe of the eye in the same direction, yet as they have different velocities, the refraction resulting from their obliquity, might be unequal; and though the image of the wire and the image of the star coincided at the retina, yet their directions out of the eye might be really different. The error resulting from this cause might be very considerable; for the refracting force of the aqueous and vitreous humours of the eye is nearly equal to that of distilled water; and that of the crystalline much greater; so that the deviation produced by these substances would seem to be important. But the experiments made with the prism, prove that even under these circumstances, the error in question does not exist; for the globe of the eye and the humours which fill it, act upon light precisely like the prism in the experiments of M. Arago; and consequently, in this case, as in the other, the obliquity of the refracting surfaces will produce no apparent deviation.

From these considerations, it appears that the aberration produced by the motion of the earth, is always independent of the instruments made use of. For when the action of the refracting media is parallel to the resultant of the velocities of light and of the earth, which is most commonly the case, theory shows that there can hence result no change in the apparent direction of the luminous particles, because the composition of the two velocities must take place at the instant the luminous particle encounters the terrestrial object; and in the other case, where the oblique action of the refracting media tends to alter this direction, it is proved by experiment, made with a prism, that the apparent deviation is still insensible, for reasons not yet well understood.

The theory of the aberration of light completes our knowledge of the general motions observed in the apparent positions of the stars. By applying a correction for this aberration, as well as for precession and nutation, all the apparent motions disappear, and the whole system of stars becomes immoveable, with the exception of some slight proper motions, which are not subject to any known law.

After this perfect agreement of theory with fact, we cannot but admit that the aberration of light is a striking proof of the earth's progressive motion. We have already mentioned that the discovery is due to the celebrated Dr Bradley, who was also the first to recognise the effects of nutation.

498. If we recollect that all the celestial phenomena are consistent with the earth's motion, and are represented upon this hypothesis much more simply than upon any other; if we call to mind all the analogies which indicate this motion, and which must be destroyed by the contrary supposition; and lastly, if we add to all this, the striking proof furnished by the phenomena of aberration, we cannot doubt that the earth has two motions, one on its axis, and the other round the sun. Accordingly, we shall henceforth consider this conclusion as an incontestible truth.

Stations and Retrogradations of the Planets.

499. When in physical inquiries we are so fortunate as to arrive at the truth, it not only enables us to predict new phenomena, but serves also to account for those which were before inexplicable; the facts already ascertained throw light upon one another, and the discovery of a new law helps to simplify those before known. This is the peculiar characteristic of truth, and the surest indication that we have discovered it.

500. We have a striking instance of what is here advanced, in the explanation of several intricate phenomena, presented by the motion of the planets, as seen from the earth. This motion does not always appear to be directed the same way, as it really is when seen from the centre of the sun. Sometimes it is from west to east, or in the order of the signs, sometimes retrograde, and sometimes the planet seems stationary. Thus, when Venus passes beyond the sun with respect to the earth, its motion, compared with the stars is direct; when it passes between the earth and the sun, it appears to be retrograde; and in going from one of these states to the other, the motion becomes insensible, and Venus seems stationary. The same is true of Mercury. The superior planets present analogous appearances; they are direct in their conjunctions, and retrograde in their oppositions. These phenomena are called the *stations* and *retrogradations* of the planets.

501. With respect to Mercury and Venus, they are easily explained upon any hypothesis; for their orbits do not embrace the earth; and as they always revolve about the sun in the

same direction, from west to east, their motion must appear to be direct when they go from their western to their eastern elongation, passing beyond the sun; and retrograde, when they return from their eastern to their western elongation, passing this side of the sun. The annual motion of the earth or that of the sun, would only modify these appearances, by varying the points of the stations.

But the phenomena presented by the other planets are not so easily explained. It would seem, indeed, that their motion must always appear to be direct, since we are placed within their orbits, and since they always move in the same direction about the sun. But it must be remembered that the sun carries with it, in the ecliptic, the orbits of these planets. This motion may, in certain cases, be contrary to the motion of the planets, and, in others, conspire with it, according as it is in the same or in an opposite direction; and as the combined effect of these two motions determines the apparent direction of the planet, this may evidently be the cause of the stations and retrogradations observed.

502. But in order to judge how far this explanation agrees with the phenomena, and determine whether the supposition of the earth's motion represents them with the greatest simplicity, we shall give them a more particular consideration. As the same explanation will apply to all the superior planets, we shall consider only one, namely, Mars.

There are times when Mars after being lost in the solar rays, appears in the horizon a few minutes before the sun in the morning. Accordingly, it is now departing from conjunction. Let us follow its motions from this epoch.

If we observe each day its meridian altitude and right ascension, we find that its motion is *direct*, that is, from west to east. This is known by its daily increase in right ascension. Mars, being therefore west of the sun, moves in the same direction, but its proper motion is exceeded by that of the sun. Hence it happens that the two bodies recede from each other, instead of approaching nearer. At the same time the motion of Mars diminishes, and becomes altogether insensible, after an interval of about one year. Mars is then about 137° distant from the sun; and for some days, when referred to the stars, appears stationary. After this its motion is retrograde, that is, from east to west; and as the sun, on the contrary, always moves from west to east, these two bodies recede more rapidly from one another. In this way Mars arrives at a distance of 180° from the sun. It now sets when the sun rises, and is in opposition with respect to the earth. At this point its retrograde velocity is greatest. Continuing its course in this direction, Mars approaches nearer and nearer the sun, and the opposite motions of the two conspire to diminish the angular distance which separates them. But, at the same time, the retrograde velocity of Mars diminishes, and becomes nothing when the angular distance is again 137°. Then Mars, compared with the stars, appears stationary. The arc of retrogradation which it describes in the heavens is about 16°, and the time employed 73 days. From this point, Mars resumes its direct motion, which tends to make it recede from the sun over the part of the heavens already traversed; but the sun, having a more rapid motion, gradually approaches it, and overtakes it in the interval of about a year. Mars, having now returned to its conjunction, is again enveloped in the sun's rays, and becomes invisible.

503. These irregularities are common to all the superior planets, with the modifications which belong to the difference of their motions. There is even a sensible difference in the extent and duration of the retrogradations of the same planet on account of the elliptical form of its orbit. The following are the mean values for the principal planets.

	Angular Distances between the Planet and the Sun at the instant of being stationary.	Mean Arc of Retrogradation.	Mean Duration of the Retrograde Motion.	Mean Duration of the Synodic Revolution.
	° ′	° ′	Days.	Days.
Mercury	18 00	13 30	23	118
Venus	28 48	16 12	42	584
Mars	136 48	16 12	73	780
Jupiter	115 12	9 54	121	399
Saturn	108 54	6 18	139	378
Uranus	103 30	3 36	151	370

504. We proceed to explain these phenomena upon the supposition of the earth's motion. Indeed they follow so naturally and so necessarily that they are alone sufficient to confirm the theory.

Let O be the centre of the planetary motions. Let the circle $BCDE$ represent the earth's orbit, and a larger one $B'C'D'E'$, the orbit of a superior planet, Jupiter, for example. Let BC, CD, DE, represent equal arcs in the first, and $B'C'$, $C'D'$, $D'E'$, equal arcs in the second, the latter being described by Jupiter, in the same time that the earth employs in describing the former. Those described by Jupiter will contain a less number of degrees than the other, in proportion as Jupiter moves slower than the earth. Suppose now that when the earth is at B, Jupiter is at B'; it will appear to us to be in B'' on the celestial sphere, in the prolongation of the straight line BB'. Now if the earth passes from B to C, and Jupiter from B' to C' in the same time, we shall see the latter pass to C'' on the celestial sphere, and its motion will appear direct. Then while the earth moves from C to D and Jupiter from C' to D', this motion will always appear direct; but gradually the earth will begin to be interposed more directly between the sun and Jupiter; when it is at F, Jupiter will appear in F'', and will appear to be returning back on the celestial sphere. In passing from one of these directions to the other, there will be a time when it appears stationary, namely, while the earth is describing EF. When the earth arrives at G and Jupiter at G' in opposition with respect to the sun, we shall see it in G'' on the celestial sphere, and it will now appear to have moved backward through the whole arc $E''G''$, although in reality it has been constantly pursuing its direct course. As Jupiter and the earth continue to move in the same direction, the apparent retrogradation will continue also in the same manner; but its velocity will diminish, according as the proper motion of the earth becomes more oblique to the visual ray drawn to the planet, and alters less the proper motion of Jupiter. Finally, when the earth has arrived at a point I in its orbit, such that the direction of its velocity is the same as that of the visual ray drawn to the planet, the retrogradation will cease. Then if Jupiter is in I', it will appear in I'' on the celestial sphere, and will a second time appear stationary. When the earth has moved to K, and Jupiter to K', it will appear to have described the arc $I''K''$ in the order of the signs. Accordingly it will have resumed its direct motion on the celestial sphere. Its whole retrogradation then will comprehend the arc $E''I''$, and this will have taken place while Jupiter was describing $E'I'$, and the earth *EI*. This illustration will apply to Mars, Saturn, and all the superior planets; only the retrogradations are more frequent in the more distant planets, as Saturn, for instance, because the motion is slower than that of Jupiter, so that the earth rejoins it sooner. In Mars, on the contrary, they are less frequent, because its motion is more rapid. Hence, we see with what wonderful simplicity the motion of the earth accounts for these perplexing phenomena. It is to Copernicus that we are indebted for the revival of the hypothesis of the earth's motion which had been already advanced by some of the ancient philosophers, but which he first established in a satisfactory manner, by showing how simply it accounted for the various phenomena of the heavens.

If all the planets moved, like the earth, in the plane of the ecliptic, and if, moreover, their orbits and that of the earth were circular, which would render all their motions uniform, the epochs of retrogradation for each planet, would be constant; they would return after a synodic revolution, and their duration and extent would also be constant. But the inclination of their orbits and their ellipticity disturb this regularity, and produce variations in the epoch, as well as in the duration, of the retrogradations. Notwithstanding this complexity, it is evident that the optical cause of these appearances being known, their exact determination is a purely geometrical problem, the solution of which is to be derived from formulas, expressing the motion of each planet, regard being had to the ellipticity and inclination of its orbit. But as the observation of these phenomena is of no real utility, we deem it unnecessary to pursue the subject farther.

True Dimensions of the Planetary Orbits.

505. We have determined the relative distances of the planets from the sun, half the major axis of the earth's orbit being unity; we have been able, moreover, by anticipating the sun's parallax to reduce these distances to known measures. We are now to explain more fully the method of making this reduction.

506. The process would be very simple, if we had, for any given instant, the distance of a planet from the earth, expressed in known measures. For the geocentric positions of the planet

and the sun being determined for the same epoch, by the theory of the motions of the planet and the earth, a simple trigonometrical calculation would give the distance of the planet from the sun, expressed likewise in the same measures. We already know the value of this distance in parts of the major axis of the earth's orbit; we can, therefore, convert this major axis into the same measures; then the second law of Kepler would give all the major axes, and all the dimensions of the orbits expressed in the same manner, merely from the duration of their revolutions.

507. The parallax of any one planet, if it were known, would give immediately its absolute distance from the earth; but these parallaxes are in general too small to be accurately observed. That of Mars becomes sensible under certain circumstances favourable to observation; that is, when it is in its perihelion; and at the same time near its opposition. Then its distance from the earth is the smallest possible, and its parallax at its maximum.

On the 6th of October 1751, for example, Mars not being far from its opposition, its parallax became sensible; and, from the observations of Wargentin and Lacaille, it was found to be 24″,6, as already stated; and hence it follows that the distance 140. of Mars from the earth at this epoch was 8371 terrestrial radii, or 8371 × 3956 miles. This same distance, as deduced from the theory of its elliptical motion, would have for its expression 0,4354, half the major axis of the earth's orbit being unity. Hence it will be readily seen that the mean distance or half the major axis of the earth's orbit, expressed in terrestrial radii, has for its value $\frac{8371}{0{,}4354}$ or 19226.

The parallaxes of the heavenly bodies being inversely proportional to their distances, that of the sun must be equal to $\frac{24'',6 \times 8371}{19226}$ or 10″,69, when at its mean distance.

The ratios of the distances of the planets from the sun being known by the laws of Kepler; these results will enable us to determine the values of all these distances in terrestrial radii or in miles.

508. Unhappily the parallax of Mars is so small, that the errors of observation may have a very considerable influence upon the result, and consequently upon the values of the dis-

tances determined by means of it. If, for example, in determining the horizontal parallax, we commit an error of 2″ or ¼ of its total value, there would be an error of ¼ in the estimate of the distance. Yet we can never be secure against so small an error in observations so delicate; for 2″ does not amount to half the breadth of the wire used in the micrometer. The preceding results, therefore, can only be regarded as a first approximation; they are sufficient, however, to show that the sun's parallax is extremely small, and that its distance from the earth is proportionably great.

A much more exact method has been derived from observations of the transit of Venus.

509. We have seen that Venus, when it passes between the earth and the sun, is projected on the sun's disc under the appearance of a black spot, and by the effect of its proper motion combined with that of the sun, it appears to describe a chord of this disc. These chords, when observed from different parts of the earth, are sensibly different; and the difference arises from the parallax of Venus, in consequence of which the different observers do not refer the planet to the same points of the sun's disc. These differences have a considerable influence upon the duration of the transit, which may be observed with great precision, and which gives, with extreme accuracy, the difference between the parallax of Venus and that of the sun.

Fig. 118. Let T represent the earth, V Venus, and S the sun; an observer placed at the centre of the earth would see Venus in the prolongation of the radius vector TV; this planet will appear to correspond to the point S, on the disc of the sun, and in its successive positions it will describe the line DS. But other observers placed at O' and O'', on the surface of the earth, will see Venus at V' and V''. To the first it will appear to describe the chord $D'V'$, and to the second the chord $D''V''$. These effects are evidently owing to the difference in the parallax of Venus and that of the sun, and this difference may be calculated from the duration of the transits by precisely the same formulas, which are employed in calculating solar eclipses.

Now, by the theory of elliptic motion, we know, for the same epoch, the ratio of the distances of Venus and the sun from the earth. We know, therefore, the ratio of the parallaxes of these planets, since they are inversely as the distances. Hence it is extremely easy to ascertain their separate values; for both quantities are determined when we know their ratio and difference.

510. In this manner the sun's parallax is ascertained to be 8″,70167, that is, about $\frac{1}{6}$ less than that derived from the parallax of Mars. This value was determined by the transit of Venus in 1769. The phenomenon was anticipated by astronomers with the liveliest interest. They distributed themselves over the surface of the earth, and undertook long voyages for the express purpose of making the observation from different points and under the most favourable circumstances. Their expectations were not disappointed; for the difference in the durations observed at Otaheite in the South Sea, and at Cajanbourg in Finland, amounted to more than 15 minutes. Such was the general desire and united exertions of different nations for the discovery of truth.

511. Laplace has shown that the sun's parallax may be ascertained in a manner still different, without observing it directly, by means of an inequality in the motions of the moon which is found to be connected with this parallax. To understand this connexion, we must remember that the inequalities of the moon's motion have a close dependance upon the positions of the earth and sun. The relations are made known by calculation, and the extent of the inequalities is determined by observation. By combining these data, we can obtain the value of the elements on which the inequalities depend, for we have the expression of their dependance, and the measure of their action. The whole is reduced to finding inequalities in which this action is, in some measure, augmented, or in which it is constantly reproduced, in such a manner that we can determine it with exactness from a great number of observations. There is in the motion of the moon an inequality of this kind which depends upon the parallax of the sun, or its distance from the earth; and by determining it from observation, Laplace found the parallax to be 8″,59896,* differing but little from that obtained above. It is probable that this result of theory is even more exact than that derived from observations of the transit of Venus.

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* This is the value given by Laplace in the fifth edition of the *Exposition du Système du Monde*, published in 1824. In the *Mécanique Céleste* it is estimated at 8″,5602 or about $\frac{1}{221}$-part less.

Another inequality of the moon has made known the oblateness of the earth which is thus found to be equal to $\frac{1}{305}$, as
105. already remarked. These results are the more to be valued as they are independent of the small irregularities of the earth's surface, which become insensible at the distance of the moon. In this point of view we may regard them as more exact than observations themselves. For the determination of these two important elements, we are indebted to Laplace, who deduced from theory alone, results which so many voyages and observations had been undertaken for the purpose of determining.

512. The parallax 8″,59896 gives 23987 terrestrial radii for the mean distance of the sun from the earth. This result is sufficient for reducing all the dimensions of the planetary orbits to parts of the same measure.

In the first place, since the apogee and perigee distances of the sun are represented by 1,016814 and 0,983186 for the year 1750, the parallaxes will vary inversely as these quantities; whence we have the following values;

	″
Parallax of the sun at the apogee	8,4549
Mean parallax	8,59896
Parallax at the perigee . .	8,7460.

Top.148. Dividing the value of the radius in seconds or 206264″,80, by the parallax, we have the corresponding distance of the sun expressed in terrestrial radii. We know, besides, that the mean radius of the earth is equal to 3956 miles.

We can thus convert these results into miles which will render their values more intelligible. They are both expressed in the following table;

			Radii of the Earth.		Miles.
Apogee distance of the sun			24390	or	96486840
Mean	"	"	23987		94892572
Perigee	"	"	23584		93298304
Eccentricity		"	403		1594268.

In the same manner the dimensions of all the planetary orbits may be reduced.

513. These results enable us also to compare the bulk of the earth with that of the sun; for the mean parallax 8″,599, is the angle under which the apparent semidiameter of the earth would be seen from the centre of the sun at its mean distance. The apparent semidiameter of the sun, at the same distance, is 961″,34. The radius of the sun, therefore, is to that of the earth, as 961,34 is to 8,599, or as 111,7851 to 1.

We have already seen that the mean distance of the moon from the earth is about 60 terrestrial radii; consequently, if we suppose the centre of the sun placed at the point which the centre of the earth now occupies, the bulk of this luminary would fill a space of nearly double the diameter of the moon's orbit.

514. On the supposition that the sun and earth are spherical bodies, which is not far from the truth, their bulks will be as the cubes of their radii. We thus find that the bulk of the sun is about 1400000 times as great as that of the earth.

If we compare, in the same manner, the distances of the other planets with the measure of their apparent diameters, we easily obtain the ratio of their bulks to that of the earth. These results are contained in the table at the end of the section.

515. To form a sensible idea of these ratios, suppose that we construct spheres, having the proportions of the different planets to each other. Then, if we represent the diameter of the earth by 1 inch, the proportions will be nearly as follows;

The sun	112
Mercury	$\frac{2}{5}$
Venus	1
Mars	$\frac{1}{2}$
Jupiter	11
Saturn	10
Uranus	$4\frac{1}{4}$.

516. These ratios would enable us to determine the masses of the planets if we could suppose them all to be of an equal density; but the theory of attraction proves that this is not the case. We estimate the masses, therefore, by the intensities of their attractions, as will be seen hereafter.

Dimensions of the Planets.*

	Mean Diameters of the Planets in Miles.	Mean Diameters of the Planets, that of Earth being 1.	Surfaces of the Planets, that of the Earth being 1.	Bulks of the Planets, that of the Earth being 1.
The Sun	884444	111,7851	12496,0	1396856,0
Mercury	3036	0,3837	0,1472	0,0565
Venus	7590	0,9593	0,9202	0,8828
Earth	7912	1,0000	0,0000	1,0000
Mars	4140	0,5233	0,2738	0,1433
Vesta	269	0,034	0,0012	0,00004
Juno	1393	0,176	0,0310	0,0055
Ceres	1582	0,2	0,0400	0,008
Pallas	2025	0,256	0,0655	0,0168
Jupiter	83937	10,8616	117,97	1281,3
Saturn	79168	9,9831	99,66	994,9
Uranus	35112	4,2630	18,77	77,5

Universal Gravitation.

517. By taking a general view of the planetary motions, we have discovered certain constant laws, by which they are all combined together. We hence conclude that these motions are not independent of each other, but the effect of a general cause acting upon all the heavenly bodies. Let us endeavour to ascend, by the laws of analogy, to this great principle.

518. If we first consider the moon, we perceive that it constantly follows the earth in its annual revolution. There is, then, a certain force, which retains it about the earth and prevents it from wandering from its course. In this respect the force in question, whatever it be, is analogous to gravity.

Gravity tends to bring towards the earth, in straight lines, the bodies which are detached from its surface. When these bodies receive, in a direction oblique to that of gravity, an impulse tending to carry them farther from the earth, this power still exerts its influence, and causes them to describe curved lines, the concave part being turned towards the surface. The greater the force of projection, the greater will be the space described, before any considerable deviation from a straight line takes place. Bodies fall by the combined effect of gravity, and the resistance of the air which gradually destroys the projectile force; but if this resistance were removed, a body projected with sufficient force from the summit of a mountain, might be made to describe the whole circuit of the globe without approaching any nearer to the surface. In this last case, the velocity of projection would remain undiminished, as there would be nothing to resist it; accordingly, when the body had returned to the point from which it set out, having still the same velocity, it would continue its course through another revolution, and so on indefinitely, like one of the satellites.

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* The basis of this table, that is, the relative apparent diameters of the several planets, as seen at the mean distance of the earth from the sun, is taken from Schubert's Astronomy, published at St Petersburgh, in 1822.

Now this is exactly the case with the moon revolving about the earth; it is therefore natural to suppose that the effect is to be ascribed to a cause of this kind, since, however high we ascend above the general surface of the earth, by means of a mountain or a balloon, we always continue to experience the influence of terrestrial gravity; hence there is reason believe that it extends still further, and as we know of nothing to limit it, we conclude that it embraces the orbit of the moon.

But at the same time, it may be that gravity, at this distance, is less intense than at the surface of the earth; we have reason to suppose this, from the experiments made with the pendulum at the tops of mountains; for it appears that even at so small a distance gravity is less than at the surface.

519. These considerations are equally applicable to all the other satellites. The circumstance of their orbits being nearly circular seems to indicate that they gravitate towards the centre of their primaries, as the moon does towards the earth, and that they are retained by this force.

520. As the motions of the planets present analogous phenomena, it is natural to conclude that they gravitate, in like manner, towards the centre of the sun, to which they serve as so many satellites. We may presume that comets also are subject to the same force, both on account of the regularity with which they describe their orbits, and the relations by which

their motions are connected with those of the planets, according to the laws of Kepler.

521. Here then we recognise the existence of a general cause, which seems sufficient to maintain the motions of the heavenly bodies. To render this intelligible and probable, we must compare the satellites and planets to heavy bodies projected into space at a certain distance from the centre to which their gravity causes them to tend. We must decompose their motions, and estimate separately the effects of the impulse by which they revolve, and of the force which retains them in their orbits. We shall thus know with certainty the action and intensity of this force, as exhibited in the observed phenomena.

It is by the principles of mechanics, that the motions of bodies are thus decomposed, and the nature of the force is deduced from the effects which it produces. It is analysis which enables us to make this decomposition for each point of the orbit, and follow exactly the variations of the force from one point to another. It is necessary, therefore, to combine these two principles in order to discover the general cause of the celestial motions; and as they could not well be introduced into an elementary treatise, we can here only indicate the course to be pursued in their application. We shall do this the more readily, since this course being extremely rigorous, is calculated to inspire the utmost confidence in the results to which it leads.

522. We suppose a planet to revolve about the sun, and form equations expressing the laws of its motion. The forces which act upon it enter into these equations, and constitute the unknown quantities to be determined. This being done, observation serves to establish, in an incontestible manner, the following facts, which are the laws of Kepler.

1st. *The areas described by the radius vector of a planet, in its motion about the sun, are proportional to the times.*

Hence it is found by calculation, that the force which acts upon the planets is directed towards the centre of the sun.

2d. *The orbits of the planets and comets are conic sections, of which the sun occupies one of the foci.*

It follows from this law, that the force which urges one of these bodies is inversely as the square of the distance between the centre of this body and that of the sun; reciprocally, since the force is in this ratio, the curve is a conic section.

3d. *The squares of the times of revolution of the planets are proportional to the cubes of the major axes of their orbits; or what amounts to the same thing, the areas described in equal times in different orbits, are proportional to the square roots of their parameters.*

We hence infer that the same force is exerted upon the planets and comets; that it only varies for the different bodies according to their distance, and that if they were urged by no projectile force, and were at equal distances from the sun, they would fall towards it with the same velocity; whence it is manifest that the force which acts upon them, penetrates each particle and is proportional to their mass.

The facts observed by Kepler lead, therefore, directly to the knowledge of the force which retains the planets and comets in their orbits, each fact developing a property. This force being exerted in such a way as to draw bodies toward the sun, we shall call it *solar attraction*, intending to express by this phrase, not the nature of the power, but only its effects.

523. The motions of the satellites presenting analogous phenomena, and being in like manner subject to the laws of Kepler, it follows that *each system of satellites is attracted towards the primary with a force inversely proportional to the square of the distance.*

524. As the ellipticity of the orbits is one of the data upon which we proceed, it would seem that the above results could not be extended to the satellites whose orbits have not yet been proved to be ellipses. But, if we suppose the eccentricity nothing, the proportionality of the squares of the times to the cubes of the mean distances, would lead to the same conclusions. The law in question is therefore general, and applicable to all the satellites.

525. In the case of the moon, in particular, it can be demonstrated in a striking manner. To obtain the force which retains this body in its orbit, it is sufficient to diminish the earth's gravity in the ratio of the square of the distance.

526. We are thus led to consider the sun and those planets that have satellites, as so many centres to an attractive force, which is exerted upon all the surrounding bodies. It is natural to suppose that this property belongs to all parts of the planetary system; and indeed the nearly spherical form, common to all these bodies, seems to indicate that their particles are united about their centres by a force which attracts them equally at

equal distances; but this truth admits of direct demonstration. It is a general law of nature, that action and reaction are equal Iron attracts the magnet, and is itself attracted in turn. The planets gravitate towards the sun; the sun, therefore, gravitates towards them by the same laws.

527. By thus comparing the phenomena of the heavens we are conducted to this great principle of nature. *All parts of matter mutually attract each other in the direct ratio of their masses, and in the inverse ratio of the squares of their distances.* The discovery of this principle is due to Newton.

528. In the study of natural phenomena there are, in general, two ways of proceeding, one consists in reasoning rigorously from exact data; the other in retracing our steps for the purpose of ascertaining whether the phenomena observed can be deduced as consequences, from the results at which we have arrived. The law of universal gravitation has been derived directly from the first method; the second is not less favourable to the same law, for, by following out the consequences to be deduced by means of it, we find that all the celestial phenomena hence derive their origin.

529. In the first place, the elliptical motion of bodies is thus completely explained. When a planet departs from its perihelion, the direction of its motion, always indicated by the tangent of its orbit, is perpendicular to the radius vector. It is at this point of the orbit that its tendency towards the sun is greatest; but the velocity of revolution is also at its maximum; the calculus shows that this second cause preponderates over the first; and consequently the planet diverges from the sun, and its radius vector increases more and more.

At the same time its gravity tends to bring it back to the sun, and opposes its motion. This cause always acting, gradually diminishes the velocity, until the planet arrives at its aphelion. Then the velocity of revolution is at its minimum; and the solar attraction being weakened by the distance, is considerably diminished; but the calculus shows that in this state it prevails over the tendency of the planet to recede from the sun. It is, therefore, brought back in an opposite direction, and from this point to the perihelion, its velocity increases, since the continually increasing attraction of the sun conspires with the direction of the motion.

The planet, having returned to this point of its orbit, is found to have all the velocity which it had at the preceding revolution; it will then commence a new revolution with the same velocity, passing through the same periods, and so on indefinitely.

530. An elliptical motion is not the only one which answers to the law that the force of gravity is reciprocally proportional to the square of the distance. It is shown by analysis that the orbit may also be a parabola, or a hyperbola, and in general, any conic section. These curves are the only ones which can be described by the force in question. In each particular case the kind of orbit is determined by the intensity of the initial impulse; and, what is exceedingly remarkable, the direction of this impulse does not influence the result; it only modifies the dimensions of the orbit.

It is probable that there are bodies in the heavens which thus describe parabolas or hyperbolas about the sun, but then these bodies can never be visible but once; and when they disappear we lose sight of them forever.

531. To represent accurately the celestial motions, it is not sufficient to consider separately the bodies which compose our system. We must at the same time have regard to the mutual attractions which these bodies exert upon one another. On this account the motion will only approximate to the elliptical hypothesis as we actually find. This deviation, therefore, is a proof in favour of universal gravitation.

532. It is thus, for example, that the elliptical motion of the moon about the earth is disturbed by the action of the sun. If this action were the same upon the moon and earth, both would yield to its influence at the same time, and while they were carried along the ecliptic with an equal motion, the moon would regularly describe its ellipse about the earth. But as the moon is sometimes nearer the sun than the earth is, and sometimes farther off, and as the sun's action upon the moon has for the most part an oblique direction to that exerted upon the earth, variations take place in the moon's gravity to the earth, and in the velocity with which it revolves about the earth; hence the inequalities of its motion.

For example, at the time of conjunction, the moon is between the earth and the sun, and consequently nearer the sun than the earth is. In this position the sun attracts it more and increases a little its distance from the earth. On the contrary, at the time of opposition, the earth is most attracted by the sun and is thus withdrawn from the moon, so that with respect to us, a similar effect is produced. At the quadratures the action of the sun, decomposed in the direction of the radius vector of the moon, tends to diminish its distance; but the calculus shows that this effect is less than the augmentation which takes place at the syzygies, so that, on the whole, the moon is maintained at a greater distance from the earth than it would be without the action of the sun.

This action, being variable in the different points of the moon's orbit, must produce periodical inequalities in its motion, which recur at each revolution, but their intensity must be different according to the different distances of the sun from the earth; it must be greatest, for instance, when the sun is at the perihelion, and least when at the aphelion; hence result some inequalities of another sort which serve as corrections to the preceding, and which, depending on the position of the earth in its orbit, recur annually.

533. The motion of the nodes of the moon's orbit and the change in its inclination to the ecliptic, are also necessary consequences of the action of the sun. When the moon, in its motion about the earth, approaches the plane of the ecliptic, the sun attracts it towards this plane and tends to bring it down to it. By the effect of this attraction, the moon crosses the ecliptic a little sooner and comes to it more directly; hence the retrograde motion of the nodes and the change of inclination of the orbit.

534. A similar action is exerted upon all the planetary bodies, by which their inequalities are produced. The motions of the nodes and perihelia, and the changes in the inclination of their orbits, are owing to these reciprocal attractions. The law of universal gravitation affords at the same time an explanation and a measure of the inequalities in question.

If it were necessary to consider simultaneously all the causes of perturbation, the calculations would become extremely complicated, and the difficulties would be insurmountable. Happily the constitution of the solar system permits us to diminish them. The bodies which compose this system are insulated in space, at considerable distances; each planet with its satellites forms, as it were, a world in miniature, in which every thing is determined by the particular attractions of the bodies which make a part of it. The other bodies being placed at great distances, exert nearly an equal action upon all these bodies, which are comparatively near together, and this common action does not disturb their relative motions. Thus, the laws of the planetary motions themselves apply to the motions of the satellites, and these bodies exhibit, in their revolutions, a rapid representation of the secular derangements which are to take place in the planetary system, and which time will develope.

But, as we have remarked in speaking of the motions of the moon, it is not always sufficient to consider each system separately; if we would attain to the utmost accuracy, regard must be had to the action of those bodies, which, by reason of their proximity or their mass, are capable of exerting a sensible influence; and we must neglect those only whose action, calculated in a rigorous manner, would always be less than the errors unavoidably incident to observation.

In the case of the moon, for example, it is sufficient to have regard to the reciprocal actions of the earth, the moon, and the sun. Those of the other heavenly bodies may be neglected, either because their mass is too small, or their distance too great, to produce a sensible effect; hence the theory of the moon has received the name of the *problem of the three bodies.*

In the case of the earth, on the other hand, regard must be paid to the united actions of the Sun, Moon, Venus, Mars, Jupiter, and Saturn. Uranus is too distant to have a sensible influence, and the other planets are so small that their action may be neglected. The effects of these mutual actions are particularly remarkable in the motions of Jupiter and Saturn. These two planets, the largest in our system, acting with great force upon each other, occasion in their respective orbits, very great inequalities, extending through several centuries; yet modern analysis has enabled us to discover their laws.

535. It will be readily perceived that perturbations of this kind, when the bodies in question have so considerable an influence upon each other, must be well adapted to the determination of their masses. Accordingly this method has been made use of, and we measure the mass of each of the heavenly bodies by the effects which it produces, its distance being taken into consideration. The bulks are ascertained, as we have seen

above, by means of the apparent diameters. Comparing these with the masses, we deduce the densities of the different planets, and the laws of gravity at their surface.

The following table, taken from the *Exposition du Système du Monde*, 5th edition, published in 1824, presents the most exact values of the masses of the principal planets, that of the sun being taken as unity.

Mercury	$\frac{1}{2025810}$
Venus	$\frac{1}{405871}$
Earth	$\frac{1}{354936}$
Mars	$\frac{1}{2546320}$
Jupiter	$\frac{1}{1070,5}$
Saturn	$\frac{1}{3512}$
Uranus	$\frac{1}{17918}$

Thus we see how small all these masses are when compared with that of the sun. The earth which appears to us so great, becomes as it were a grain of sand by the side of this vast body, which is itself but a point in the immensity of space.

The perturbations to which the satellites of Jupiter are subject, enable us to determine their masses in the same manner. The values are as follows; that of the primary being taken for unity.

First satellite . . .	0,0000173281
Second "	0,0000232355
Third "	0,0000884972
Fourth "	0,0000426591

Although these masses are very small, the influence which they exert in producing the inequalities of the satellites, afford reason for believing that the above values are very exact.

The comets hitherto observed have not produced any sensible perturbations in the bodies of the planetary system, and accordingly we have not yet been able to determine the mass of any of them. But we conclude for this very reason, that their masses are very small.

536. The perturbations which take place in the heavenly bodies, may sometimes be so great as to affect an entire change

in their orbits. This is probably the case with respect to comets which describe parabolas and hyperbolas. These bodies are supposed to wander from system to system, yielding continually to new attractions, until they are destroyed, or their orbits become invariably fixed.

537. It has been thought that the orbit of the comet of 1770 was changed in this way. It has been presumed, with great probability, that this comet, moving towards its perihelion, was made to take an elliptical orbit by the action of Jupiter, near which it passed at this time; but, that afterwards, having described one revolution in this orbit, it again passed near Jupiter, and by a contrary attraction, the effects of this first perturbation were destroyed, and its orbit again became infinitely elongated. Laplace has carefully considered the possibility of such a phenomenon, and given in the fourth volume of the *Mécanique Céleste*, considerations which render it extremely probable.

This comet approached nearer to the earth than any other hitherto observed, and the earth must consequently have experienced a sensible action, if the mass of the comet bears any proportion to that of the terrestrial globe. On the supposition that the masses are equal the action of the comet would have increased the duration of the year more than six minutes. Now it is well ascertained from the comparison of numerous observations, made by MM. Delambre and Burckhardt in constructing their solar tables, that since 1770, the sidereal year has not been increased one second; the mass of the comet, therefore, is not $\frac{1}{5000}$ part of that of the earth. Moreover, if we consider that this comet in 1767 and 1779, traversed the system of Jupiter's satellites without producing the slightest disturbance, we shall perceive that it is still less. We may add that the smallness of the masses of the comets is indicated by the circumstance that they exert no sensible influence upon the planetary motions. These motions are represented by the action simply of the bodies of this system, with such a degree of precision that the small deviations belonging to our best tables are attributable to errors of observation and calculation.

538. From the mere circumstance that the planetary motions take place in the same direction and in ellipses of small eccentricity and little inclined to each other, it may be demonstrated that the present state of the system is stable; that no one of the

planets was originally a comet; and that the solar system only oscillates about a mean state of ellipticity and inclination, from which the deviations are inconsiderable.

539. We have seen in universal gravitation the general cause of the celestial motions. It is the same cause which determines the rounded figure of the planets, and which, combined with the centrifugal force derived from the rotatory motion, gives rise to the protuberance at the equator, and the flattening at the poles; it is moreover the cause, which, in connexion with the oblateness, produces the balancing or nutation of these bodies about their centre of gravity. Let us take the earth as an example. If it were exactly spherical, the particles of matter situated on opposite sides of the centre would be equally attracted by the sun, and there would result no motion about this centre. But the protuberance at the equator disturbs this inequality. The particles which compose it may be considered as so many small moons connected firmly with each other and with the terrestrial globe. Each of them must experience inequalities analogous to those of the real moon; that is, its nodes must retrograde on the ecliptic by the action of the sun. Now these particles adhering to the body of the earth, cannot have any motion independent of the globe itself. They will tend, therefore, to draw the earth with them in a retrograde direction; and although their motion, when transmitted to so large a mass, must be greatly weakened, yet it is not wholly insensible. The entire body of the earth gradually yields to it, and the terrestrial equator retrogrades slowly on the ecliptic, and hence the precession of the equinoxes.

The moon acting upon the earth, like the sun, must produce analogous effects; and the smallness of its mass is even more than compensated by its proximity. But as its position about the earth is continually changing, the effects are equally variable. Accordingly the action of the moon is not confined, like that of the sun, to producing a motion in the equinoxes. It changes the obliquity of the ecliptic and produces the nutation of the earth's axis; and these inequalities which are peculiar to it, have periods which depend upon its motions.

The mean value of the precession being the result of the combined actions of the sun and moon, while nutation is produced by the moon alone, these phenomena are well calculated to make known the relative masses of these bodies; accordingly they have been employed, if not in finding this ratio, at least in verifying the value assigned to it by means of other phenomena. The same remark is applicable to the oblateness of the earth, without which there would be neither precession nor nutation.

540. Lastly, we perceive the effects of universal gravitation in the phenomena of the tides. The liquid particles which are situated on the side of the terrestrial spheroid, next to the sun are more attracted by it than the centre of the earth; their weight is therefore somewhat diminished, as we observed with respect to the moon when in conjunction. Hence a greater mass of water will necessarily accumulate on this side to compensate for the diminution of gravity. The same effect is produced also in the particles of water situated in the opposite hemisphere; since these are less attracted than the centre of the earth, which is drawn away from them; and thus their weight is diminished, as is the case with respect to the moon when in opposition. Accordingly by the union of these circumstances, the ocean must be elevated under the body which attracts it, and form on the earth's surface two opposite protuberances, situated nearly in the straight line which joins the centre of the earth and that of the sun. The total mass of waters remaining the same, there must be a corresponding depression in the parts of the ocean which are 90° from the sun.

The action of the moon must produce analogous phenomena; this also raises the waters of the ocean; and the oscillations which it occasions follow the periods of its motion.

The total height of the tides depends, therefore, upon the combined action of these two bodies; it is the greatest possible when the moon is in the syzygies, at which time the two actions conspire, and the smallest at the quadratures where they are opposed to each other.

The difference between the periods of the lunar and solar tides has furnished the means of estimating the separate effects of these two bodies and the measure of their action. That of the moon is the greatest, since the smallness of its mass is more than compensated by its proximity. In this way we have obtained the ratio of the masses of the moon and sun. This result agrees with that obtained from other phenomena upon which the mass of the moon has a sensible influence. From all these facts taken together, it appears that the mass of the moon is $\frac{1}{75}$[†] part of that of the earth. We know also that the diameter of the moon is expressed by 0,273, that of the earth being 1; whence it follows that its bulk is $\frac{1}{49,149}$ of that of the earth; and as its mass is only $\frac{1}{75}$, it follows that its density is less than that of the earth, as already remarked.

According to the foregoing theory, each tide ought to take place at the instant when the body producing it passes the meridian. But this is not according to what we actually observe in our ports; owing to local circumstances, as the accumulation of water, friction at the bottom of the sea, and the configuration of the shores, the oscillations are impeded and retarded. This retardation does not take place in the open sea. But as the circumstances which produce it in any place are nearly constant, so the effect is also constant, and can be predicted after having been once observed.

541. Such is a brief account of the phenomena that are deduced as natural consequences from universal gravitation. We have been able only to indicate their mutual connexion and dependence. With the aid of the calculus we might enter into the minutest details, and develope the most hidden relations of these phenomena; we might even estimate them with more exactness than by actual observation. These profound researches form the subject of the *Mécanique Céleste.*

Being restricted by the plan we proposed to ourselves in this work, to a developement of the mere elements of the science, we are able to advance no further. But having reached this point, let us for a moment look back upon the course we have pursued. We set out with those fundamental ideas which the aspect of the earth and the heavens suggests to every man; we were surrounded by all the false appearances which take place in the atmosphere and in the heavens. By degrees we detected our erroneous judgments, escaped from the illusion of the senses, and removed the veil from nature. We thus attained to more correct ideas of the system of the world. Distrusting our first impressions, we resorted to actual measurement. We investigated the laws which govern the motions of the heavenly bodies, and by comparing these laws together we have been led to the discovery of a few general principles which completely embrace all the rest. Lastly, we have seen that these results may all be reduced to one, namely, universal gravitation. This is the fundamental principle of all the celestial motions, all being derived from it as from a common source. We have thus, as it were, decomposed the system of the world, reduced it to its simple elements, and then recomposed it. In this view astronomy is, of all the sciences, the most complete, the most sublime, and that which most exalts the human mind. But the circumstance which gives it an inestimable value is its perfect certainty. Whatever may be the progress of science, the principle of universal gravitation is unalterably fixed, since it rests upon undeniable facts, and its duration will be eternal. It is certainly wonderful to see the human mind arriving at conclusions so bold and extensive; but truths which are deduced from facts by rigorous reasoning, cannot be called in question. And here we may apply those noble words of Cicero, which seem as if intended expressly for our purpose. *Hæc mirabilia videri intelligo; sed cùm certè superiora firma ac vera sint, his autem ea consentanea et consequentia; nec de eorum quidem veritate est dubitandum.*—De Fin. lib. 3, cap. 15.

[†] This is the value given by Laplace in the last edition of the *Exposition du Système du Monde.* In the *Mécanique Céleste* it is estimated at $\frac{1}{68,4}$.

FIXED STARS.

542. We have been able to determine the distance of the earth from the sun, and thence the distances and dimensions of the other planets. All these determinations have for their basis the radius of the earth, that is, a line which, seen from the sun, would appear less than the diameter of a hair. Let us now endeavour to apply our measures to what is still more remote.

543. It would be idle to inquire whether the fixed stars have a sensible parallax when observed from different parts of the earth. We have already had abundant evidence that their distance is almost infinite. It is only by taking the longest base accessible to us that we can hope to arrive at any satisfactory result.

544. Accordingly we employ the major axis of the earth's orbit which is nearly 200000000 miles in extent. By observing a star from the two extremities of this axis, at intervals of six months, and applying a correction for all the small inequalities, the effect of which we have calculated, we shall know whether the longitude and latitude are the same or not at these two epochs.

It is obvious, indeed, that the star must appear more elevated above the plane of the ecliptic when the earth is in the part of its orbit which is nearest to the star, and more depressed when the contrary takes place. The visual rays drawn from the earth to the star, in these two positions, differ from the straight line drawn from the star to the centre of the earth's orbit; and the angle which either of them forms with this straight line, is called the *annual parallax*.

As the earth does not pass suddenly from one point of its orbit to the opposite, but proceeds gradually, if we observe the positions of a star at the intermediate epochs, we ought, if the annual parallax is sensible, to see its effects developed in the same gradual manner. For example, if the star is placed at the pole of the ecliptic, the visual rays drawn from it to the earth, will form a conical surface, having its apex at the star, and for its base, the earth's orbit. This conical surface being produced beyond the star, will form another opposite to the first, and the intersection of this last with the celestial sphere, will constitute a small ellipse, in which the star will always appear diametrically opposite to the earth, and in the prolongation of the visual rays drawn to the apex of the cones. This result is very different from that of aberration, which causes the apparent position of the star to vary perpendicularly to the radius of the earth's orbit, and not in the direction of this radius.

545. It is, therefore, very easy to distinguish the results which belong to the annual parallax from the other inequalities. But notwithstanding all the pains that have been taken to multiply observations, and all the care that has been used to render them perfectly exact, we have been able to discover nothing which indicates with certainty even the existence of an annual parallax, to say nothing of its magnitude. Yet the precision of modern observations is such that if this parallax were only 1″, it is altogether probable that it would not have escaped the multiplied efforts of observers, and especially those of Dr Bradley, who made so many observations to discover it, and who, in this undertaking, fell unexpectedly upon the phenomena of aberration and nutation. These admirable discoveries have themselves served to show, by the perfect agreement which is thus found to take place among observations, that it is hardly to be supposed that the annual parallax can amount to 1″. The numerous observations of the pole star, recently employed in measuring an arc of the meridian through France, have been attended with a similar result, as to the amount of the annual parallax. From all this we may conclude that as yet there are strong reasons for believing that the annual parallax is less than 1″, at least with respect to the stars hitherto observed.

Thus the semidiameter of the earth's orbit, seen from the nearest star, would not appear to subtend an angle of 1″; and to an observer placed at this distance, our sun, with the whole planetary system, would occupy a space scarcely exceeding the thickness of a spider's thread.

546. If these results do not make known the distance of the stars from the earth, they at least teach us the limit beyond which the stars must necessarily be situated. If we conceive a right-angled triangle, having for its base half the major axis of the earth's orbit, and for its vertex an angle of 1″, the distance of this vertex from the earth, or the length of the visual ray, will be expressed by 212207, the radius of the earth's orbit being unity; and as this radius contains 23987 times the semidiameter of the earth, it follows that if the annual parallax of a star were only 1″, its distance from the earth would be equal to 5090209309 radii of the earth or 20086868036404 miles, that is, more than 20 billions. But if the annual parallax is less than 1″, the stars are beyond the limit which we have assigned.

547. We have already seen that light employs 8′ 13″ in traversing the radius of the earth's orbit. Hence it is easy to calculate the time it would require to travel a distance equal to that of the stars, if their parallax were sensible and equal to 1″; for this time would be 212207 × 8′ 13″, or about 3¼ years. Light employs then more than three years in passing from the stars to us. The stars whose light seems to break forth at once in the heavens, must have existed long before they are perceived; and in some instances their light is still seen long after they have vanished from their places. Perhaps there are some stars so distant that their light has not yet reached us; others perhaps ceased to be millions of ages ago, while their light is still seen in the places they once occupied in the expanse of the heavens.

548. It is evident that the stars undergo considerable changes, since these changes are sensible even at the distance at which we are placed. There are some which gradually lose their light, as the star δ of Ursa Major. Others, as β of Cetus, become more brilliant. Finally, there are some which have been observed to assume suddenly a new splendor, and then gradually fade away. Such was the new star which appeared in 1572, in the constellation Cassiopeia. It became all at once so brilliant that it surpassed the brightest stars, and even Venus and Jupiter when nearest the earth. It could be seen at mid-day. Gradually this great brilliancy began to diminish, and the star disappeared in sixteen months from the time it was first

seen, without having changed its place in the heavens. Its colour, during this time, suffered great variations. At first it was of a dazzling white, like Venus; then of a reddish yellow, like Mars and Aldebaran; and lastly, of a leaden white, like Saturn. Another star which appeared suddenly in 1604, in the constellation Serpentarius, presented similar variations, and disappeared after several months. These phenomena seem to indicate vast flames which burst forth suddenly in these great bodies. Who knows that our sun may not be subject to similar changes, by which great revolutions have perhaps taken place in the state of our globe, and are yet to take place.

549. Some stars, without entirely disappearing, exhibit variations not less remarkable. Their light increases and decreases alternately in regular periods. They are called for this reason *variable stars*. Such is the star Algol in the head of Medusa, which has a period of about 3 days; δ of Cepheus, which has one of 5 days; β of Lyra, 6; μ of Antinous, 7; o of Cetus, 334; and many others.

Several attempts have been made to explain these periodical variations. It is supposed that the stars which are subject to them, are, like all the others stars, self-luminous bodies, or true suns, turning on their axes, and having their surfaces partly covered with dark spots, which may be supposed to present themselves to us at certain times only, in consequence of their rotation. Other astronomers have attempted to account for the facts under consideration, by supposing these stars to have a form extremely oblate, by which a great difference would take place in the light emitted by them under different aspects. Lastly, it has been supposed that the effect in question is owing to large opaque bodies, revolving about these stars, and occasionally intercepting a part of their light. Time and the multiplication of observations may perhaps decide which of these hypotheses is the true one.

550. One of the best methods of observing these phenomena is to compare the stars together, designating them by letters or numbers, and disposing them in the order of their brilliancy. If we find, by observation, that this order changes, it is a proof that one of the stars thus compared, has likewise changed; and a few trials of this kind will enable to ascertain which it is that has undergone a variation. In this manner, we can only com-

pare each star with those which are in the neighbourhood and visible at the same time. But by afterwards comparing these with others, we can, by a series of intermediate terms, connect together the most distant extremes. This method, which is now practised, is far preferable to that of the ancient astronomers who classed the stars after a very vague comparison, according to what they called the *order of their magnitudes*, but which was in reality, nothing but that of their brightness, estimated in a very imperfect manner.

551. By comparing the places of some of the fixed stars, as determined from ancient and modern observations, Dr Halley discovered that they had a proper motion, which could not arise from parallax, precession, or aberration. This remarkable circumstance was afterwards noticed by Cassini and Le Monnier, and was completely confirmed by Tobias Mayer, who compared the places of 80 stars, as determined by Roemer, with his own observations, and found that the greater part of them had a proper motion. He suggested that the change of place might arise from a progressive motion of the sun towards one quarter of the heavens; but as the result of his observation did not accord with his theory, he remarks that many centuries must elapse before the true cause of this motion could be explained.

The probability of a progressive motion of the sun was suggested upon theoretical principles by the late Dr Wilson of Glasgow; and Lalande deduced a similar opinion from the rotatory motion of the sun, by supposing, that the same mechanical force which gives it a motion round its axis, would also displace its centre, and give it a motion of translation in absolute space.

If the sun has a motion in absolute space, directed towards any quarter of the heavens, it is obvious that the stars in that quarter must appear to recede from each other, while those in the opposite region would seem gradually to approach, in the same manner as when walking through a forest, the trees toward which we advance are constantly separating, while the distance of those which we leave behind is gradually contracting. The proper motion of the stars, therefore, in opposite regions, as ascertained by a comparison of ancient with modern observations, ought to correspond with this hypothesis; and Sir W. Herschel found, that the greater part of them are nearly in the direction which would result from a motion of the sun towards the constellation Hercules, or rather to a part of the heavens whose right ascension is 250° 52′ 30″, and whose north polar distance is 40° 22′. Klugel found the right ascension of this point to be 260°, and Prevot made it 230°, with 65° of north polar distance. Sir W. Herschel supposes that the motion of the sun, and the solar system, is not slower than that of the earth in its orbit, and that it is performed round some distant centre. The attractive force capable of producing such an effect, he does not suppose to be lodged in one large body, but in the centre of gravity of a cluster of stars, or the common centre of gravity of several clusters.

552. There are stars which, when viewed by the naked eye, and even by the help of a telescope of moderate power, have the appearance of only a single star, but being seen through a a good telescope they are found to be double, and in some cases a very marked difference is perceptible, both as to their brilliancy and the colour of their light. These Sir W. Herschel supposed to be so near each other as to obey reciprocally the power of each other's attraction, revolving about their common centre of gravity, in certain determinate periods.

The two stars, for example, which form the double star Castor, have varied in their angular situation more than 45° since they were observed by Dr Bradley in 1759, and appear to perform a retrograde revolution in 342 years, in a plane perpendicular to the direction of the sun. Sir W. Herschel found them in intermediate angular positions, at intermediate times, but never could perceive any change in their distance. The retrograde revolution of γ in Leo, another double star, is supposed to be in a plane considerably inclined to the line in which we view it, and to be completed in 1200 years. The stars ϵ of Bootes, perform a direct revolution in 1681 years, in a plane oblique to the sun. The stars ξ of Serpens, perform a retrograde revolution in about 375 years; and those of γ in Virgo in 708 years, without any change of their distance. In 1802, the large star ζ of Hercules, eclipsed the smaller one, though they were separate in 1782. Other stars are supposed to be united in triple, quadruple, and still more complicated systems.

553. With respect to the determination of the real magnitude of the stars, and their respective distances, we have as yet made

but little progress. Researches of this kind must be left to future astronomers. It appears, however, that the stars are not uniformly distributed through the heavens, but collected into groups, each containing many millions of stars. We can form some idea of them from those small whitish spots called *nebulæ*, which appear in the heavens. By means of the telescope, we distinguish in these collections an almost infinite number of small stars so near each other, that their rays are ordinarily blended by irradiation, and thus present to the eye only a faint uniform sheet of light. That large, white, luminous track, which traverses the heavens from one pole to the other, under the name of the *Milky Way*, is probably nothing but a nebula of this kind, which appears larger than the others, because it is nearer to us. With the aid of the telescope we discover in this zone of light such a prodigious number of stars that the imagination is bewildered in attempting to represent them. Yet from the angular distances of these stars, it is certain that the space which separates those which seem nearest to each other, is at least a hundred thousand times as great as the radius of the earth's orbit. This will give us some idea of the immense extent of the group. To what distance then must we withdraw, in order that this whole collection may appear as small as the other nebulæ which we perceive, some of which cannot, by the assistance of the best telescopes, be made to present any thing but a bright speck or a simple mass of light, of the nature of which we are able to form some idea only by analogy? When we attempt, in imagination, to fathom this abyss, it is in vain to think of prescribing any limits to the universe, and the mind reverts involuntarily to the insignificant portion of it which we are destined to occupy.

APPENDIX.

I.—*Note to Article* 30, *Page* 16.

THE micrometer here referred to is different from either of the instruments bearing this name, which are described in the treatise on optics. It consists of hairs, spiders' threads, or wires of extreme fineness, placed at the focus of the eye-glass so as to divide the field of view into several parallel portions of the same width. By this means the position of any heavenly body at the instant of its transit is determined with great precision. In order that the wires may be kept well stretched and in their places, they are fastened to a metallic plate with a circular opening called a *diaphragm*, which is attached to the interior of the telescope by means of a lateral screw, the proper position being previously determined by trial, so that the object and the wires may be seen through the eye-glass with equal distinctness. The eye-glass and object-glass of the telescope, are usually attached to tubes capable of sliding in the principal tube of the instrument, in order that the observer may regulate the distance of the glasses from each other and from the wires of the micrometer, and adapt them to his eye. In this way the eye-glass must be so adjusted, that the image of each wire may be perfectly distinct, single, and well defined. Opt. 188.

There are several kinds of wire micrometers. The most simple and the one most in use, is represented by *M*, in figure 4. It is composed of five parallel wires, and a sixth which crosses them at right angles; the plate which carries the wires, is moveable in a grove, by means of a screw. The micrometer is placed in an opening in the tube of the telescope, which allows the plate and the wires attached to it, to be moved till the centre *C* of the wires is brought into the optic axis or *line of collimation* of the instrument, that is, the straight line drawn from the centre of the eye-glass to the centre of the object-glass.

Sometimes the micrometer consists of two parallel wires, one fixed, the other moveable by means of a screw, which carries it nearer to or farther from the first, and a third wire crossing the two first at right angles. This apparatus serves to measure the apparent diameters of the heavenly bodies.

The *astronomical quadrant*, represented in figure 6, and other instruments for measuring angles, are provided with two *cross hairs* or wires, placed at the focus of the eye-glass, at right angles to each other, and whose intersection, like C in figure 4, is in the common axis of the object and eye-glass, and serves as a guide to the eye.

II.—*Note to Article* 137, *Page* 71.

Fig. 24. Let D be the distance of the heavenly body from the centre of the earth, a distance which we suppose constant while the body is rising to its greatest height. Let R be the radius of the earth, COZ the vertical, Z the zenith distance or angle LOZ, and π the parallax in altitude or angle OLC. This being supposed, in the triangle LOC the sines of the angles being proportional to the opposite sides, we have

Trig. 21

$$\sin \pi = \frac{R \sin Z}{D}.$$

Suppose now the heavenly body in the horizon, the zenith distance being 90°, we have $\sin Z = 1$; and, calling Π the corresponding value of π, that is, the horizontal parallax, we have, by the preceding formula,

$$\sin \Pi = \frac{R}{D};$$

this is the ratio of the sines to the opposite sides, in the triangle $L'OC$, in which OL' is horizontal, and a tangent to the earth at O. As the sine of an angle is always a fraction less than unity, the product $\frac{R}{D}$ into $\sin Z$, or the parallax in altitude will always be less than $\frac{R}{D}$, which is the horizontal parallax. The horizontal parallax is therefore a maximum. If we eliminate D by means of the two preceding equations, we shall obtain

$$\sin \pi = \sin \Pi \sin Z.$$

The distance of the heavenly bodies is so great that Π and π are always very small angles, even in the case of the moon, whose horizontal parallax never exceeds 1°. Now for such very small angles, the ratio of the arcs is very nearly equal to that of the sines; accordingly, by substituting $\frac{\pi}{\Pi}$ for $\frac{\sin \pi}{\sin \Pi}$ in the preceding ratio, we have

$$\pi = \Pi \sin Z;$$

that is, *the parallax in altitude is equal to the horizontal parallax multiplied by the sine of the zenith distance.* Strictly speaking, this ratio takes place between the sines of the parallaxes, but in order to judge how nearly the arcs approach to the same ratio, it is sufficient to remark, that the sine of 1° is equal to 0,01745240, and its tangent 0,01745507, radius being unity. Now, as the arc is always less than Trig. 16. its tangent and greater than its sine, it is comprehended between the two preceding expressions, and differs from either of these values less than they differ from each other. The error which we commit in substituting the arc for the sine in such a case, is less than 0,00000267, which, divided by 0,01745240, gives about $\frac{1}{6536}$ of the total value of the arc.

From the preceding expression of the sine of π, it will be seen that the parallax of a heavenly body may be calculated for all altitudes, when we know its distance from the centre of the earth in parts of the terrestrial radius, or the ratio $\frac{R}{D}$. Reciprocally, we can calculate the distance, when we know the parallax by observation.

Knowing the parallax in altitude by the preceding formula, if we call Z'' the *true* zenith distance, as observed from the centre of the earth, we shall have

$$\sin \pi = \sin \Pi \sin Z, \quad Z'' = Z - \pi.$$

These formulas suppose Z known. In astronomical tables, which are constructed for the true places, Z'' is supposed to be known, and we must deduce Z from it in order to find the apparent place. For this purpose, we must express the parallax π in a function of Z'' This is easily done; for, as Z is equal to $Z'' + \pi$, by eliminating Z, we shall have

$$\sin \pi = \sin \Pi \sin (Z' + \pi) = \sin \Pi (\sin Z' \cos \pi + \cos Z' \sin \pi); \qquad \text{Trig. 11.}$$

whence we deduce

$$\operatorname{tang} \pi = \frac{\sin \Pi \sin Z''}{1 - \sin \Pi \cos Z''}. \qquad \text{Trig. 8.}$$

As the horizontal parallax Π is always a very small arc, this expression may by be reduced to a converging series, arranged according to the powers of $\sin \Pi$. It is, therefore, only necessary to perform the division indicated, and we shall have

$$\text{tang}\,\pi = \sin \Pi \sin Z'' + \frac{\sin^2 \Pi \sin^2 Z''}{2} + \&c.$$

But the two first terms will always be sufficient.

If we would have rigorously the value of π, we can easily obtain it by means of an auxiliary angle φ, of such a value as to give

$$\cos \varphi = \sqrt{\sin \Pi \cos Z''},$$

from which we deduce

Trig. 10. $$\text{tang}\,\pi = \frac{\sin \Pi \sin Z''}{\sin^2 \varphi},$$

a formula adapted to calculation by logarithms.

III.—*Note to Article* 138, *Page* 72.

The process by which this result is obtained, is very simple. Let us consider the case of figure 25. Let π be the parallax in altitude, with respect to an observer at O, which is equal to the angle OLC; in like manner, let π' be the angle $O'LC$, or the parallax in altitude for an observer at O'; lastly, let Π be the horizontal parallax of the same body, which would be the same for the two observers, on the supposition that the earth is spherical. This being supposed, if we call Z and Z' the two zenith distances, we shall have, according to the preceding note,

$$\pi = \Pi \sin Z, \qquad \pi' = \Pi \sin Z';$$

which give, in case of figure 25,

$$\pi + \pi' = \Pi (\sin Z + \sin Z').$$

Now, $\pi + \pi'$ is simply the angle OLO', under which is seen from the centre of the heavenly body the chord of the terrestrial arc, which joins the two observers. This angle being the fourth in the quadrilateral $COLO'$, is easily calculated; indeed the angle at O is equal to $180° - Z$, and the angle at O equal to $180° - Z'$. Let φ be the angle at the centre of the earth comprehended between the verticals of the two observers, which is known by means of their latitude; the sum of the four angles of the quadrilateral being equal to four right angles, we shall have

Geo. 80. $$\varphi + 180° - Z + 180° - Z' + OLO' = 360°,$$

consequently,

$$OLO' = Z + Z' - \varphi,$$

and, since this angle OLO' is equal to $\pi + \pi'$, we shall have, putting these two values equal to each other,

$$\Pi = \frac{Z + Z' - \varphi}{\sin Z + \sin Z'};$$

on the supposition that the heavenly body is on the same side of the zenith with respect to the two observers, as in figure 26, the reasoning will be altogether similar, only the angle OLO' will no longer be $\pi' + \pi$, but $\pi' - \pi$, or $\Pi \sin (Z' - \sin Z)$, π' being the greatest of the two parallaxes; we should have moreover in the triangle OKO''

$$Z + OLO' + 180° - Z' + \varphi = 180°,$$

which gives

$$OLO' = Z' - Z - \varphi.$$

The whole is reduced, therefore, to making Z negative, as well as $\sin Z$ in the preceding value of π. This formula, in which every thing is known, will give the value of the horizontal parallax of the heavenly body. We shall then have the distance by the formula

$$\sin \Pi = \frac{R}{D},$$

which gives

$$D = \frac{R}{\sin \Pi}.$$

The angle OLO' or $Z + Z' - \varphi$, may also be deduced immediately from the difference of the observed declinations between the heavenly body and a fixed star, as remarked in the text. 139.

It will be seen in the same article, that the parallax of Mars is found, by this method, to be 24″,64 ; we have, therefore, for Mars

$$\sin \Pi = 0{,}000119458 = \frac{1}{8371{,}153},$$

and, consequently,

$$D = \frac{R}{\sin \Pi} = R\ 8371{,}15,$$

that is, the distance of Mars from the earth was then equal to 8371,15 terrestrial radii.

We can even calculate the value of D without the aid of the table of sines, for the angle Π being very small, we can substitute it for its sine without any material error; but then radius, which is here considered as unity, must be converted into parts of the same nature with Π, that is, into seconds. Now the value of the radius, thus reduced, is 206264″,8 ; accordingly, we have Top.148.

$$D = R\,\frac{206264''8}{\Pi},$$

and in the present case

$$D = 8371{,}15\,R.$$

This conversion of the radius is often resorted to in astronomy, and it will be seen, that it is necessary whenever we substitute a small arc for its sine. It is on this account, that I have made use of the present occasion to exemplify it.

IV.—*Note to Article* 143, *Page* 74.

Fig. 28. Let C be the centre of the earth, C' that of the heavenly body, Π the horizontal parallax of the heavenly body, or the angle $OC'C$, D its distance CC' from the centre of the earth; according to what precedes, we shall have

$$\sin\Pi = \frac{R}{D}.$$

$2\,\Pi$ is the apparent diameter of the earth, with respect to an observer at the heavenly body; reciprocally, the apparent diameter of the heavenly body, seen from the earth, is simply $O''CO''$, that is, double the horizontal parallax of the earth with respect to the same observer. Let R', therefore, be the radius of the heavenly body, supposed to be spherical, and Δ its observed apparent diameter, we shall have, in like manner,

$$\sin\tfrac{1}{2}\Delta = \frac{R'}{D};$$

By dividing these two equations the one by the other, we deduce

$$\frac{\sin\frac{1}{2}\Delta}{\sin\Pi} = \frac{R'}{R};$$

substituting for the ratio of the arcs, that of their sines, as we may do, since the arcs are very small, we have

$$\frac{\Delta}{2\,\Pi} = \frac{R'}{R},$$

which is the theorem enunciated in the text. We shall thus have the ratio of the radius of the heavenly body to that of the earth, and by cubing this ratio we obtain $\frac{(R')^3}{R^3}$ as the ratio of their bulks.

V.—*Note to Article* 144, *Page* 75.

To find these corrections, let us consider successively our two spherical triangles; let Z be the apparent zenith distance or ZS; Fig. 29. Δ the apparent polar distance, or PS; P the apparent horary angle, or the angle ZPS; A the azimuth of the heavenly body or the angle PZS, PZ, the third side of the triangle, will be the zenith distance of the pole, or the complement of the latitude; let us represent it by D. This being supposed, by the formulas of spherical trigonometry, the angle P may be expressed in a function of the azimuth and the sides of the triangle; for we have

$$\cot P = \frac{\cos Z \sin D - \cos A \sin Z \cos D}{\sin A \sin Z}; \quad \text{Trig. 51.}$$

whence

$$\cot P = \frac{\cot Z \sin D - \cos A \cos D}{\sin A}.$$

Let us call Z', Δ', P', the corresponding quantities in the triangle ZPS', referred to the true places; we shall have, in like manner,

$$\cot P' = \frac{\cot Z' \sin D - \cos A \cos D}{\sin A},$$

for the azimuth and the zenith distance of the pole remain the same for both triangles; subtracting one of these equations from the other, we find

$$\cot P - \cot P' = \frac{(\cot Z - \cot Z') \sin D}{\sin A}. \quad \text{Trig. 11.}$$

Substituting for the cotangents their trigonometrical expression $\frac{\cos}{\sin}$, and reducing to a common denominator, we obtain

$$\sin (P - P') = \frac{\sin (Z - Z') \sin D \sin P \sin P'}{\sin A \sin Z \sin Z'};$$

$Z - Z'$ precisely the parallax in altitude π, the sine of which is equal to $\sin \Pi \sin Z$, Π being the horizontal parallax; substituting this value, we have 137.

$$\sin (P - P') = \frac{\sin \Pi \sin D \sin P \sin P'}{\sin A \sin Z'}.$$

We can facilitate the use of this formula, by eliminating the azimuth A which is not commonly observed, and which only serves to connect our two triangles. The process is simple, for in every spherical triangle the sines of the angles are proportional to the sines of the opposite sides. Thus, in our triangle *ZPS* referred to the true places, the ratio $\frac{\sin P'}{\sin \Delta}$ is equal to $\frac{\sin Z'}{\sin \Delta'}$, Δ', being the true polar distance; we have then, by substituting this value,

$$\sin (P - P') = \frac{\sin \Pi \sin D \sin P}{\sin \Delta'}.$$

$P - P'$ is the difference between the true horary angle and the apparent horary angle. It is indeed what is called the ***parallax in right ascension;*** if we represent it by α, we shall have $P = P' + \alpha$, and by eliminating P, it will become

$$\sin \alpha = \frac{\sin \Pi \sin D \sin (P' + \alpha)}{\sin \Delta'};$$

and developing $\sin (P' + \alpha)$, we obtain

$$\tan \alpha = \frac{\dfrac{\sin \Pi \sin D \sin P'}{\sin \Delta'}}{1 - \dfrac{\sin \Pi \sin D}{\sin \Delta'} \cos P'}.$$

This formula corresponds exactly to that for the parallax in altitude. See note II. It can be developed in a series by the same method, and confining ourselves to the two first terms, which will always be sufficient, we shall have

$$\tan \alpha = \frac{\sin \Pi \sin D \sin P'}{\sin \Delta'} + \frac{\sin^2 \Pi \sin^2 D}{2 \sin^2 \Delta'} \sin^2 P' + \&c.$$

We may very often neglect even the second term, and then substituting the ratio $\frac{\alpha}{\Pi}$ for $\frac{\tan \alpha}{\sin \Pi}$, we shall have, by approximation,

$$\text{Parallax in right ascension} = \frac{\Pi \sin D \sin P'}{\sin \Delta'}.$$

Lastly, we can still find the value of tang α rigorously by a transformation analogous to that in the note above referred to, by means of an auxiliary angle φ, of such a value as to give

$$\cos \varphi = \sqrt{\frac{\sin \Pi \sin D \cos P'}{\sin \Delta'}};$$

we then obtain

$$\tan \alpha = \frac{\sin \Pi \sin D \sin P'}{\sin \Delta' \sin^2 \varphi}.$$

All these formulas, giving the parallax by means of the true places, are adapted to astronomical tables; they agree in showing that the parallax in right ascension is nothing at the meridian where P' is nothing.

The parallax in declination is found in a similar way, by comparing the polar distances in our two triangles; indeed, if we call A the azimuth SZP, reckoned from the pole, we can express the polar distances Δ, Δ', in a function of A, of D the zenith distance of the pole, and of the true or apparent horary angle, thus,

$$\cot \Delta = \frac{\cos A \sin P + \cos D \cos P \sin A}{\sin D \sin A};$$

Trig. 51.

whence

$$\cot \Delta = \frac{\cot A \sin P + \cos D \cos P}{\sin D};$$

also

$$\cot \Delta' = \frac{\cot A \sin P' + \cos D \cos P'}{\sin D},$$

subtracting one of these equations from the other, we obtain

$$\sin (\Delta - \Delta') = - \frac{\sin \Delta \sin \Delta'}{\sin D} \left(\cot A (\sin P - \sin P') + \cos D (\cos P - \cos P') \right)$$

Since the azimuth has been employed as constant, we eliminate it by substituting its value in the true triangle, derived from the formula

$$\cot A = \frac{\cot \Delta' \sin D - \cos P' \cos D}{\sin P'},$$

which is perfectly similar to those above obtained, and belongs to a spherical triangle in which we know two sides Δ' and D, and their included angle P'; by means of this expression, the preceding formula becomes

$$\sin(\Delta - \Delta') = \sin \Delta \left(\frac{\sin \Delta' \cos D \sin (P - P')}{\sin D \sin P'} - \frac{2 \cos \Delta' \cos \frac{1}{2}(P + P') \sin \frac{1}{2}(P - P')}{\sin P'} \right)$$

We have called α the parallax in right ascension $P - P'$; let us now call δ the parallax in declination $\Delta - \Delta'$; we shall have $\Delta = \Delta' + \delta$ as we had $P = P' + \alpha$; and hence the formula becomes

$$\sin \delta = \left(\frac{\sin \Delta' \cos D \sin \alpha}{\sin D \sin P'} - \frac{2 \cos \Delta' \cos (P' + \frac{1}{2}\alpha) \sin \frac{1}{2} \alpha}{\sin P'} \right) \sin (\Delta' + \delta);$$

substituting, in the first term, for sin α its value

$$\frac{\sin \Pi \sin D \sin (P' + \alpha)}{\sin \Delta},$$

which will simplify it a little, we shall have

$$\sin \delta = \left(\frac{\sin \Pi \cos D \sin (P' + \alpha) - 2 \cos \Delta' \cos (P' + \tfrac{1}{2} \alpha) \sin \tfrac{1}{2} \alpha}{\sin P'}\right) \sin (\Delta' + \delta).$$

The coefficient contained within the parentheses, depends simply on the true places, and will be entirely known as soon as we shall have calculated the parallax in right ascension; representing it by Q, the preceding formula will become

$$\sin \delta = Q \sin (\Delta' + \delta).$$

It is then still analogous to that which gave us the parallax in altitude, as well as the parallax in right ascension; it may be resolved in the same manner, either exactly, by means of an auxiliary angle, or approximately, by means of the series

$$\text{tang } \delta = Q \sin \Delta' + \frac{Q^2}{2} \sin 2 \Delta' + \&c.$$

In practice we can always, without any sensible error, substitute the ratios of the arcs α, δ, and Π, for those of their sines and tangents; which amounts to neglecting the cubes of the parallaxes. We can then take for $\frac{1}{2} \alpha$, the value $\frac{1}{2} \frac{\Pi \sin D \sin (P' + \alpha)}{\sin \Delta'}$. Lastly, if we take only the first power of the parallax, which will always be sufficient, we can make α and δ nothing in the terms of the second member, which have been already multipled by Π, and then it becomes

$$\text{Parallax in declination} = \Pi (\sin \Delta' \cos D - \cos \Delta' \sin D \cos P').$$

VI.—*Note to Article* 146, *Page* 76.

In general, let Π' be the horizontal parallax of a heavenly body at the equator, Π its horizontal parallax in latitude L; calling D the distance of the heavenly body, A the radius of the equator, and R the terrestrial radius at the latitude L, we shall have

$$\sin \Pi' = \frac{A}{D}, \qquad \sin \Pi = \frac{R}{D},$$

from which we obtain

$$\sin \Pi = \frac{R}{A} \sin \Pi';$$

or, substituting the ratio of the angles Π' and Π for that of their sines

$$\Pi = \frac{R}{A}\,\Pi'.$$

By means of this formula, we see that the equatorial parallax is the greatest of all, and the polar the smallest. In general, when we know Π', we can calculate Π by means of the values R.† It will be seen, moreover, that it is sufficient to know Π, by observation, with respect to any single latitude, in order to deduce from it the value of Π', or the equatorial parallax.

VII.—*Note to Article* 149, *Page* 77.

Let Z be the apparent observed zenith distance, Z' that distance reduced to the true zenith, and Z'' the distance Z' reduced to the centre of the earth. Let us call ω the angle ZOZ' formed by the radius with the vertical, and π the parallax in altitude for the distance Z'. This being supposed, if the heavenly body is in the meridian of the place on the equator, we shall have Fig. 34

$$Z = Z - \omega; \quad Z'' = Z' - \pi; \quad \sin\pi = \sin\Pi \sin Z.$$

Π is here the horizontal parallax for the radius OC. These formulas suppose Z to be known; but if it was the true distance Z'' which was given, we could easily obtain Z in the manner already described; for the two last equations would give π, in a function of Z'', by the series referred to, and then we should hence deduce

$$Z = Z' + \pi; \; Z = Z' + \omega = Z' + \omega + \pi.$$

ω must have been taken negative, if the heavenly body had been observed on the side toward the pole. It must not be forgotten that these formulas are applicable only in the plane of the meridian.

The calculation of the parallax, by two observations made on the meridian at great distances, is just as easy in the case of the spheroid as in that of the sphere; for, denoting always the positions of the two observers by O, O', figure 31, analogous to figure 25, the quadrilateral $COLO'$ will present the same relations. Only as the observed zenith distances are LOZ, $LO'Z'$, about the verticals ON, ON', we must, in order to obtain the distances $LOZ_{,}$, $LOZ_{,}$, relative to the true zenith, subtract the angles formed by the radii CO, CO', with the corresponding verticals, angles which we shall call ω, ω', and which can be calculated by the formula given below.† Thus, calling always π, π', the angles CLO, CLO', or the parallaxes in altitude; R, R' the two terrestrial radii at O and O'; Π, Π', the two horizontal parallaxes, and D the distance CL of the heavenly body from the centre of the earth, we shall have, as before,

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† On the supposition that the terrestrial meridian is an ellipse, and that the oblateness is $\frac{1}{308{,}65}$, the length of a degree, in any latitude L, according to Delambre, is equal to $\overset{\text{metre.}}{11111{,}111}, - \overset{\text{metre.}}{540{,}848} \cos 2L$, and $\log R = 6{,}8038861 + 0{,}0007150 \cos 2L$. But the above metre is to the legal metre as 1 is to 1,0000723; and the legal metre is equal to 39,371 English inches.

$$\sin \Pi = \frac{R}{D}; \ \sin \Pi' = \frac{R'}{D}; \ \pi = \Pi \sin (Z - \omega); \ \pi' = \Pi' \sin (Z - \omega').$$

We understand here by ***horizontal parallax***, that which takes place when the visual ray is perpendicular to the terrestrial radius. The visual ray is not then a tangent to the spheroid; so that it would be more proper to call the angle in question ***the greatest parallax***, or the ***maximum parallax***; but custom has prevailed. The ratio of the sines of these parallaxes being very nearly the same as that of the arcs, we have $\frac{\Pi'}{\Pi} = \frac{R'}{R}$, that is, they are proportional to the terrestrial radii, and availing ourselves of this ratio to eliminate Π', if we add together the expressions for π and π', we shall have

$$\pi + \pi' = \Pi \left(\frac{R \sin (Z - \omega) + R' \sin (Z' - \omega')}{R} \right).$$

But the sum of the parallaxes $\pi + \pi'$, is equal to the angle OLO'; moreover, the sum of the four angles of the quadrilateral $COLO'$, must always be equal to four right angles; hence we shall have as before

$$OLO' = \pi + \pi' = Z - \omega + Z' - \omega' - \varphi;$$

φ being the angle OCO', formed by the two terrestrial radii. Let us produce the normals ON, ON', till they meet the straight line OE which represents the plane of the equator; the angles ENO, $EN'O'$, formed by the normals with this plane, will be the latitudes of the two observers; we shall call them λ, λ'; now these angles being the exterior angles of the triangles NOC, $N'O'C$, we shall evidently have

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† L being the latitude, and $\frac{1}{308,65}$ the oblateness,

$$\omega = 11' \, 9'',39 \sin 2L - 1'',086 \sin 4L.$$

$$\lambda = NCO + \omega, \quad \lambda' = N'CO' + \omega';$$

whence we obtain

$$NCO + N'CO' = \varphi = \lambda + \lambda' - \omega - \omega';$$

a value, which, being substituted for φ in the expression for OLO', gives

$$OLO' = Z + Z - \lambda - \lambda';$$

a result altogether analogous to that which takes place on the sphere. This value of OLO or of $\pi + \pi'$, being equal to the first which we obtained, we deduce from it the value of the *maximum* parallax for the observer O, which will be

$$\Pi = \frac{R\,(Z + Z' - \lambda - \lambda')}{R \sin\,(Z - \omega) + R' \sin\,(Z' - \omega')}.$$

For the observer O', we should have

$$\Pi' = \frac{R'\,(Z + Z' - \lambda - \lambda')}{R \sin\,(Z - \omega) + R' \sin\,(Z - \omega')}.$$

These expressions are altogether analogous to those before obtained, and they are not more difficult to calculate. We have supposed, in the calculation, that the two observers were situated on opposite sides of the equator; if they were on the same side, we should have to regard the least latitude as negative in the preceding formula, and also make the corresponding value of ω or of ω' negative. In this case, if the heavenly body, instead of being situated between the two zeniths, were situated on the same side of the zenith, with respect to the two observers, we should be obliged, moreover, to regard as negative the least of the two zenith distances.

VIII.—*Note to Article* 150, *Page* 78.

Let A be the apparent azimuth of the heavenly body, reckoned from the pole. This will be the supplement of the angle $Z'ZS$; let Z be the observed distance from the apparent zenith, Z' the apparent distance from the true zenith, ω the angle formed by the radius with the vertical. This being supposed, if we observe that $\cos\,(180^\circ - A) = -\cos A$, the spherical triangle $ZZ'S$ will give Fig. 38.

$$\cos Z' = -\sin Z \sin \omega \cos A + \cos \omega \cos Z;$$ Trig. 47.

taking an auxiliary angle φ, such as to give

$$\text{tang}\ \varphi = \cos A\ \text{tang}\ \omega,$$

we shall have

$$\cos Z' = \frac{\cos \omega}{\cos \varphi} \cos (Z + \varphi).$$

If we would determine the azimuth by means of the observed horary angle, which is easier than to observe it, nothing is more simple. For, if we imagine a new spherical triangle *ZPS*, figure 34, referred to the three visual rays drawn from the observer to the pole, to the apparent zenith, and to the heavenly body, we shall know, in this triangle, the observed apparent distance or Z, the distance from the pole to the apparent zenith or D; lastly, the apparent horary angle or P. The angle *SZP*, comprehended between the two sides Z and D of this triangle, is exactly equal to the azimuth A. We can then calculate this azimuth by combining the following formulas deduced from spherical trigonometry,

$$\text{tang}\, \psi = \cos D \,\text{tang}\, P, \qquad \sin (A + \psi) = \frac{\sin \psi \,\text{tang}\, D}{\text{tang}\, Z};$$

in which ψ is an auxiliary angle. When we know A by these formulas, the two first equations will make known Z', or the apparent distance of the true zenith. The parallax for this distance, or $\Pi \sin Z'$, will thus be known. Subtracting it from Z', we shall have the true zenith distance $Z'' = Z' - \Pi \sin Z'$.

If, on the contrary, Z'' were given, we might hence deduce Z' by means of the formulas of note III, in which π would represent the parallax in altitude $Z' - Z''$, referred to the true zenith. Then we might obtain Z, by resolving the spherical triangle $SZ'Z$ in which the angle Z', the true azimuth of the heavenly body, might be easily calculated by the elements of its true place. But there is another method much more simple for finding Z directly. It will be seen in the following note.

When Z is very considerable with respect to ω, we can easily obtain the value of $Z' - Z$ in a converging series arranged according to the powers of $\sin \omega$. But, generally speaking, this reduction is not possible, since the arcs Z, Z', ω, can be compared together, when the body L is observed very near the true or apparent zenith.

IX.—*Note to Article* 151, *Page* 78.

According to what we have said, calling always ω the angle formed by the terrestrial radius with the vertical, and denoting by α the parallax in right ascension, and by δ the parallax in declination, the formulas investigated in note v, the earth being supposed to be spherical, will give, on the supposition that it is spheroidal,

$$\sin \alpha = \frac{\sin \Pi \sin (D + \omega) \sin (P' + \alpha)}{\sin \Delta'}$$

$$\sin \delta = \left(\frac{\sin \Pi \cos (D + \omega) \sin (P' + \alpha) - 2 \cos \Delta' \cos (P' + \tfrac{1}{2} \alpha) \sin \tfrac{1}{2} \alpha}{\sin P'} \right) \sin (\Delta' + \delta)$$

or, with an approximation almost always sufficient,

$$\alpha = \frac{\Pi \sin (D + \omega) \sin P'}{\sin \Delta'},$$

$$\delta = \Pi (\sin \Delta' \cos (D + \omega) - \cos \Delta' \sin (D + \omega) \cos P').$$

Δ' is the true polar distance, and P' the true horary angle as seen from the centre of the earth. α and δ being calculated by these formulas, if we call Δ the apparent polar distance, and P the apparent horary angle, seen from the surface of the terrestrial spheroid, we shall have

$$P = P' + \alpha, \qquad \Delta = \Delta' + \delta;$$

with P, Δ, and D, we can calculate directly the apparent distance Z. For, if we suppose the spherical triangle SZP, figure 34, referred to the three visual rays, drawn from the observer to the apparent zenith, to the heavenly body, and to the pole, we shall know in this triangle the side $PS = \Delta$, $PZ = D$, and the angle $SPZ = P$. We can then easily determine the third side, or the apparent zenith distance $ZS = Z$. We can likewise calculate the angle SZP or the apparent azimuth of the heavenly body. We shall thus have all the elements of the apparent place of the heavenly body, in the most simple manner, by means of the elements of its true place. In like manner, if we suppose the spherical triangle $S'ZP$ referred to the three visual rays, drawn from the centre of the earth to the true zenith, to the heavenly body, and to the pole, we shall know, in this triangle, the sides $S'P = \Delta'$, $Z'P = D + \omega$, and the angle $S'PZ' = P'$. We can then calculate the third side $S'Z' = Z'$, from which we shall be able to determine Z' or ZS, by the formulas given in note II. We can also calculate the angle $S'Z'P$ which is the true azimuth of the heavenly body. It will be seen that the circumstance of the earth's oblateness does not render these results at all more complicated.

X.—*Note to Article* 152, *Page* 78.

Fig. 32. Let us call r the semidiameter OC of the earth, R the radius CL, or CZ, drawn from its centre to that of the moon. The line OL, the side of the right-angled triangle COL, will have for its length $\sqrt{R^2 - r^2}$; the line OZ will have for its length $R - r$; and the difference of these two lines, or the excess of OL above OZ, will be expressed by

$$\sqrt{R^2 - r^2} - (R - r);$$

if we suppose, as is the fact, that r is a very small fraction compared with R, it will be sufficient to extract, by approximation, the indicated square root, employing for this purpose, the binomial formula of Newton; and, rejecting all but the first powers of R, we shall have for the root in question $R - \frac{r^2}{2R}$. The above expression, which represents the difference of the lines OZ and OL, will then be expressed by $r - \frac{r^2}{2R}$, that is, it is nearly equal to r, for the term $-\frac{r^2}{2R}$ is very small, in comparison with r, since it is equal to the product of r by $\frac{r}{2R}$, which is a very small fraction, and differs but little from $\frac{1}{120}$.

XI.—*Note to Article* 173, *Page* 98.

Let D be the sun's declination, observed near the time of the solstice, ω the obliquity of the ecliptic, and L the longitude of the sun corresponding to the declination D; then the right-angled spherical triangle, formed by the longitude, the declination, and right ascension, gives

Top. 82.

$$\sin D = \sin \omega \sin L.$$

If the observation were made at the moment of the solstice, the declination of the sun would be exactly equal to the obliquity of the ecliptic, and the longitude equal to 90°. We should then have $D = \omega$, and $L = 90°$; but since, by supposition, the observation is made near the time of the solstice, we shall have

$$D = \omega - D', \quad L = 90° - L',$$

D' and L' being very small quantities. But substituting these values in the preceding equation, it becomes

$$\sin \omega \cos D' - \cos \omega \sin D' = \sin \omega \cos L'.$$ Trig. 11.

By putting for cos D' and cos L' their values $1 - 2 \sin^2 \frac{1}{2} D'$, and $1 - 2 \sin^2 \frac{1}{2} L'$, changing all the signs of the equation, and dividing by cos ω, we have Trig. 27, 4.

$$2 \tan \omega \sin^2 \tfrac{1}{2} D' + \sin D' = 2 \tan \omega \sin^2 \tfrac{1}{2} L'.$$

It is evident from inspection that sin D' is a very small fraction of the order of $\sin^2 \frac{1}{2} L'$; consequently, $\sin^2 \frac{1}{2} D'$ is a fraction much smaller. And by neglecting it, we have very nearly

$$\sin D' = 2 \tan \omega \sin^2 \tfrac{1}{2} L'.$$

But if we would obtain a nearer approximation, which is sometimes necessary when the observed declination is distant eight or nine degrees from the solstice; we have only to consider that the exact value of sin D' is

$$\sin D' = 2 \tan \omega \sin^2 \tfrac{1}{2} L' - 2 \tan \omega \sin^2 \tfrac{1}{2} D'.$$

If the value of sin D' given by the first term only, is not perfectly exact, it will at least be near enough to be substituted in the last term $2 \tan \omega \sin^2 \frac{1}{2} D'$, which is very much smaller than the first. We can even, in this substitution, take for $\sin \frac{1}{2} D'$ the value $\frac{1}{2} \sin D'$, the arcs, when very small, differing but little from their sines; we shall then have with respect to the last term, $\sin \frac{1}{2} D' = \tan \omega \sin^2 \frac{1}{2} L'$; and this value being substituted in the preceding equation, we shall have, by this second approximation,

$$\sin D' = 2 \tan \omega \sin^2 \tfrac{1}{2} L' - 2 \tan^3 \omega \sin^4 \tfrac{1}{2} L'.$$

This value of sin D' will be sufficiently exact, even when $L' = 18°$.

By means of this formula, an observation of the meridian altitude of the sun, made before or after the solstice, may be readily reduced to this epoch; the observation, however, should be near the solstice. In order to this we calculate, by means of the astronomical tables already formed, the sun's longitude at the time of the observation; this will be the value of L; substracting this from 90°, the remainder will be L', or the sun's distance from the solstice at the time of the observation. Knowing L', we have D' by the preceding formula; and D' added to the observed declination will give the solsticial declination $D + D'$, precisely as if it had been observed at the moment of the solstice.

It is evident, by proceeding in this way, that the only uncertainty will be in the value of L', given by the tables, which we suppose imperfect. Suppose then, that this value, instead of being L', were $L' + e = L''$, e being the error arising from the imperfection of the tables; in order to know what will be the effect upon the value of D', let us limit ourselves to the first term of the value of sin D', which gives

$$\sin D' = 2 \operatorname{tang} \omega \sin^2 \tfrac{1}{2} L'.$$

Now, if we suppose that the substitution of L'' for L', changes D' into D'', we shall also have

$$\sin D'' = 2 \operatorname{tang} \omega \sin^2 \tfrac{1}{2} L''.$$

Subtracting the first of these equations from the second, and putting for the difference of the sines, its value in a function of the difference of the arcs, we shall have

$$\sin \tfrac{1}{2}(D'' - D') \cos \tfrac{1}{2}(D'' + D') = \operatorname{tang} \omega \sin \tfrac{1}{2}(L' - L'') \sin \tfrac{1}{2}(L + L').$$

$D'' - D'$ is the error in the solsticial declination corresponding to the error $L'' - L'$ in the longitude. If we limit ourselves to the first powers of these small arcs, we may substitute their ratio for that of their sines. We may then put $L'' = L'$ and $D'' = D'$, in the other terms of the equation; we shall thus have

$$D'' - D' = \frac{e \operatorname{tang} \omega \sin L''}{\cos D''}.$$

This expression may be somewhat simplified by substituting unity for the denominator cos D''; since, in the calculation, we confine ourselves to the first power of sin D'', and as $\cos D'' = 1 - 2 \sin^2 \tfrac{1}{2} D''$, it will be seen that the difference between cos D'' and unity is of the order of terms which we propose to neglect. We shall then have, by an approximation sufficiently exact,

$$D'' - D' = e \operatorname{tang} \omega \sin L''.$$

It will be seen by this formula that the error in the declination is much less than the error e of the longitude, since this is multiplied by the factor sin L'', which is very small in observations made near the solstice, and which becomes nothing at the solstice itself. To appreciate the influence of this, suppose that we have only imperfect tables for determining the value of L'', which are liable to an error of half a day in the time of the solstice, and consequently an error of 0°,495 or 29′ 42″, in the longitude; so that $e = 29' 42''$; it will then be necessary to make the observation as near as possible to the solstice, for instance, on the very day it takes place. In this case the distance from the solstice being near half a day, it is clear that L'' will be nearly equal to 29′ 42″, by supposing the daily motion of the sun equal to 59′ 24″, as we have done in the text. By these suppositions we have

$$D'' - D' = 0^\circ\ 29'\ 42''\ \sin 29'\ 42''\ \text{tang}\ 23^\circ\ 27'\ 53'',$$

taking 23° 27′ 53″ for the obliquity of the ecliptic. The numerical value of this formula derived from the trigonometrical tables is

$$D'' - D' = 6'',7.$$

But if instead of making use of such tables in calculating the value of L'', we were to employ others less imperfect and liable to an error only $\frac{1}{100}$ part as great or 0°,00495, the error in the solsticial declination, resulting from an observation made on the day of the solstice, would become less in the same proportion; it will thus be reduced to 0″,067, a quantity nearly insensible.

An error of 0°,00495, in the sun's longitude, answers to 7¼ minutes, or 432 seconds in time. With respect to the astronomical tables in actual use the error in time rarely amounts to 4 seconds; the error e, in the longitude, is accordingly reduced in the same proportion; so that the distance from the solstice being the same, it becomes only $\frac{1}{100}$ of the above; and the corresponding error in the solsticial declination would amount only to 0″,00067.

XII.—*Note to Article* 187, *Page* 107.

We have referred in the text to article 105 of Topography for the method of finding the latitude and longitude of a heavenly body, when the declination and right ascension are known. We shall here give general formulas adapted to the same purpose that will often be found more convenient.

Through the centre of the celestial sphere draw the two visual rays CP, CP', the first to the pole of the equator, the second to the Fig. 39. pole of the ecliptic. The angle $P'CP$ will be equal to the obliquity of the ecliptic to the equator. Produce indefinitely the circle of 169. declination AS, and the circle of latitude SL. The first will pass through the pole P, and the second through the pole P'; so that in the spherical triangle $P'PS$, $P'P$ will be the obliquity of the ecliptic, PS the complement of the declination, and $P'S$ the complement of the latitude; since the arcs PA, $P'L$, are each 90°. Let us now attend to the angles at P and P'. If we draw through the pole P the circle of declination $P♈$, which passes through the equinox, the angle $♈PP'$ will be a right angle; for the plane of the circle $P'P$ or $CP'P$, being perpendicular to the line of the equinoxes, all the planes which pass through this line will be perpendicular to that of $P'P$. Moreover, the arcs $P♈$, PA, being each 90°, the angle $♈PA$ will have for its measure the arc $♈A$, and will consequently be equal to the right ascension of the heavenly body S; thus the whole angle $P'PS$ will be equal to $90^\circ + ♈A$. In like manner, if through the pole P' we draw the circle of latitude $P'♈$, the angle $♈P'P$ will be a right angle, and the arcs $P'♈$, $P'L$, will be each 90°. The angle $♈P'L$ will, therefore, have for its measure the arc $♈L$ reckoned on the ecliptic, and will be equal to the longitude of the heavenly body. Consequently, the angle $SP'P$ will be equal to $90 - ♈L$, that is, it will be equal to the complement of the longitude.

Let ω be the obliquity of the ecliptic, d the declination of the heavenly body, a its right ascension, r its latitude, l its longitude. Then, if the right ascension and declination are known, we have the longitude and latitude by the following formulas

Trig. 47.
$$\cos P'S = \sin P'P \sin SP \cos P'PS + \cos P'P \cos SP,$$

Trig. 51.
$$\cot PP'S = \frac{\cot SP \sin P'P - \cos P'PS \cos P'P}{\sin P'PS}.$$

By adopting the denominations stated above, and observing that $\cos P'PS = \cos (90^\circ + a) = - \sin a$, we have

$$\sin \lambda = - \sin \omega \cos d \sin a + \cos \omega \sin d,$$

$$\operatorname{tang} l = \frac{\operatorname{tang} d \sin \omega + \sin a \cos \omega}{\cos a}.$$

These two formulas may be adapted to calculation by logarithms, if we take an auxiliary angle φ, such that

$$(1.) \qquad \operatorname{tang} \varphi = \frac{\sin a}{\operatorname{tang} d};$$

for, by eliminating $\sin a$ from the first, and $\operatorname{tang} d$ from the second, they become

$$(2.) \qquad \sin \lambda = \sin d \, \frac{\cos (\varphi + \omega)}{\cos \varphi},$$

$$(3.) \qquad \operatorname{tang} l = \operatorname{tang} a \, \frac{\sin (\varphi + \omega)}{\sin \varphi};$$

on the contrary, if the latitude and longitude are given, we have the declination and right ascension by these formulas,

Trig. 48.
$$\cos PS = \sin P'P \sin P'S \cos PP'S + \cos P'P \cos P'S,$$

$$\cot P'PS = \frac{\cot P'S \sin P'P - \cos PP'S \cos P'P}{\sin PP'S}.$$

Or, by putting for these quantities their values, and observing that $\cot P'PS = -\operatorname{tang} a$, we have

$$\sin d = \sin \omega \cos \lambda \sin l + \cos \omega \sin \lambda,$$

$$\operatorname{tang} a = \frac{-\operatorname{tang} \lambda \sin \omega + \sin l \cos \omega}{\cos l}.$$

These equations are similar to those which we gave at first for the latitude and longitude. The latitude answers to the declination, and the longitude to the right ascension; the only difference is in the obliquity ω, which has here a negative value. Following this analogy, we can resolve these new equations by means of an auxiliary angle φ', such that

$$(1.) \qquad \operatorname{tang} \varphi' = \frac{\sin l}{\operatorname{tang} \lambda};$$

for, by eliminating $\sin l$ from the first, and $\operatorname{tang} \lambda$ from the second, we have

$$(2.) \qquad \sin d = \frac{\sin \lambda \cos (\varphi' - \omega)}{\cos \varphi'},$$

$$(3.) \qquad \operatorname{tang} a = \operatorname{tang} l \, \frac{\sin (\varphi' - \omega)}{\sin \varphi'}.$$

If we would obtain the angle of position *PSP'*, or *S*, we can easily find it by recollecting that in every spherical triangle the sines of the sides are as the sines of the opposite angles, which gives

$$\sin S = \frac{\sin \omega \cos a}{\cos \lambda}, \quad \text{or} \quad \sin S = \frac{\sin \omega \cos l}{\cos d}.$$

These formulas are calculated on the supposition that the body in question is in the first quadrant of the ecliptic, reckoning from the vernal equinox; but to extend them to all other positions, it will be sufficient to observe carefully the use of the signs plus and minus, making the sines, cosines, and tangents, positive or negative, according to the value of the arcs to which they correspond. There is no difficulty with respect to the right ascension and longitude, which are both reckoned from the same point, and always in the same direction from 0° to 360°; but with respect to the declination and latitude, which are reckoned from a plane, since they are considered as positive on one side, they should be regarded as negative on the other. That is, since we have considered the northern declinations as positive, we must consider the southern declinations as negative. Fig. 40.

XIII.—*Note to Article* 188, *Page* 108.

Fig. 40. Let L be the longitude of the zenith, Λ its latitude referred to the plane of the ecliptic; let l and λ represent the corresponding quantities for any observed heavenly body, these quantities being taken at the surface of the earth. If it be supposed, in the figure, that the heavenly body is west of the zenith, beyond the plane EZP', and that the equinoctial point ♈ is also beyond this plane to the west of the heavenly body, the difference of longitude $L - l$, will represent the angle $S'P'Z$, reckoned from the apparent meridian and from east to west, like the horary angles. This angle is analogous to the apparent horary angle P in the case in which the right ascension and declination were considered. To complete the analogy, let D' be the distance of the zenith from the pole of the ecliptic, P the difference of the apparent longitudes $L - l$, and Δ the apparent distance of the heavenly body from the same pole, which gives $D' = 90^\circ - \Lambda$, and $\Delta = 90^\circ - \lambda$. It will be sufficient to substitute these quantities for the corresponding ones in the formulas of pp. 372, 374, and mark with an accent the elements of the true place, as we have done there; we shall then have, by approximation,

$$\text{Parallax in longitude } P - P' = \frac{\Pi \sin D' \sin P'}{\sin \Delta'},$$

$$\text{Parallax in latitude } \Delta - \Delta' = \Pi(\sin \Delta' \cos D' - \cos \Delta' \sin D' \cos P');$$

and, as $P - P' = L - l - (L - l') = l' - l$, we shall have the apparent longitude l, and the apparent distance of the heavenly body from the pole of the ecliptic, or Δ, by the following formulas,

$$l = l' - \frac{\Pi \sin D' \sin(L - l')}{\sin \Delta'},$$

$$\Delta = \Delta' + \Pi(\sin \Delta' \cos D' - \cos \Delta' \sin D' \cos(L - l')).$$

Π is the horizontal parallax at the place of observation. For the sake of simplicity in the calculation, we have retained the polar distances in these formulas, which, as they may be reckoned from 0° to 90°, require, in the different cases, no particular care, except with regard to the algebraic signs in the trigonometrical expressions. In this respect they are much more convenient than the declinations and latitudes, which, being reckoned from a plane, require to be made positive on one side and negative on the other. As to the difference of longitude $l' - L$, the use of it occasions no difficulty and requires no particular construction, provided we always reckon the longitudes l' and L from west to east, referring to the same equinoctial point from which we reckon the right ascension on the equator. Finally the distance D from the zenith to the pole of the ecliptic, must not be reckoned from the vertical, but from the true zenith, determined by prolonging the radius of the earth as we did for the parallax in right ascension and declination.

A careful attention to these general rules will suffice for any particular case. It will also be seen from the above expressions, that the parallax must be added to the elements of the true place with the sign which the formula gives, to obtain the elements of the apparent place. It should not be forgotten that the preceding formulas are only approximate, and that recourse must be had to a series in order to obtain exact ones.

XIV.—*Note to Article* 192, *Page* 113.

To express the secular variation of the obliquity of the ecliptic, we begin from the commencement of the year 1750, which was rendered memorable by the labors of Lacaille, and has become the origin of almost all astronomical determinations. The mean obliquity of the ecliptic, that is, the obliquity corrected for the small oscillations comprehended in short periods, was, at that time, $23°,4698$. Let V' express the value after a number t of Julian years, reckoned from that epoch, t being considered as negative for any previous period; this being supposed, by having regard only to the secular change of the obliquity, we shall have, by the formulas of the *Mécanique Céleste*, tom. iii. p. 158,

$$V' = 23°,4731 - (0°,93154) \sin t\, 32'',1157 - (0°,66179) \sin^2 t\, 6'',9732.$$

At the epoch of 1750, we have $t = 0$. All the terms vanish except the first, and there remains $V' = 23°,4698$; as is evident, since this epoch is taken for the origin. At the time of Hipparchus, 128 years before the Christian era, we have

$$t = -(1750 + 128) = -1878.$$

By employing this value we find

$$V' = 23°,4731 + 0°,26534.$$

The mean obliquity, therefore, at that time, exceeded that of 1750 by $0°,26534$ or $15'\ 55',2$.

Making t vary by unity in the expression for V', we shall have the annual change of the mean obliquity in different centuries, which will be

$$-0'',0805 \sin t\, 13'',9465 - 0'',5211 \cos t\, 32'',1158.$$

This expression being negative when t is positive, indicates a diminution of the obliquity. At the commencement of 1750, we have $t = 0$. The preceding formula gives, for the annual diminution, $0'',5210892$, or $52'',10892$ for a century.

We come now to the periodical inequalities. Let V' be the value of the obliquity at the commencement of any year, and D' the annual diminution for the same year, V' and D' being calculated from the preceding formulas. This being premised, after the lapse of a number n of days the apparent obliquity E' will be

$$E' = V' - \frac{D' n}{365,25} + 0'',4345 \cos 2L + 9'',63 \cos N,$$

L being the longitude of the sun, and N the longitude of the *moon's ascending node*, that is, the point of its orbit where it crosses the ecliptic, as it passes from the south towards the north. This node moves upon the ecliptic, and makes an entire revolution in eighteen years; which constitutes the period of what is called the *lunar nutation*. The obliquity is found to have another small oscillation depending upon the position of the sun in the ecliptic. The period of this is evidently a half year, since, after this interval of time, the longitude of the sun having increased to 180°, the arc $2L$ will have increased to 360°; whence it follows that $\cos 2L$ resumes periodically the same value. This second inequality is occasioned by the action of the sun producing upon the equator a libration analogous to that which the action of the moon produces; it is called, by analogy, the *solar nutation*, as the other is called the *lunar nutation*. We have given these explanations and formulas, that we might leave nothing vague in relation to the inequalities of which we have spoken; and also that, in future when we shall have developed their laws by observation, the expression for them may be found for the places to which they are naturally referred.

XV.—*Note to Article* 196, *Page* 115.

Let T be the tropical revolution which is given, S the sidereal revolution which is unknown. Since the equinoxes retrograde $50'',1$ in a tropical year, their motion in a sidereal year will be $50'',1\,\frac{S}{T}$. Such then is the arc which the sun will have to describe over and above 360° to complete a sidereal year. The time employed for this, in virtue of the sun's mean motion, will be expressed by $50'',1 \times \frac{S}{T} \times \frac{T}{360^\circ}$, or simply $\frac{50'',1}{360^\circ} \times S$. The sidereal year

S will therefore be equal to a tropical year plus this quantity, that is, we shall have

$$S = T + \frac{50'',1}{360^\circ} \times S\text{; therefore } S = \frac{T}{1 - \frac{50'',1}{360^\circ}}.$$

By reducing the second member to a series, it becomes

$$S = T + \frac{T.50'',1}{360^\circ} + T\left(\frac{50'',1}{360^\circ}\right)^2 \text{ \&c.}$$

The two first terms express the value given in the text. The third term will be but asm all fraction of a second.

XVI.—*Note to Article* 201, *Page* 121.

To obtain the motion of the equinoxes upon the ecliptic, it is necessary to refer them to some fixed line in this plane. We take for the origin the position of the equinoctial point ♈, at the commencement of the year 1750, and call $+\psi'$ the quantity which the equinox will have retrograded upon the ecliptic from this point, after a number t of years; t must be taken negatively for any previous period. This being supposed, by regarding only the secular part of the precession, we shall have, from the formulas of the *Mécanique Céleste*, tom. iii, p. 158,

$$\psi' = t\,50'',412 - 1^\circ,2854 \sin t\,13'',94645 + 5^\circ,59834 \sin^2 t\,16'',05788.$$

By making $t = 0$, ψ' becomes zero as it ought. By making $t = -1878$, which carries it back to the time of Hipparchus, 128 years before the Christian era, we have

$$\psi' = -26^\circ,29829 + 0^\circ,16278 + 0^\circ,11882 = -26^\circ,01669.$$

This result being negative, indicates that the equinoctial point was then to the east of the equinox of 1750, and shows that the equinoxes retrograded 26° 1′ 0″, from this ancient epoch down to 1750.

The annual value of the mean precession for different centuries is, from the same formulas,

$$= 60'',899 + 1'',4973 \sin t\,32'',1158 + 0'',6257 \sin^2 t\,6'',9732.$$

This is the value by which ψ' varies when t is increased by unity. In 1750, we have $t = 0$, the precession was then 50″,0942128. The change of the annual precession since 1750, is expressed by the other terms of the formula. It is, therefore, equal to

$$1'',5693 \sin t\,32'',1158 + 0'',6257 \sin^2 t\,6'',973.$$

Both terms of this expression are positive when t is positive. The annual precession, therefore, increases continually from century to century; whence it follows that the tropical year is diminished. To estimate this diminution, it is necessary to reduce the preceding expression to time, according to the ratio of the entire circumference to the tropical year; that is, it is necessary to multiply it by $\frac{365^{d},242264}{360^{\circ}}$; when it will be expressed in days, and its value will be

$$0^{d},000442 \sin t\ 32'',11575 + 0^{d},00017632 \sin^{2} t\ 6'',97323.$$

For periods preceding 1750, t changing its sign, the value of this expression becomes negative, and then it shows the increase of the year. By thus making $t = -1878$, which answers to the time of Hipparchus, we shall find 4'',15368 for the excess of the tropical year at that time, over that of 1750, that is, 11'',076 in time. By making the same supposition in the formula which expresses the change of the annual precession in parts of a degree, we shall find 0'',4549, for the corresponding arc; this is the quantity by which the annual precession, in 1750, exceeded that at the time of Hipparchus. This last, therefore, was 49'',6443.

As we have said that the extent of the oscillations of the ecliptic is not very exactly determined, it will be perceived that the preceding formulas can only be approximations. But the approximation extends to 2000 years before and after the epoch of 1750, which is employed as the origin.

We come now to the periodical inequalities. Let ψ' be the mean retrogradation of the equinoctial point ♈, from 1750 to the commencement of any year whatever, and M the value of the annual precession at that time; ψ' and M being calculated by the preceding formulas. This being premised, after the lapse of a number n of days, the retrogradation of the equinoctial point will be

$$\psi' + \frac{Mn}{365,25} - \frac{0'',4345}{\operatorname{tang} \omega} \times \sin 2L - \frac{19'',26}{\operatorname{tang} 2\omega} \times \sin N;$$

ω being the obliquity of the ecliptic, L the longitude of the sun, N the *ascending node of the moon's orbit.* The two last inequalities proceed from the phenomena of the solar and lunar nutation. We have already shown their influence upon the obliquity of the ecliptic; only they were then expressed by the cosine of $2L$ and that of N, whereas they are here expressed by the sines of the same arcs. As to their numerical coefficients, the first is that of cos $2L$ divided by tang ω; the second is that of cos N multiplied by $\frac{2}{\operatorname{tang} 2\omega}$; we can easily verify this by the preceding expressions. These results depend upon the laws according to which the phenomena of nutation. take place. They can be demonstrated only by the theory of attraction which makes known their cause. We state them here as simple facts derived from observation. If, in these expressions, we put for ω its value relative to the actual obliquity, for the purpose of reducing all the coefficients to numbers, we shall have, for the retrogradation of the equin-octial point after the lapse of n days from the commencement of any given year,

$$\psi' + \frac{Mn}{365{,}25} - 1'',0001 \times \sin 2L + 18'',0032 \times \sin N.$$

When this expression becomes negative it indicates that the equinoctial point, instead of being farther west than at the commencement of the year, is, on the contrary, farther east; so that it will have been carried back in a direction contrary to that of its mean motion, by the effect of periodical perturbations which it undergoes. In reducing the coefficients to numbers, we employ for ω the value 23°,4659 or 23° 27′ 57″, which was that of the obliquity at the beginning of the year 1800. But as the coefficients are very small, this same value will answer for their reduction during several centuries, without the necessity of regarding the secular variation of the obliquity. Thus in all the calculations of modern observations, we can always employ the numerical formula which we have just determined.

XVII.—*Note to Article* 202, *Page* 121.

The precession in declination and right ascension is easily deduced from the equations which we have given in the note to page 107. By calling ω the obliquity of the ecliptic, l the longitude, λ the latitude, a the right ascension, and d the declination, we found

$$\sin d = \sin \omega \cos \lambda \sin l + \cos \omega \sin \lambda,$$

and

$$\operatorname{tang} a = \frac{-\operatorname{tang} \lambda \sin \omega + \sin l \cos \omega}{\cos l}.$$

We shall begin with the first of these equations. By the effect of the precession, the longitude l changes and becomes l', for example. Let us suppose that the latitude and the obliquity of the ecliptic do not change, and let d' represent the value of the declination corresponding to the longitude l'; we shall then have

$$\sin d' = \sin \omega \cos \lambda \sin l' + \cos \omega \sin \lambda.$$

By subtracting the first of these equations from the second, we obtain

$$\sin d' - \sin d = \sin \omega \cos \lambda (\sin l' - \sin l);$$

or,

$$\sin \tfrac{1}{2} (d' - d) \cos \tfrac{1}{2} (d' + d) = \sin \omega \cos \lambda \sin \tfrac{1}{2} (l' - l) \cos \tfrac{1}{2} (l' + l).$$

The difference of longitude $l' - l$, is equal to the precession in longitude; we call this p; the difference of declination $d' - d$, is the precession in declination; this we call δ; and we shall then have

$$\sin \tfrac{1}{2} \delta \cos (d + \tfrac{1}{2} \delta) = \sin \omega \cos \lambda \sin \tfrac{1}{2} p \cos (l + \tfrac{1}{2} p).$$

This formula is rigorous, and we can deduce δ from it, in a manner analogous to that pointed out in the note to article 149, for the calculation of parallaxes. But, on account of the smallness of the arcs p and δ, we may confine ourselves to the first power; and, by substituting their ratio for that of their sines, we have, by approximation,

$$\delta = p \frac{\sin \omega \cos \lambda \cos l}{\cos d}.$$

This formula may still be simplified; for, if we recur again to the spherical triangle *SPP'*, figure 39, which we have considered page 383, and the vertices of which are referred to the pole of the ecliptic, the pole of the equator, and the heavenly body, we have evidently in this triangle, the side *P'S*, the complement of the latitude, opposite to the angle *P'PS*, or $90 + a$; and the side *PS*, the complement of the declination, opposite to the angle *PP'S*, or $90 - l$; and as, in every spherical triangle, the sines of the angles are proportional to the sines of the opposite sides, this proportionality will give, in the case before us,

$$\cos a = \frac{\cos \lambda \cos l}{\cos d};$$

so that, by substituting this value, we shall have simply

$$\text{(1.)} \qquad \text{Precession in declination } \delta = p \sin \omega \cos a.$$

This is a very simple expression, but only an approximation to the value of δ; and as we have supposed $\delta = d' - d$, we obtain

$$d' = d + p \sin \omega \cos a.$$

By introducing, instead of the declinations of the heavenly body, the north polar distances Δ, Δ', which is always more simple, on account of the signs, we shall have $\Delta = 90° - d$, $\Delta' = 90° - d'$; whence we obtain $d' - d = \Delta - \Delta'$, and consequently,

$$\Delta' = \Delta - p \sin \omega \cos a.$$

The effect of the precession in declination changes, therefore, its sign at the same time with cos a. By following the variations of this term, it will be seen that if we suppose the plane of a great circle to pass through the poles of the equator and the ecliptic and which is called the *solsticial colure*, the precession causes those stars to approach the north pole of the equator, which are situated with respect to this colure on the same side with the vernal equinox; and, on the contrary, causes those to depart from the same pole, that are on the same side with the autumnal equinox; which is evidently conformable to what ought to take place from a motion parallel to the ecliptic, like that of the precession.

We shall now investigate the precession in right ascension; and for this purpose, we resume the second formula,

$$\text{tang } a = \frac{-\text{tang } \lambda \sin \omega + \sin l \cos \omega}{\cos l}.$$

By calling a' the right ascension, changed by the precession, we shall have

$$\text{tang } a' = \frac{-\text{tang } \lambda \sin \omega + \sin l' \cos \omega}{\cos l'};$$

and, by subtracting the first of these equations from the last, we have

$$\text{tang } a' - \text{tang } a = \frac{-\text{tang } \lambda \sin \omega \,(\cos l - \cos l') + \sin (l' - l) \cos \omega}{\cos l \cos l'};$$

or, by putting for cos l — cos l' its value,

$$\frac{\sin (a' - a)}{\cos a \cos a'} = \frac{-2 \,\text{tang } \lambda \sin \omega \sin \tfrac{1}{2} (l' + l) \sin \tfrac{1}{2} (l' - l) + \sin (l' - l) \cos \omega}{\cos l \cos l'}.$$ Trig. 8, 11.

$a' - a$ is the precession in right ascension. From the preceding expression it will be seen that its value is small compared with $l' - l$ or p. Thus, by restricting ourselves to the first powers of these quantities, we may substitute the ratio of the small arcs $a' - a$ and $l' - l$ for that of their sines. We can moreover suppose $l = l'$, and $a = a'$, in all the other terms which already have these small arcs for factors; and the expression becomes

$$a' - a = \frac{p \cos^2 a \,(-\text{tang } \lambda \sin \omega \sin l + \cos \omega)}{\cos^2 l}.$$

It is now necessary to eliminate from this expression λ and l, so as to have $a' - a$ expressed in a function of the declination and right ascension. In order to this, put in the denominator of the second member, instead of cos l its value $\frac{\cos a \cos d}{\cos \lambda}$; the equation will then become

$$a' - a = \frac{p\,(-\sin\lambda\cos\lambda\sin\omega\sin l + \cos\lambda^2\cos\omega)}{\cos^2 d}.$$

Now the fundamental equation which expresses the value of sin d, page 391, will give

$$\sin\omega\cos\lambda\sin l = \sin d - \cos\omega\sin\lambda;$$

and, by substituting this value in the expression for $a' - a$, we have

$$a' - a = \frac{p\,(-\sin\lambda\sin d + \cos\omega)}{\cos^2 d}.$$

It only remains to substitute for sin λ its value in a function of the declination and right ascension which, from the fundamental formulas, page 384, is

$$\sin\lambda = \cos\omega\sin d - \sin\omega\cos d\sin a,$$

and we shall have, after reduction,

(2.) Precession in right ascension $a' - a = p\,(\cos\omega + \sin\omega\sin a \operatorname{tang} d)$;

consequently, by introducing instead of d, the polar distance

$$\Delta = 90^\circ - d,$$

we shall have

$$a' = a + p\,(\cos\omega + \sin\omega\sin a\cot\Delta).$$

In this century the annual precession in longitude is equal to $50'',0992$. This is the value of p, which must be employed in transferring the right ascensions and declinations from one year to the next.

But if we wish to calculate the variation of the declination and right ascension for several years, it will be necessary to take, instead of $p = 50'',0992$, the value $2p$, $3p$, &c., we should then have for $d' - d$, $\Delta - \Delta'$, and $a' - a$, values double, triple, &c.; but by continuing to extend this proportionality it finally becomes inaccurate, particularly for stars in the neighbourhood of the pole of the equator, with respect to which d differs little from 90°. In fact, for these stars, a small change in the declination d produces a very considerable effect upon the value of tang d, because tang d becoming infinite when d is equal 90°, increases rapidly as it approaches that limit; so that the small variations produced in d by the precession, would become sensible with respect to $a' - a$. Thus, for the circumpolar stars, it is necessary to have recourse to the rigorous method which we have explained in article 204 of the text, or if we would make use of the preceding formulas, it is necessary to calculate the variations in right ascension only for short intervals of time.

It must be recollected, that in these formulas we have supposed the obliquity ω constant, while it in fact varies, as we have shown above. Besides, the equinoctial point is by no means fixed upon the equator, as we shall soon prove; the change of place of the ecliptic causes it to advance from west to east. The formulas which we have now obtained are not therefore altogether complete. We shall presently see what must be added to them.

XVII.—*Note to Article* 204, *Page* 123.

We take always for the origin the equinox of 1750, and call $+\psi$ the retrogradation of the equinoctial point upon the fixed ecliptic, after any number t of years, reckoned from this epoch; t must be supposed negative for previous years. We also designate by V the obliquity of the equator to the fixed ecliptic, and we shall have, by the formulas of the *Mécanique Céleste*,

$$\psi = t\,50'',4120 + 2^{\circ},79917 + 3^{\circ},83006 \sin\left(t\,50'',4120 + 85^{\circ},56597\right)$$
$$- 6^{\circ},61777 \cos t\,32'',1158 - 1^{\circ},58148 \sin t\,13'',9465,$$
$$V = 23^{\circ},47308 - 0^{\circ},33089 - 1^{\circ},63688 \cos\left(t\,50'',4120 + 85^{\circ}56597\right)$$
$$+ 0^{\circ},45744 \cos t\,13'',9465 - 2^{\circ},56172 \sin t\,32'',1158.$$

It has already been shown that, by calling ψ' the corresponding retrogradation of the equinoxes upon the moveable ecliptic, and designating by V' the apparent obliquity of the equator to the moveable ecliptic, we shall have for any year since 1750,

$$\psi' = t\,50'',4120 - 1^{\circ},28541 \sin t\,13'',9465 + 5^{\circ},59834 \sin^2 t\,16'',0579,$$
$$V' = 23^{\circ},47308 - 0^{\circ},93154 \sin t\,32'',1158 - 0^{\circ},66179 \sin^2 t\,6'',9732.$$

When we have calculated ψ, ψ', and V, we shall have $\dfrac{\psi - \psi'}{\cos V}$ for the motion of the equinoctial point in right ascension ♈′ ♈″, and by following the course indicated in the text, we shall have, from these values, all the changes necessary for reducing the apparent positions of the stars from the epoch of 1750 to any other epoch whatever, whether before or after, even for two thousand years, which is sufficient for the purposes of astronomy.

From the preceding value of ψ, by making t vary a unit, we find the annual motion of the equinoxes upon the fixed ecliptic equal to

$$50'',4120 + 3'',3696 \cos\left(t\,50'',4120 + 85^{\circ},56597\right)$$

$$+ 3'',7094 \sin t\, 32'',1158 - 0'',3849 \cos t\, 13'',9465\,;$$

for 1750 we have $t = 0$; this expression then becomes

$$50'',4120 + 0'',2603 - 0'',3849 = 50'',2874.$$

In like manner, by making t vary by unity in the value of V, we find the annual change of the obliquity of the equator to the fixed ecliptic of 1750 equal to

$$1'',4400 \sin (t\, 50'',4120 + 85^\circ,56597) - 0'',1114 \sin t\, 13'',9465$$
$$- 1'',4359 \cos t\, 31'',1438.$$

By developing the first term, this expression takes the following form;

$$0'',1114 (\sin t\, 50'',4120 - \sin t\, 13'',9465) + 1'',4359 (\cos t\, 50'',4120$$
$$- \cos t\, 32'',1158.)$$

Under this form we see that it is nothing when $t = 0$, positive when t is positive, and negative when t is negative. The annual variation of the equator to the fixed ecliptic does not then begin by being constant. It is at first nothing, and afterwards is accelerated in proportion to the time; for if we develope the preceding expression according to the powers of the times, by confining ourselves to the first, the terms affected by the cosines destroy each other, and the rest is reduced to

$$t\, 0'',1114 (\sin 50'',4120 - \sin 13'',9465),$$

a result, which, being estimated numerically, will become

$$+ t\, 0'',00001968.$$

According to the theory of uniformly accelerated motions, it follows that the total change of obliquity for the time t will be equal to the product of the annual acceleration by half of t, that is, for the time t the obliquity V will become

$$V + t^2\, 0'',00000984.$$

So in the descent of heavy bodies, the acceleration being proportional to the time, the spaces passed through are proportional to the squares of the times.

These results differ much from those which we obtained for the annual change of the obliquity of the equator to the moveable ecliptic; the expression for this change was

$$- 0'',0605 \sin t\, 13'',9465 - 0'',5211 \cos t\, 32'',1158,$$

or, by transforming the last term to make evident the constant part,

$$- 0'',5211 - 0'',0605 \sin t\, 13'',9465 + 1'',0422 \sin^2 t\, 16'',0579\,;$$

thus, besides the terms proportional to the times, and to the different powers of the times, it contains the constant term — 0″,5211, to which we shall not find any thing analogous in the variations of the obliquity of the equator to the fixed ecliptic.

The reason of this difference belongs to the causes which produce the two phenomena. The attraction of the sun and moon, if that were the only cause, would produce a constant precession equal to 50″,4120, and it would not change the obliquity of the equator to the ecliptic, which in this case would be fixed. But by the attraction of the planets, the true ecliptic is displaced in the heavens, and draws these two bodies with it. Then their action changes and produces a small variation in the obliquity of the equator to the fixed ecliptic. This variation, at first insensible, is accelerated proportionally to the time, and the absolute change of obliquity which hence results, is proportional to the square of the time. But still, the attraction of the planets which displaces the true ecliptic, inclines it likewise to the fixed ecliptic. This other annual variation is at first constant, and its effect is proportional to the time. Now, the apparent obliquity which we observe is the difference of the two inclinations of the equator and the true ecliptic to the fixed ecliptic. It is the excess of the first over the second; it is then the difference of the two preceding results, and this is the reason why the expression, which we have developed, contains the two kinds of variations which characterize them.

XVIII.—*Note to Article* 205, *Page* 124.

As the point ♈, which is here used as the origin on the moveable ecliptic, leaves no trace on this ecliptic, we may perhaps be asked how it happens that the arc ♈,♈″ is the apparent precession which we observe. In order to conceive of this, let us recollect what this apparent precession is; it is the difference of longitude of any point in the heavens, for example, of a star, on the moveable ecliptic at two different epochs. Let us suppose that in the first position of the ecliptic, a star is found precisely at the equinoctial point ♈. The longitude and declination of this star would be nothing at this epoch. But the ecliptic being displaced, the equinoctial point is displaced, and passes to ♈″. Then the coordinates of the star ♈, referred to the true equinox, are no longer nothing. If from this star we draw the arc ♈P' perpendicular to the new ecliptic, this arc will be the apparent latitude of the star ♈, and the arc ♈″P' will be the apparent longitude. Now, in the spherical triangle N♈P' which is right- Fig. 41.

angled at P', if we call $N♈ = h$ $NP' = c$, and the angle $N = n$, we shall have

$$\text{tang } c = \text{tang } h \cos n,$$

or

$$\text{tang } h - \text{tang } c = 2 \text{ tang } h \sin^2 \tfrac{1}{2} n,$$

or, lastly,

$$\sin (h - c) = 2 \cos c \sin h \sin^2 \tfrac{1}{2} n.$$

We thus see that if the angle n, formed by the two ecliptics, is very small, the sine of $h - c$, and consequently the difference $h - c$ of the two sides $N♈$ and NP', will be still smaller, because $\sin^2 \frac{1}{2} n$ is a fraction much smaller than $\sin n$. We may then, by neglecting the quantities of this order, suppose these sides equal; and as by construction $N♈ = N♈_,$, we shall have $NP' = N♈_,$, that is, the point P' will coincide with the point $♈_,$; consequently $♈♈_,$ will be the difference of the apparent longitude of the star ♈ at the two epochs under consideration. This then will be the apparent precession for the interval.

XIX.—*Note to Article* 206, *Page* 125.

We have already by construction $N♈ = N♈_,$; it will be seen that in the spherical triangle $N♈''P$, right-angled at P, the side NP differs very little from the hypothenuse $N♈''$, and may be considered as equal to it when we neglect quantities of the order $\sin^2 \frac{1}{2} n$. Now, if from the equal arcs $N♈$, $N♈_,$, we take the equal arcs NP, $N♈''$, the remainders will also be equal, that is, we shall have $♈P = ♈_,♈''$. This is the proposition stated in the text.

XX.—*Note to Article* 206, *Page* 125.

As in the reductions which we shall soon make known, it is necessary to have regard to the variation of obliquity which is produced by the displacement of the equator and the ecliptic; it will now be important to calculate the variations which such changes produce in the angular coordinates, by which the place of a heavenly body is determined.

Let us first suppose the ecliptic fixed; let us give to the equator a small motion of rotation about the line of the equinoxes, so as to increase the obliquity by a small quantity ω'; there would hence result no change in the longitude and latitude, since the plane of the ecliptic and the line of the equinoxes, from which the arcs are reckoned, are not displaced; but the declinations and right ascensions would be affected by a small quantity. In order to estimate the effect thus produced, let us resume the formulas of page 384, which give their values in a function of the longitude and latitude. We shall have, by preserving the same denominations,

$$\sin d = \frac{\sin \lambda \cos (\varphi' - \omega)}{\cos \varphi'}, \qquad \text{tang } a = \text{tang } l \frac{\sin (\varphi' - \omega)}{\sin \varphi'},$$

in which φ' is an auxiliary angle of such a value as to give

$$\text{tang } \varphi' = \frac{\sin l}{\text{tang } \lambda}.$$

Now, if the obliquity increases by the small quantity ω', l and λ remaining the same, the angle φ' will not vary, but d and a will change, and by representing their new values by d' and a', we shall have

$$\sin d' = \frac{\sin \lambda \cos (\varphi' - \omega - \omega')}{\cos \varphi'}, \quad \text{tang } a' = \text{tang } l \frac{\sin (\varphi' - \omega - \omega')}{\sin \varphi'},$$

the angle φ' being the same as before. The preceding equations being subtracted from these, we have

$$\sin d' - \sin d = \sin \lambda \times \left(\frac{\cos (\varphi' - \omega - \omega') - \cos (\varphi' - \omega)}{\cos \varphi'} \right),$$

$$\text{tang } a' - \text{tang } a = \tan l \times \left(\frac{\sin (\varphi' - \omega - \omega') - \sin (\varphi' - \omega)}{\sin \varphi'} \right);$$

or, by employing our usual transformations,

$$\sin \tfrac{1}{2} (d' - d) \cos \tfrac{1}{2} (d' + d) = \frac{\sin \lambda \sin \frac{1}{2} \omega' \sin (\varphi' - \omega - \frac{1}{2} \omega')}{\cos \varphi'},$$

$$\sin (a' - a) = \frac{-2 \text{ tang } l \cos a \cos a' \sin \frac{1}{2} \omega' \cos (\varphi' - \omega - \frac{1}{2} \omega')}{\sin \varphi'}.$$

These formulas are rigorous, and we might deduce from them $d' - d$, by series analogous to those which we used for the parallaxes; but since we suppose ω' to be very small, it will be perceived that the variations $d' - d$, $a' - a$, of the declination and right ascension, are small quantities of the same order. If we confine ourselves to the first powers of these variations, we may substitute their ratio for that of their sines; we can, moreover, suppose $d = d'$, $a = a'$, and $\omega' = 0$, in the other terms of the equation. We shall thus have, by approximation,

$$d' - d = \frac{\omega' \sin \lambda \sin (\varphi - \omega)}{\cos d \cos \varphi'}, a' - a = \frac{- \omega' \operatorname{tang} l \cos a \cos (\varphi' - \omega)}{\sin \varphi'}.$$

We can eliminate from the first sin $(\varphi' - \omega)$, and from the second cos $(\varphi' - \omega)$, by means of their values deduced from the first equations with which we set out; we shall then have

$$d' - d = \frac{\omega' \tan \omega \sin \lambda \operatorname{tang} \varphi'}{\cos d \operatorname{tang} l}, \quad a' - a = \frac{\omega' \cos^2 a \sin d \operatorname{tang} l}{\sin \lambda \operatorname{tang} \varphi'},$$

or, by substituting for $\sin \lambda \operatorname{tang} \varphi'$ its value $\sin l \cos \lambda$,

$$d' - d = \frac{\omega' \operatorname{tang} a}{\cos d} \times \cos \lambda \cos l, \; a' - a = \frac{- \omega' \cos^2 a \sin d}{\cos \lambda \cos l}.$$

Now it only remains to observe that, in the spherical triangle referred to the pole of the ecliptic, the pole of the equator, and the heavenly body, we shall have as before,

$$\frac{\cos a}{\cos l} = \frac{\cos \lambda}{\cos d},$$

which gives

$$\cos a \cos d = \cos \lambda \cos l.$$

Eliminating, therefore, $\cos \lambda \cos l$, by this value, there remains

$$d' - d = \omega' \sin a, \qquad a' - a = - \omega' \cos a \operatorname{tang} d;$$

whence we deduce

$$d' = d + \omega' \sin a, \qquad a' = a - \omega' \cos a \operatorname{tang} d.$$

It will be seen that the variation of the declination is nothing at the equinoctial point, and that it is the greatest possible when the right ascension is 90°; then we have $d' = d + \omega'$, and the whole effect of the variation in the obliquity belongs to the declination.

Let us now suppose the equator fixed, the ecliptic being made to turn about the line of the equinoxes, so as to increase the obliquity by a small quantity ω''; the right ascension and declination will continue the same, but the latitudes and longitudes will change. We might calculate these variations by the preceding method, but we can immediately obtain their values according to a remark which we have already frequently made, which is, that the latitude is entirely analogous to the declination, and the longitude to the right ascension; so that these quantities are expressed the one by the other in the same manner, merely by changing the sign of the obliquity ω. Since then we here suppose an increase of the obliquity as in the preceding case, we shall have, by analogy,

$$\lambda' = \lambda - \omega'' \sin l, \qquad l' = l + \omega'' \cos l \operatorname{tang} \lambda,$$

which gives the new values for the latitude and the longitude. We have supposed an increase of the obliquity; in the case of a diminution, we have only to make φ' and ω'' negative in the results.

XXI.—*Note to Article* 207, *Page* 126.

In order to obtain the approximate formulas, let us call l and λ the longitudes and latitudes of the heavenly body at the first of the two epochs which we are considering, and propose to calculate them for another epoch later by the number of years t. According to what has been said in the text, and agreeably to figure 41, we must first add to l the motion of the equinoctial point upon the fixed ecliptic during the interval under consideration. We shall thus have the longitudes $l + \psi$ and the latitudes λ, referred to the new equinoctial point ♈′ on the fixed ecliptic. In this new position, the obliquity of the equator to the fixed ecliptic will become $\omega + \omega'$, ω being this obliquity for the first epoch. With these data, we calculate the declinations d' and the right ascensions a', referred to the equinoctial point ♈′, and according to the general formulas of note XI, page 384, their values will be

$$\sin d' = \sin(\omega + \omega') \cos\lambda \sin(l + \psi) + \cos(\omega + \omega') \sin\lambda,$$

$$\text{tang } a' = \frac{-\text{tang }\lambda \sin(\omega + \omega') + \sin(l + \psi)\cos(\omega + \omega')}{\cos(l + \psi)}.$$

But it will be seen that according to these formulas, the problem amounts to increasing the longitudes by the quantity ψ, and the obliquity by the quantity ω', this increase being the effect of the displacement of the equator. Since these variations are supposed to be very small, we can obtain them separately, and add them together. We have then only to apply here the approximate formulas before found for the effect of the precession and the change of the obliquity; we shall thus have immediately

$$d' = d + \psi \sin\omega \cos a + \omega' \sin a,$$

$$a' - a + \psi(\cos\omega + \sin\omega \sin a \text{ tang } d) - \omega' \cos a \text{ tang } d.$$

The coordinates a' and d' have their origin in the equinoctial point ♈′ on the fixed ecliptic; in order to reduce them to the true equinox ♈″ it is sufficient to subtract from a' the motion of the equinoctial point in right ascension, which we shall call μ; consequently, if we call d'' and a'' the declination and right ascension deduced for the new position of the equator and for the true equinox ♈'', we shall then have

$$d'' - \mu = d + \psi \sin \omega \cos a + \omega' \sin a,$$
$$a'' = a - \mu + \psi (\cos \omega + \sin \omega \sin a \operatorname{tang} d) - \omega' \cos a \operatorname{tang} d.$$

We call ♈'' the true equinox, since we leave out of consideration the periodical changes produced by nutation.

It may be well here to make a suggestion calculated to render these formulas more exact. When we found, by approximation, the different terms which compose the values of d'' and a', we supposed $a = a'$, $d = d'$, in all the terms which were already multiplied by the small quantities ψ and ω'. This amounts to neglecting the powers of ψ and ω' above the first. But in reality, the terms in question would contain $\frac{a + a'}{2}$, $\frac{d + d'}{2}$, $\frac{2\omega + \omega'}{2}$; they agree, therefore, with the mean value of these quantities; we shall thus be able to secure to them this exactness by taking for a, d, and ω, the values corresponding to the mean epoch between those under consideration. But, as we do not know these mean values, we shall begin with deducing, by approximation, a' and d' from the values of a and d, by means of the preceding formulas, after which, when we know them, we put $\frac{a + a'}{2}$ in the place of a, and $\frac{d + d'}{2}$ in the place of d, in these same formulas, and by recommencing the calculation with these new elements, we shall obtain the values of a'' and d'' more exactly. The advantage of this process consists in adapting the approximation to the smallest interval of time.

Knowing the right ascension and declination for the new epoch, with the apparent obliquity of the equator to the moveable ecliptic, an obliquity which we shall in general express by $\omega + \omega''$, it is very easy hence to deduce the longitude l'' and the latitude λ'', for this same epoch; for, according to the formulas of note XII, page 384, we shall evidently have

$$\sin \lambda'' = \frac{\sin d'' \cos (\varphi + \omega + \omega'')}{\cos \varphi}, \ \operatorname{tang} l'' = \frac{\operatorname{tang} a'' \sin (\varphi + \omega + \omega'')}{\sin \varphi},$$

φ being an auxiliary angle of such a value as to give

$$\operatorname{tang} \varphi = \frac{\sin a''}{\operatorname{tang} d''}.$$

As for the variations ψ, μ, ω', ω'', which enter as data into the formulas, their values are here deduced by rigorous formulas given above.

t represents the number of years reckoned from 1750.

Retrogradation of the equinoctial point on the fixed ecliptic of 1750, after t years . . $\psi = t\,50'',2874$.

Retrogradation of the equinoctial point on the true ecliptic . . . . $\psi' = t\,50'',0992$.

Difference of these values, or direct motion of the equinoctial point in longitude, occasioned by the displacement of the ecliptic $\psi - \psi' = t\,0'',1882$.

Thence by employing the mean obliquity $V = 23^\circ,473080$, as it was in 1750, we deduce the retrogradation from the equinoctial point in right ascension, which has for its expression $\dfrac{\psi - \psi'}{\cos V}$; and designating it by μ, we find . . . . $\mu = t\,0'',2052$.

The motion of the equinoctial point in latitude, in a direction towards the south pole of the ecliptic of 1750, will be $(\psi - \psi')$ tang V. Designating it by ν, we find . . . . $\nu = t\,0'',0817$.

Change in the obliquity of the equator to the fixed ecliptic of 1750 . . . . $\omega' = + t^2 0'',00000984$.

Change of the apparent obliquity of the equator to the moveable ecliptic . . . . $\omega'' = - t\,0'',5111$.

We shall take the differences of these values for the two epochs which we wish to consider, and substitute these differences in the formulas which express the corresponding reductions of the coordinates. As all these quantities except ω', are proportional to the time, we shall immediately form their differences, by multiplying the numerical coefficient which expresses their annual variation, by the number of years comprised between the epochs under consideration. But for ω', which is proportional to the square of the time, it will be necessary to multiply its numerical coefficient by the difference of the squares of the times elapsed since 1750. Lastly, the variation ω' is so small, that it will be $0'',0984$ only in a century, and consequently it may generally be neglected, when we compare intervals which are not distant. This is done by astronomers; but as the introduction of this correction does not render the formulas more complicated, we have preserved it, in order to leave to the reasoning all its generality.

XXII.—*Note to Article* 209, *Page* 129.

The question here is precisely the same as in the note to page 125. It is to calculate the effect produced on the mean declination d, and the mean right ascension a, by a small increase ω' in the obliquity of the ecliptic, and by a small increase ψ in the longitude; thus, by calling d' and a' the values of the declination and right ascension after these changes, we shall have,

$$\sin d' = \sin\ (\omega + \omega') \cos \lambda \sin (l + \psi) + \cos (\omega + \omega') \sin \lambda,$$

$$\text{tang}\, a' = \frac{-\text{tang}\, \lambda \sin (\omega + \omega') + \sin (l + \psi) \cos (\omega + \omega')}{\cos (l + \psi)},$$

ω being the obliquity of the ecliptic, λ the latitude of the heavenly body, l its longitude. These formulas are rigorous, and express only a simple transformation of coordinates; but if we would consider the variations ω' and ψ as very small, and confine ourselves to their first power, we shall have, the following approximate values of d' and a',

$$d' = d + \psi \sin \omega \cos a + \omega' \sin a,$$

$$a' = a + \psi (\cos \omega + \sin \omega \sin a \text{ tang } d) - \omega' \cos a \text{ tang } d.$$

d and a are the coordinates referred to the mean equinoctial point; d' and a' are the coordinates referred to the equinoctial point as it varies by the effect of nutation. If the first are given, we shall immediately find the second by these formulas; but if the apparent coordinates d' and a' were given, and we wished to deduce the mean coordinates, we should obtain from the equations the values of d and a, namely,

$$d = d' - \psi \sin \omega \cos a - \omega' \sin a,$$

$$a = a' - \psi (\cos \omega + \sin \omega \sin a \text{ tang } d) + \omega' \cos a \text{ tang } d.$$

In truth, the second member still contains the quantities a and d which we are seeking; but by reason of the small difference of these coordinates with respect a' and d', we may substitute these last for the others in the second member, all of whose terms are already multiplied by the small quantities ω' and ψ; we shall have, in this way,

$$d = d' - \psi \sin \omega \cos a' - \omega' \sin a',$$

$$a = a' - \psi (\cos \omega + \sin \omega \sin a' \text{ tang } d') + \omega' \cos a' \text{ tang } d'.$$

These formulas will give the mean place of the heavenly body when we know its true place.

It only remains to substitute, in these expressions for ω' and ψ, their values such as result from observation and theory; now these values are

$$\omega' = 9'',63 \cos N, \quad \psi = -\frac{2 \times 9'',63}{\text{tang } 2\,\omega} \sin N,$$

N being the longitude of the ascending node of the moon. The opposition of the sign of ψ and ω' is easily imagined from the construction which we have given in the text; for when the node N is in the first quarter of the ecliptic from the first of Aries to the first of Cancer, the true pole is in the following quadrant of its ellipse, that is, between Cancer and Libra. In this position, the apparent obliquity is greater than the mean obliquity, and the equinox is carried back toward the sign Taurus, that is, the longitudes which are reckoned from this equinox, in the order of the signs, are diminished; this is indicated by the values of ω' and ψ; for sin N and cos N being both positive in this part of the ecliptic, these values indicate an increase of the obliquity and a diminution of the longitude. We may, in like manner, observe in the other quadrants, the use of the algebraic signs of sin N and cos N; we shall always find it to conform to the kind of oscillation which we have ascribed to the pole; it only remains then to substitute these numerical values of ω' and ψ in the preceding values of d' or d. In making these substitutions, it is necessary to take for ω the mean obliquity of the ecliptic at the epoch for which we are calculating; but by reason of the extreme smallness of ψ and ω', the secular variations of ω have here very little influence, and the results calculated with the obliquity of one year, are still sufficiently exact, for many years before and after; for this reason, we shall adopt, in our numerical calculation, the mean obliquity as it was in 1810, that is, 23°,4644 (23° 27′ 51″,84), and with these values we shall find

$$\omega' = 9'',63 \cos N, \quad \psi = -18'',00 \sin N.$$

Substituting these values in the general expressions for d', and for a', and performing numerically the multiplications by sin ω and cos ω, we shall have

$$d' = d - 7'',169 \sin N \cos a - 9'',63 \cos N \sin a,$$

$$a' = a - 16'',515 \sin N - (7'',169 \sin N \sin a$$
$$+ 9'',63 \cos N \cos a) \text{ tang } d,$$

values which for calculation by logarithms, may be more conveniently taken in this form,

$d' = d + 8'',399 \sin (a - N) + 1'',231 \sin (a + N)$,

$a' = a - 16'',515 \sin N - (8'',399 \cos (a - N) + 1'',231 \cos (a + N))$.

These variations of d and a are called the ***lunar nutation in declination*** and the ***lunar nutation in right ascension.*** It is clear that the solar nutation will give analogous values, for the variations which it produces in the obliquity and in the longitude, are of the same form as the preceding, and their values are

$$\omega' = 0'',4345 \cos 2\, L,$$

$$\psi = -\frac{0'',4345}{\operatorname{tang} \omega} \sin 2\, L,$$

L being the longitude of the sun. By always taking $\omega = 23°,4644$, which is the value of the obliquity for 1810, and making in the same way the substitutions of ω' and ψ in our general formulas, we find that they produce the following terms, analogous to those which we have calculated;

For the true declination,

$$+ 0'',417 \sin (a - 2\, L) + 0'',018 \sin (a + 2\, L);$$

For the true right ascension,

$$- 0'',918 \sin 2L - (0'',417 \cos (a - 2L) + 0'',018 \cos (a + 2L)) \operatorname{tang} d.$$

These terms which compose the ***solar nutation in declination and right ascension,*** are added to those which compose the lunar nutation, and together form what is called the ***luni-solar nutation.*** We may remark that the values ω' and ψ have not among themselves the same relation in the two nutations, as well on account of the coefficient depending on the angle $2\, L$, as on account of the denominator, which is $\operatorname{tang} 2\, \omega$ in the first, and simply $\operatorname{tang} \omega$ in the second. This difference is a result of the theory which we content ourselves with pointing out, lest, being guided by the analogy of the formulas, we should be tempted to regard it as an error.

When we make use of these formulas, it will be sufficient to put in them for a and d the values of the mean right ascension and declination the point which we are considering, and we shall know the variations which these two elements undergo in consequence of the effect of the luni-solar nutation.

For example, if we suppose $a = 0$, $d = 0$, which are the coordinates of the mean vernal equinox, the formula will give the motions of this equinox about the true equinox; we shall then find

$$d' = - 7'',169 \sin N - 0'',397 \sin 2\, L,$$

$$a' = - 16'',514 \sin N - 0'',918 \sin 2\, L.$$

d' and a' are the coordinates of the mean vernal equinox, referred to the true equinox, by having regard only to the periodical derangements caused by the luni-solar nutation. In order to have the complete values of d' and a', we must also add their secular variations, whose values we have given in a preceding note. These results are of constant use in astronomy.

It only remains for us to deduce from our general formulas the geometrical construction which we have made use of in the text, and according to which the apparent pole of the equator describes a small ellipse about the mean pole. This is extremely easy. We may even generalize the problem, and determine the apparent orbit which each point of the celestial sphere thus describes about its mean place in consequence of nutation.

This inquiry will be much simplified, if we observe that the curve in question having a very small extent, may be projected on the concave surface of the heavens as on a plane surface; and, moreover, the oscillation being made about the mean place, as a centre, it is proper to take this point for the origin of the coordinates. Then the most simple method is to draw through the mean place an arc of a great circle perpendicular to the meridian, and to take these two circles, or rather their tangents, for the axes of the rectangular coordinates; for, by reason of the smallness of the orbit, the arc and tangent are confounded. Our two coordinates will then be the difference of declination $d' - d$, reckoned on the meridian, and the difference of right ascension $a' - a$, referred to the declination of the mean place, on the great circle perpendicular to the meridian, that is, multiplied by the cosine of the declination of the mean place; in this manner, by making

$$X = d' - d, \quad Y = (a' - a)\cos d,$$

the coordinates X and Y will depart from the same point, will be rectangular to one another, and, considering the smallness of the orbit, may be regarded as rectilinear. Now, if we substitute these values in our general formulas, we shall find, by multiplying $a' - a$ by $\cos d$,

$$X = -7'',17 \sin N \cos a + 9'',63 \cos N \sin a,$$

$$Y = -16'',51 \sin N \cos d - (7'',17 \sin N \sin a + 9'',63 \cos N \cos a) \sin d.$$

If, from these two equations, we eliminate the angle N, we shall have the equation of the orbit described by the apparent place about the mean place.

If we wish this apparent place to be the north pole of the equator, we have only to suppose $d = +90^\circ$, which gives

$$\cos d = 0, \text{ and } \sin d = +1;$$

then, the values of X and Y will become

$$X = -7'',17 \sin N \cos a + 9'',63 \cos N \sin a,$$

$$Y = -7'',17 \sin N \sin a - 9'',63 \cos N \cos a;$$

the right ascension a remains still undetermined, because all the meridians pass through the pole of the equator. The value of a will depend on the choice which we make of a particular meridian for the axis of the Ys. The preceding equations give

$$X \cos a + Y \sin a = -7'',17 \sin N,$$

$$X \sin a - Y \cos a = +9'',63 \cos N;$$

consequently,

$$\left(\frac{X \cos a + Y \sin a}{7'',17}\right)^2 + \left(\frac{X \sin a - Y \cos a}{9'',63}\right)^2 = 1.$$

This equation is that of an ellipse referred to its centre, but not to its axes. In order to reduce it to this system of coordinates, we must make $a = 90°$, that is, take for the central meridian that which passes through the poles of the equator and the ecliptic; then, in fact, we shall have $\sin a = 1$, and the preceding equation will become

$$Y^2 (9'',63)^2 + X^2 (7'',17)^2 = (7'',17)^2 (9'',63)^2,$$

which is that of an ellipse whose greater axis is directed according to the coordinates X tangentially to the central meridian, and the smaller axis according to the coordinates Y, perpendicularly to the first; the first has for its length 19'',26, the second 14'',34. These lengths are both expressed in parts of a great circle of the celestial sphere. If we wish to reckon the second on the parallel to the ecliptic which passes through the mean pole of the equator, and to which the smaller axis of our ellipse will be a tangent, we must divide 14'',34 by the cosine of the distance of this parallel from the pole of the ecliptic, that is, by the sine of the obliquity ω. The smaller axis, thus expressed, will have for its value 36'',01 of the parallel to the ecliptic upon which it is found.

It will be seen, then, by the calculation which we have made, that the ellipse of nutation has it greater axis situated in the plane of a great circle passing through the mean poles of the equator and ecliptic; for we found that, in order to refer the Ys to this axis, we must make $a = 90°$. Here we might dispose of a at pleasure, since, all the meridians passing through the pole of the equator, we might choose at pleasure that which we wish to take for the origin of the Ys.

Let us now determine the position of the true pole on this ellipse for any instant. It will be given by the simultaneous values of X and Y, which, by making $\cos a = 0$, $\sin a = 1$, become

$$X = +\,9'',63 \cos N, \quad Y = -\,7'',17 \sin N.$$

When the moon's node coincides with the vernal equinox, its longitude is nothing; we have, therefore, $\sin N = 0$, $\cos N = 1$; consequently,

$$X = 9'',63, \quad Y = 0.$$

The apparent pole is then found in the central meridian, on the axis of the Xs, and at the summit of the nearest ellipse to the equator, that is, at the point π. Fig. 43, 44.

As the longitude N increases, Y increases remaining negative; at the same time X diminishes and remains positive; the apparent pole is then found to be comprised in the first quadrant of its ellipse, on the side of the autumnal equinox, for example, at π''.

In general, it may be easily seen that X and Y perform the periods of their values in the same time that the angle N performs an entire circumference, that is, the pole will turn on its ellipse in the same time that the moon's node turns on the ecliptic.

About the mean pole as a centre, and with a radius equal to half the major axis of the ellipse, let us describe the circumference of the circle $\pi K \pi''$. Upon this circumference, let us conceive a point K Fig. 44. moving uniformly and completing a revolution in the same time in which the node completes its revolution on the ecliptic, and which also moves like it, from east to west, that is, in the direction $\pi \pi' \pi''$. If from the centre of the circle, which is also that of the ellipse, we draw a radius PK to the moveable point at any instant, the angle $KP\pi$, formed by this radius with the axis of the positive Xs, $P\pi$, will always be equal to the angle N, that is, to the longitude of the moon's node on the ecliptic; so that if we call X' the abscissa PR of the moveable point K at any instant, we shall have

$$X' = 9'',63 \cos N, \quad Y' = -\,9'',63 \sin N;$$

the abscissa of this point will then be the same as that of the true pole. Consequently, this pole will be found on the ellipse at the point π''' where this curve is cut by the ordinate KP'. This is the construction enunciated in the text.

XXIII.—*Note to Article* 371, *Page* 225.

The heights of the lunar mountains have been differently estimated by different observers. Sir W. Herschel supposed, that with the exception of a few, they did not exceed half a mile. Schroeter seems to have made the most careful and exact observations both as to their perpendicular elevation and the length of their bases; from which it would appear, that there are three whose height a little exceeds five miles, and ten that are between four and five miles. The bases of these vary from twenty to sixty miles. A long catalogue is given of others of inferior dimensions. Schroeter measured also the depths and lengths of the lunar cavities. The depression in one instance exceeded three miles, and in four others amounted to more than two, the breadth or horizontal extent, being from ten to forty miles.

The excavations here referred to are the dark spots. They have the appearance of bodies of water, and they have received names accordingly. But it is found, that when the boundary between the illuminated and dark parts of the moon, passes over one of these spots, it is rough and serrated, and not a regular, even curve, as it ought to be on the supposition of an extended, smooth, convex surface. It is hence inferred that there is nothing in the moon analogous to our seas and lakes.

There has been much dispute respecting a lunar atmosphere. The question seems at length to be decided, by the circumstance of a real twilight, which has been detected, and which can be explained only on the supposition of such a medium.

By observing the moon when her phases were extremely falcated, Schroeter discovered a faint glimmering light of a pyramidal form, extending from both cusps into the dark hemisphere. The greatest breadth of this crepuscular light was 2″ and its length 1′ 20″; from which Schroeter computed, that the breadth of the lunar twilight, from the boundary of light and darkness to where it loses itself in the light reflected from our earth, measures in a direction perpendicular to the boundary 2° 34′ 25″; and, therefore, that the inferior or more dense part of the moon's atmosphere is not more than 1500 English feet high, and that the height of the atmosphere, where it could affect the brightness of a fixed star, or inflect the solar rays, does not exceed 5742 English feet. This space subtends, at our earth, an angle of only 0″,94, which will be passed over by the star in less than 2″ of time. The occultation of Jupiter on the 7th of

April 1792, was observed by Schroeter for the purpose of confirming the preceding discovery. Some of the satellites became indistinct at the limb of the moon, while others did not suffer any change of colour. The belts and spots of Jupiter appeared perfectly distinct when close to the limb of the moon, and a small luminous spot, though by no means very perceptible, could be plainly distinguished when close to the moon's limb.

By means of a similar twilight, observed in Venus, Schroeter concluded that this planet has an atmosphere. Its height is computed to be 16020 feet, or a little more than three miles.

XXIV.—*Note to Article* 377, *Page* 228.

Let us draw TF parallel to CB, and let $TCB = STF = C$. We Fig. 87. shall have $\sin C = \frac{SA - TB}{ST} = \sin \frac{D}{2} - \sin p$, D being the apparent diameter of the sun, and p its horizontal parallax; whence it will be seen that by substituting the arcs for the sines, which may be done in the case of such small angles, we shall have $C = \frac{D}{2} - p$.

XXV.—*Note to Article* 377, *Page* 228.

Let D be the apparent diameter of the sun, p its horizontal parallax, R the radius of the earth. From what has been demonstrated in the text, we have

$$CT = \frac{R}{\sin\left(\frac{D}{2} - p\right)}.$$

With respect to the perigee, mean, and apogee distances, the values of D and p are as follows;

	D.	p.	$\frac{D}{2} - p$.
Perigee distance . .	32′ 35″,56	8″,92	16′ 08″,86
Mean distance . .	32′ 02″,71	8″,78	15′ 52″,57
Apogee distance . .	31′ 30″,96	8″,62	15′ 36″,86.

By calculating CT with these values we shall find the results given in the text.

XXVI.—*Note to Article 379, Page 229.*

With respect to the moon, let us designate by R', D', and p', the quantities which with reference to the earth were designated by R, D, and p, that is, the radius at its surface, the apparent diameter of the sun, and its horizontal parallax. From the formula, found in the preceding note, the length of the perfect shadow, projected behind the moon, would have for its value, $\dfrac{R'}{\sin\left(\frac{D'}{2}-p'\right)}$; but this result, thus presented, is expressed in parts of the radius of the moon R'. In order to have it in parts of the earth's radius, we must put it under the form

$$\frac{R\left(\frac{R'}{R}\right)}{\sin\left(\frac{D'}{2}-p'\right)}.$$

Now, if we call $\varDelta$ and δ the distances of the sun and moon from the earth, we shall have

$$D' = \frac{D\times\varDelta}{\varDelta-\delta},\ p' = p\frac{R'}{R}\times\frac{\varDelta}{\varDelta-\delta},$$

consequently,

$$\frac{D'}{2}-p' = \left(\frac{D}{2}-p\right)\left(\frac{R'}{R}\right)\frac{\varDelta}{\varDelta-\delta};$$

now, since p is the sun's horizontal parallax with respect to the earth, we have $\sin p = \frac{R}{\varDelta}$; consequently, $\varDelta = \frac{R}{\sin p}$; in like manner, by calling P the horizontal parallax of the moon, we shall have $\delta = \frac{R}{\sin P}$; whence we deduce $\frac{\varDelta}{\varDelta-\delta} = \frac{\sin P}{\sin P - \sin p}$, or simply $\frac{\varDelta}{\varDelta-\delta} = \frac{P}{P-p}$, since, without a sensible error, we may substitute the ratios of these small arcs instead of those of their sines.

With these reductions the length of the conical shadow, projected behind the moon, will have for its expression,

$$\frac{R\left(\frac{R'}{R}\right)}{\sin\left(\frac{D}{2}-\frac{pR'}{R}\right)\frac{P}{P-p}} \quad \ldots\ldots (1.)$$

We have seen that $\frac{R'}{R} = 0{,}27293$; and we have in the extreme cases considered in the text, 324

	D.	*p.*	*P.*
Sun in the apogee, maxim. parallax	31′ 30″,96	8″,62	1° 01′ 29″,32
Sun in the perigee, minim. parallax	32′ 35″,56	8″,92	53′ 50″,93

By substituting these data in formula (1) we shall have the values given in the text. As to the corresponding distances of the moon, expressed in terrestrial radii, we deduce them from the formula $\delta = \frac{R}{\sin P}$.

XXVII.—*Note to Article* 380, *Page* 229.

By calling p the parallax of the sun, P that of the moon, and D the apparent diameter of the sun, we shall have

$$\text{Semidiameter of the perfect shadow} = P + p - \frac{D}{2}.$$

XXVIII.—*Note to Article* 381, *Page* 231.

The same denominations being preserved which we have made use of in the preceding note, the sun's parallax at the moon will be $p\frac{R'}{R} \times \frac{P}{P-p}$. The apparent diameter of the sun, as seen from the same position, will be $D \times \frac{P}{P-p}$. So that, by calling D'' the apparent diameter of the moon, seen from the earth, we shall have,

Semidiameter of the lunar shadow, as seen from the moon,

$$= \frac{D''}{2} + \frac{p\,R'}{R}, \frac{P}{P-p} - \frac{DP}{2(P-p)}.$$

This expression will be rendered more simple if we recollect that, by dividing the apparent semidiameter of a heavenly body by its horizontal parallax, we shall have the ratio of its radius to the radius of the earth, a proportion which has been demonstrated. By applying 143. this result to the moon, we shall have $\frac{R'}{R} = \frac{D''}{2P}$. If we substitute this value of $\frac{R'}{R}$ in the preceding formula, we shall find, after reduction, Semidiameter of the lunar shadow, as seen from the moon,

$$= \frac{D'' - D}{2} \times \frac{P}{P - p}.$$

Following the data which we have made use of, we have

$$\frac{D''}{2} = 16'\ 45'',69,\ \frac{D}{2} = 15'\ 45'',48,\ \ P = 1^\circ\ 01'\ 29'',\ \ p = 8'',62,$$

which gives,

$$\frac{D'' - D}{2} = 1'\ 00'',21,\ \text{and}\ \frac{P}{P-p} = 1,002336\,;$$

consequently,

$$\left(\frac{D'' - D}{2}\right) \frac{P}{P-p} = 60''.$$

This is the value which we have given in the text. The apparent semidiameter $\frac{D''}{2}$ is found by multiplying the parallax P by the constant ratio 0,2762, which is nearly equal to $_{11}$. It will be perceived that the semidiameter of the shadow becomes negative when D exceeds D''; whence it follows that a total eclipse is impossible, except when the apparent diameter of the moon exceeds that of the sun.

XXIX.—*Note to Article* 382, *Page* 232.

Calling P the parallax of the moon, p that of the sun, D the apparent diameter of the sun at the moment of the eclipse, we shall have

Distance of the centre of perfect shadow from the extreme part of the penumbra $= P + p + \frac{D}{2}$;

we have had already

Semidiameter of the perfect shadow . . . . $= P + p - \frac{D}{2}$.

Difference of these values, or magnitude of the penumbra . $= D$.

XXX.—*Note to Article* 383, *Page* 233.

Making use of the same denominations as in the preceding note, the diameter of the lunar penumbra, as seen from the moon, will be

$$\frac{D''}{2} + \frac{pR'}{R} \times \frac{P}{P-p} + \frac{D \times P}{2\,(P-p)},\ \text{and as}\ \frac{R'}{R} = \frac{D''}{2P},$$

the two first terms are reduced to $\frac{D'' P}{2(P-p)}$, so that we shall have the semidiameter of the lunar penumbra, as seen from the moon,

$$= \frac{(D''+D)}{2} \times \frac{P}{P-p};$$

we have found above

Semidiameter of the lunar shadow, as seen from the moon,

$$= \frac{(D''-D)}{2} \times \frac{P}{(P-p)};$$

subtracting the first of these expressions from the second, we shall have the magnitude of the lunar penumbra $= \frac{D \times P}{P-p}$.

In the case where the sun is in apogee and the moon in perigee, at its least distance, we obtain, as we have seen above,

$$\frac{D''}{2} = 16' \, 45'',69, \quad \frac{D}{2} = 15' \, 45'',48, \quad p = 8'',62, \quad P = 1^\circ \, 01' \, 29'',$$

whence we deduce

$$\frac{(D''+D)\,P}{2\,(P-p)} = 32' \, 35'',56.$$

The apparent semidiameter of the earth, as seen from the moon, under the same circumstances, is 1° 1′ 29″, since it is represented by the horizontal parallax of the moon, so that under the most favourable circumstances the lunar penumbra covers a little more than half the disc of the earth.

XXXI.—*Note to Article* 386, *Page* 285.

If we designate by r the horizontal refraction, preserving also the other denominations which we have made use of, we shall have,

Distance of the summit of the shadow from the centre of the earth

$$= \frac{R}{\sin\left(\frac{D}{2} + 2r - p\right)}.$$

This is the formula of note 25, in which we have substituted $\frac{D}{2} + 2r$ instead of $\frac{D}{2}$. Furthermore, we shall have

Extreme radius of the penumbra . . . . $P+p+\frac{D}{2}+2r$;

Semidiameter of the perfect shadow at the distance of the moon . . . . $P+p-\frac{D}{2}-2r$.

If we would understand by $\frac{D}{2}$ the distance of any point whatever of the sun's disc from its centre, we shall find, by the first formula, the distance at which this point begins to be visible. So that by making $\frac{D}{2}=0$, we shall have the distance at which the centre of the sun begins to appear by refraction, at the surface of the earth; we then make $\frac{D}{2}$ negative for the opposite points of the disc. On the opposite edge we make

$$\frac{D}{2} = -\tfrac{1}{2} \text{ apparent diameter of the sun};$$

and then, by always leaving $\frac{D}{2}$ negative, we shall obtain it greater than this semidiameter. We shall thus have the distance at which the points on the other side of the disc become visible by rays touching the surface of the earth. It is by this method that we have calculated the table given in the text. It is taken from a work of *Dionis du Séjour* upon the Celestial Motions, page 661.

If we wish to know what part of the disc of the sun is thus visible, by refraction, to an observer situated at the moon, at a given distance from the earth, it is only necessary to calculate the value of the parallax P for this distance, and then to make the semidiameter of the shadow equal to zero, that is, to make

$$P+p-\frac{D}{2}-2r=0,$$

which gives

$$\frac{D}{2}=P+p-2r.$$

This value of $\frac{D}{2}$ being subtracted from the apparent semidiameter of the sun, the remainder will be the portion visible.

XXXII.—*Note to Article* 423, *Page* 269.

The notation employed in the text being preserved, the triangle *TPS* will give the equation Fig. 92

$$R \sin (P - S) = r \sin (N - P).$$

For the second passage through the same node, we shall, in like manner, have

$$R' \sin (P' - S') = r (N - P').$$

I use R' and not R, because the radius vector of the sun may not be of the same length in the two observations; but it is easy to calculate it by the tables; whereas r, as it answers to the same node, and consequently to the same point of the orbit of the planet, may be supposed to be constant.

These two equations are sufficient for determining N and r.

Dividing them, member by member, we shall have

$$\frac{R' \sin (P' - S')}{R \sin (P - S)} = \frac{\sin (N - P')}{\sin (N - P)},$$

or

$$R' \sin (P' - S') \sin (N - P) = R \sin (P - S) \sin (N - P').$$

Developing the terms which contain N, we deduce

$$\text{tang } N = \frac{R' \sin P \sin (P' - S') - R \sin P' \sin (P - S)}{R' \cos P \sin (P' - S') - R \cos P' \sin (P - S)}.$$

N being known, r will be so by one of the two fundamental equations.

If one of the two passages, the first, for example, happens exactly at the opposition or conjunction of the planet, $P - S$ will be nothing or equal to two right angles; in either case, we shall have tang $N =$ tang P, consequently $N = P$, that is, the longitude of the node will be equal to the geocentric longitude of the planet, as must evidently be the case under the circumstances supposed. But then we could no longer make use of the first equation for determining r. In general, it will be seen that the value of r will have little influence on that of N, when $N - P$ is very small, that is, when the observation is made near the node. Such observations will then be proper for determining the node, and not the distance of the planet from the sun. The most favourable for the last determination are those in which $N - P$ differs but little from a right angle.

We shall also give the formula for the case in which it is proposed to combine two passages through the opposite nodes; which, as we have remarked, can be admitted only by supposing a circular orbit, or at least by supposing it such that the distances of the planet from the sun are equal at its two nodes. Then calling these nodes N and N', we shall have

for the first, $R \sin (P - S) = r \sin (N - P)$,
for the second, $R' \sin (P' - S') = r \sin (N' - P')$.

But since these nodes are opposite, $N' = N + 180^\circ$. Eliminating N' by means of this relation, our two equations become

$$R \sin (P - S) = r \sin (N - P) \quad (1),$$
$$R' \sin (P' - S') = - r \sin (N - P');$$

and, proceeding as we have done just now, we deduce

$$\tan N = \frac{R' \sin P \sin (P' - S') + R \sin P' \sin (P - S)}{R' \cos P \sin (P' - S') + R \cos P' \sin (P - S)}.$$

As the planetary orbits differ but little from circles, this formula may be employed without occasioning any considerable error. Let us apply it to the following observations made by Lacaille on the passage of Mars through its two nodes, and given in his valuable work entitled *Astronomiæ Fundamenta.*

Years.	Solar Time at Paris reckoned from Noon.	Geocentric Longitude of ♂.	Geocentric Latitude of ♂.	Nodes.
1747	14 May 10^h 50′ 43″	216° 15′ 20″	0° 0′ 25″,5 N.	Descend.
1753	3 Nov. 9 31′ 46″	59° 29′ 38″,5	0° 0′ 27″,5 S.	Ascend.

It will be seen that the latitude of Mars was not exactly zero. If we had the series of Lacaille's observations, we should easily find by interpolation, the instant when this phenomenon must have happened. But in the want of such aids, we will remark that these latitudes, which are extremely small, must correspond to points not far from the node, and consequently have but little influence on the result. Now if we calculate, by the astronomical tables, the longitudes of the sun and the logarithms of its radius vector, which correspond to these two observations, we find

for 1747, 53° 38′ 20″ $\log R = 10{,}0050864$,
for 1753, 221° 39′ 16″ $\log R' = 9{,}9961542$.

I subtract from this last 6′ 16″, in order to carry back the equinox to the month of May, 1747, at the rate of 50″,1 a year. I also subtract the same quantity from the second observed longitude of the planet, and in this way I have the following elements;

$$P = 216^\circ\ 15'\ 20'',\quad P' = 59^\circ\ 23'\ 23'',$$
$$S = 53^\circ\ 38'\ 20'',\quad S' = 221^\circ\ 33'\ 1'',$$

which gives

$$P - S = 252^\circ\ 37'\ 0'',\ P' - S' = 197^\circ\ 50'\ 22''.$$

Substituting these numbers in the formula, we find

$$N = 180^\circ + 47^\circ\ 47'\ 45'',\ \text{or}\ N = 47^\circ\ 47'\ 45'',$$

for the same tangent may correspond to the two arcs N and $180^\circ + N$. The first value is that which must be adopted in the present case, because it is the only one which, substituted in the equations (1), gives r positive, as we have supposed it. The longitude of the node in which the planet was at the first observation, was then equal to this value of N, or $227^\circ\ 47'\ 45''$.

The direction of the motion of the planet showed that this was its descending node, for it was then departing from the north pole of the ecliptic; consequently, the other root, opposite to the preceding, is the longitude of the ascending node, which is found to be $47^\circ\ 47'\ 45''$, at the epoch of these observations. Taking account, by calculation, of the small latitude of the planet at its two passages, Lacaille arrived at a result $10'\ 34''$ less; from which he determined its distance from the node. But we might avoid this correction, which already supposes an approximate knowledge of the motion of the planet, if we had the series of observations made by Lacaille; for we could, by means of them, deduce the exact geocentric longitude of the planet at its passage through the node. Finally, the small difference which exists between our two results is of no importance to our present purpose, and we may always conclude from this example, that the position of the nodes of the orbits are capable of being very exactly determined by geocentric observations without the aid of any anticipated supposition.

If from the value of $N = 227^\circ\ 47'\ 45''$, we take that of P, we shall have $N - P = 11^\circ\ 32'\ 25''$; and by substituting this value in the first of the equations (1), it will make known r, that is, the distance of Mars from the sun at the epoch of its passage through the node. This distance will be expressed in the same kind of units as R and R', that is, in parts of half the transverse axis of the solar orbit. Making the calculation, we find

$$r = 1{,}510986.$$

The error of this result is less than 0,02. We may, however, consider it as an approximation, since we have supposed the two values of *r* equal at the two nodes of Mars, which is not strictly true. We should not be liable to this inconvenience if we could have employed two passages of Mars through the same node. But the above example will be sufficient at least to show the use of the method.

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